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DAILY SYSTEM CHECK OF ELECTRONIC POSITION SYSTEMS

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ABSTRACT

The use of electronic navigation systems onboard NOS survey vessels necessitates one or more daily system checks to ensure the operational precision of the systems. In most instances, a position fix by visual methods is used for the daily check. The paper will first establish precision estimates for four fundamental measurements used in visual fixes and then present a discussion of six basic visual methods for daily system checks. The visual methods will be examined for their individual merits, drawbacks and estimated precision.

INTRODUCTION

Electronic navigation systems are used almost exclusively by the National Ocean Service (NOS) to control the position of hydrographic soundings. In order to ensure the precision of these systems, a daily independent positional check is performed. In most cases, the check is a comparison of simultaneously obtained visual fix information and electronic fix information, with the visual fix having a higher precision than the electronic position. A mathematical solution for the position by visual methods can be obtained by resection, intersection or directly by geodetic azimuth and distance. In each of these solutions, a combination of four fundamental measurements is used. The measurements are : horizontal angles, measured distances, directly observed geodetic azimuths, and visual ranges (zero horizontal angle). The choice of measured quantities will depend on available

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established horizontal geodetic control (hereafter referred to as control stations). The proper choice will ensure the precision of the fix as well as the efficiency of obtaining the fix information. The resulting visual fix precision will be accurately estimable once an estimate for each of the fundamental components is determined. The daily system check will then meet the necessary precision required for electronic navigation systems. In addition to discussing the precision of the fundamental components, methods of position fixing by visual methods will be endorsed according to their economy of established control, dependency on environmental conditions, and time necessary to obtain a complete daily system check.

FUNDAMENTAL MEASUREMENTS

The statistic used for determining the precision of any of the four fundamental measurements is the root mean square (RMS) standard deviation. It is assumed that the individual measurements have had all systematic and otherwise correctable errors removed, leaving only small uncorrectable random errors. In this way, it can be assumed that the random errors will be normally distributed and that 99 percent of the observations will lie within three RMS standard deviations of the mean.

Horizontal angles and geodetic azimuths

All physically observed measurements have more than one source of random error, each of which can be estimated and combined by squaring and summing with the others. The final estimate for the standard deviation can then be obtained by evaluating the square root of the sum of the squares (the resulting standard deviation will also be normally distributed). Three optical instruments were evaluated in this manner : a one-second theodolite, a six-second optical transit, and a sextant. In each case, the standard deviation for target alignment, minimum reading resolution and pointing error was considered. In the case of the theodolite and transit, errors induced by miscentering the instrument over a control station were also considered. The statistics quickly revealed that miscentering by only 3 millimeters (mm) induced random errors as great as 20 arcseconds for targets less than 100 meters away. Therefore, when establishing standard deviations for theodolites and transits, it was assumed that no targets would be closer than approximately 200 meters from the instrument station. The following table and discussion summarize the RMS standard deviations for horizontal angle measurements.

Instrument	Standard deviations (seconds)				
	Center	Target	Point	Read	RMS
Theodolite	6	3	12	2	13.9
Sextant	—	6	30	120	124.0
Transit	6	3	12	12	18.2

The above values represent the most reasonable values considering that several factors may influence a single component of the overall standard deviation.

In each case where a compromise was necessary, the worst case value was given more weight than a mean value.

Centering error is the result of not aligning the observing instrument's axis of rotation directly over the center of the control station. The most commonly accepted tolerance for centering in control surveying is 2 millimeters. To be a bit more conservative, 3 millimeters was chosen because the optical plumbing capability and precision of levelling an instrument on a tripod is not as fine as when the instrument is situated on the more stable platforms used for control surveying. The standard deviation for centering is then obtained from the formula :

$$SD_{center} = 3\text{ mm} \times D3 / (D1 \times D2)$$

where $D3 = \sqrt{D1^2 + D2^2 - 2\,D1\,D2\cos\theta}$

The formula for D 3 is the cosine law where sides D 1 and D 2 and its measured angle θ are known. Therefore, centering error is dependent on the angle observed and the distance to the targets. As mentioned earlier, the distance to the nearest target has the greatest effect on the standard deviation.

Target misalignment error is similar to centering error in that the target is not centered over the control station. The resulting standard deviation for target alignment is computed from the formula :

$$SD_{target} = SD_t \times \sqrt{D1^2 + D2^2} / (D1 \times D2)$$

SD_t is chosen to be 5 millimeters because most targets are even less sensitive to levelling and plumbing than transits or theodolites. In this case, once again, targets at less than 200 meters distance resulted in large errors. For targets greater than 200 meters, however, little difference was found.

Pointing error for a given instrument is dependent on its optical quality and stability. In addition, if one of the targets is moving, as in the case when observing a moving survey vessel, additional pointing error will be introduced.

For a given pointing its standard deviation can be computed from the formula :

$$SD_{point} = SD_p/\sqrt{N}$$

N is the number of pointings taken for determining a mean. In our case, N is assumed to be one. SD_p is a variable depending on the situation of the targets. For a non-moving target, SD_p is normally accepted as 2 arcseconds for theodolites and transits. In the case of a moving object, experience has shown that it is possible to track a 10-centimeter target (such as the post used to attach the short-range navigation equipment to the survey vessel). Considering the half diameter of the target as the estimated pointing error, the error can be expressed in arcseconds versus centimeters for a given distance from the observing station. A good estimate for distances between 500 and 2 000 meters is 11.8 seconds. Since the angle is composed of a pointing between one fixed and one moving target, the combined standard deviation becomes $\sqrt{11.8^2 + 2^2}$ or 12 arcseconds for a theodolite and transit. For a sextant, both targets appear to move because the angle is normally measured from a moving object. In this case, due to a lower precision in the observing optics of a sextant combined with the survey vessel motion, a different set of criteria was necessary for evaluating the standard deviation of a pointing. Based on experience, it was felt possible to center objects in the optics of a sextant to within 20 centimeters of each target center at nominal ranges. A 20-centimeter estimate was considered reasonable due to the types of targets normally constructed or used for sextant observations and the ability to center such a target over a survey mark. The standard deviation for a pointing is determined considering that the targets will usually be about 2 000 meters away or that the pointing can be made increasingly more precise as the distance decreases. Therefore, 20 centimeters at 2 000 meters equals approximately 21 seconds of arc. The combined standard deviation for two targets is equal to $\sqrt{21^2 + 21^2}$ or 30 arcseconds.

Reading error for a given instrument is based on the least resolution that can be directly read on the observing instrument. In this case, the least reading was doubled in order to make the estimate more conservative considering that at least one target is moving.

As can be seen from the above table, a very large contribution for the combined standard deviations results from a moving target or observing platform. The Table omits a centering error for sextants.

It is assumed that this correction will be approximated when reducing the sextant fix to the location of the electronic navigation system. When combining sextant angles, it will be seen that very large errors in position can result from the eccentricity between sextant angles.

Measured distances

The standard deviation for a measured distance using an infrared laser instrument can be determined in a manner similar to angle measuring devices. A combination of five standard deviations including estimates for centering, fixed, scaler, weather and reading precisions is used to determine the RMS standard deviation. A centering precision of 3 millimeters is used as before for theodolites. The fixed standard deviation is given by the equipment manufacturer and in this case is assumed to be 5 millimeters. The scaler estimate results from assuming a given velocity of light in a particular atmosphere at a constant frequency. The error is expressed in parts per million (PPM) of the distance measured and is a function of the variation in frequency (in this case assume the value to be 2 PPM). The weather estimate is similar to the scaler estimate in that it is expressed in PPM, but results from the non-application of appropriate meteorological corrections to the measured distance. Since system checks are performed in real time, it is not possible to correct the measured distances prior to their use. This error can have a maximum effect of 20 PPM. The reading error estimate is a combination of the effects of a moving target and rounding off the displayed reading to the nearest meter. The combined standard deviation for a 6 000 meter-distance is as follows :

$$((3\ \text{mm})^2 + (5\ \text{mm})^2 + (6\,000 \times 2\ \text{PPM})^2 + (500)^2)^{1/2}$$

or the RMS standard deviation for this measured distance is 514 mm. Therefore, even under a worst case situation, the combined standard deviation for a measured distance will be less than 1 meter (at present maximum ranging capabilities are limited to 6 000 m).

Visual ranges

The standard deviation for vessel alignment on a visual range will involve a different derivation. First, a visual range is formed when two targets centered over established control points appear in line as viewed from the survey vessel. The uniqueness of a range results from the fact that a precise geodetic azimuth can be determined without the need for any angle measurement. Ranges are normally "sensitive" only within a limited distance from the front range and the sensitivity is directly related to the distance between the front and rear range. Two criteria were used to determine range sensitivity. The first criteria was to have no more than 2.5 meters of error caused by being either right or left of the range. The second criteria was to limit the maximum distance from the front range by simultaneously allowing 2.5 m of displacement and requiring the horizontal angle between the ranges to be no less than 30 arcseconds. By meeting both criteria (see Fig. 1) one can be assured of not being so far from the range as to make the resolution of the targets beyond the threshold of the human eye (with or without the use of magnification). The 30-arcsecond limit is directly related to the pointing standard deviation for a sextant. In other words, if 30 arcseconds can be resolved by a sextant, it is possible to resolve the minimum angular separation of the range markers. The following formula was derived by fitting a second-order curve by least squares to a data set satisfying the two criteria. The formula is useful for range marker separation of 250 meters to 6 000 meters :

$$R = -171.3 \text{ meters} + 0.1203\,(X + R) + 0.000028\,(X + R)^2$$

where R = range marker separation and X = distance to the front range.

In order to solve this equation, one must either know the length of the range or the maximum distance between the front range and area to be used for the daily system check. In the latter case, an appropriate range may be chosen or constructed.

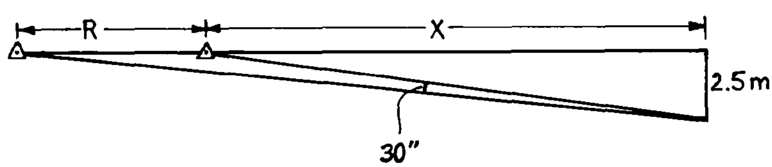

FIG. 1

METHODS OF CALIBRATION

Once estimates are determined for the fundamental measurements, one can evaluate the methods of daily system checks by visual fixes. The most common methods are static, sextant resection (three-point fix), theodolite intersection, range and cut-off angle and range-azimuth using a Total Station device or a range and Electronic Distance Measuring Instrument (EDMI). There are several variations of the above methods which will be commented on within the discussion of the above basic methods. Each method has its own merits and drawbacks, therefore, the order of discussion will be based on the efficiency of time, accuracy, and necessary personnel.

Static method

The most efficient method is the static calibration. This method requires only one established geodetic station usually located on a pier face, piling or fixed aid to navigation such as a day beacon. The survey vessel is maneuvered alongside the mark and the received electronic navigation rates are compared against pre-computed rates for the control station. Care must be taken to ensure that all eccentricities are removed. There are several drawbacks to this method such as : the static points are rarely available along open coasts or in uninhabited areas, large survey ships cannot usually be maneuvered alongside most static points, but, most important, static points are seldom available near the center of the survey area. If any of the above structures are not available, but an exposed rock is appropriately situated and positionable by geodetic methods, then it is often possible to make the rock into a static point by mechanizing an eccentric point to one of its sides by the following method. A length of lumber is attached to the rock so that the orientation and distance of the overhanging end can be determined from the control station. The extended end of the lumber then becomes the static point. Unfortunately, exposed rocks in unprotected waters are often subjected to unpredictable weather conditions or extreme tidal fluctuations making them not usable when they are most needed. The expected error for this method will be a function of how well the survey vessel can be maneuvered alongside the point. In most cases, the error should be less than 2.5 meters if all eccentricities are accounted for.

Range and cut-off angle method

Another efficient method utilizes a range and sextant cut-off angle. In this method, at least two established control stations must exist and be intervisible. A range, if not available, can be constructed by establishing a range marker (similar to an azimuth mark) from one of the control stations using the other control station for a known azimuth. The range marker should be set in a location which will then allow it to be used as the second station of the range. The position of the range marker need not be known; therefore only a theodolite is required for establishing the range marker. If the range is set up correctly, its extension will cross the center of the working area while another control station can be used for the cut-off angle. Table 1 lists various range lengths (as defined in the section on Fundamental Measurements) and minimum sextant angles to be used for cut-off angles. The Table will enable the user to keep range errors to less than 2.5 meters (per previous discussion) and also allow no more than 2.5 meters of angular error (as defined below), therefore the RMS error will be less than or equal to $\sqrt{(2.5)^2 + (2.5)^2}$ or 3.5 meters.

The minimum sextant angle was determined by mathematically analyzing the circle of position defined by a sextant angle and the distance between the two

TABLE 1

Range length or signal spacing "R"	Minimum angle	Maximum distance for angle	Maximum distance for range "X"
250	10	1 443	2 035
500	14	2 031	2 698
750	17	2 479	3 227
1 000	20	2 851	3 666
1 250	22	3 147	4 043
1 500	24	3 423	4 370
1 750	26	3 669	4 659
2 000	28	3 898	4 916
2 250	29	4 116	5 145
2 500	31	4 302	5 352
2 750	32	4 488	5 539
3 000	33	4 650	5 708
3 250	35	4 789	5 862
3 500	36	4 938	6 002
3 750	37	5 068	6 129
4 000	38	5 213	6 244
4 250	39	5 343	6 350
4 500	40	5 459	6 445
4 750	41	5 562	6 532
5 000	42	5 652	6 610

points observed. A lack of sufficient space in this paper prevents a complete discussion of this procedure which will however be contained in a future NOS Technical Publication. Once the range and cut-off angle stations are determined, a selection of cut-off angles is made (usually at two-degree increments) through the center of the survey area. Rates for the electronic navigation system are then computed for the range and cut-off angle positions. A nice feature of this method is that once the index correction is determined for the sextant, its value is left on the micrometer while successive cut-off angles are set on the sextant by moving just the arm of the sextant. For this reason, it is best to select only whole angles for the cut-off angles. To perform the system check, the survey vessel navigates down the range while a sextant is used by the hydrographer (set to one of the predetermined angles) to view both the range markers and the cut-off station. When all three targets are aligned, the displayed navigation rates are recorded. In this manner, the hydrographer can verify that the range has been adequately navigated, giving the helmsman supplemental conning instructions as necessary. The sextant is then set to the next angle and the process is repeated. The range and cut-off angle method requires only two control points, no supplemental personnel and no computations either during or after the daily system check. The check occurs within the working area and is dynamic in nature, thus it is more conducive to detection of problems related to vessel motion and vibration. It may even be possible to detect, skip, or null zones in the survey area. The method is not overly sensitive to sea state or tidal fluctuations. With some practice, it is possible to complete a daily check in less than 10 minutes. The only major drawback over the static method occurs during periods of restricted visibility.

Range-Azimuth method

The next method of daily system checks is the range-azimuth or range-distance method, which requires only two control stations and is also very efficient. If a Total Station device is available, the instrument is set over one control station and sets its azimuth (horizontal) plate into a geodetic azimuth by using the other control station. The instrument is then aimed at a special reflector board mounted on the survey vessel. The Total Station measures the range and azimuth to the launch mounted reflector. At the same instant, a "mark" is relayed to the survey vessel and the navigation rates are recorded. By using the standard deviations obtained earlier, a maximum error can be determined. Since most of these devices will measure a maximum range of 6 000 meters, the distance error will be less than 1 meter and the angular error (for a 6-second transit) will be less than 0.5 meter. The total error will then be less than $\sqrt{(1.0)^2 + (0.5)^2}$ or 1.2 meter.

This method requires an extra person ashore and some computations; however, several launches can be calibrated in a short period of time. The configuration of the control is not very critical, the method is the most accurate available and can be performed day or night if the azimuth target is lighted. This method is dynamic in nature and can be performed anywhere in the survey area. Because of the system's high accuracy, it most nearly satisfies the true definition of system calibration that NOS attempts to obtain from static base line methods.

The cost differential is small between a Total Station and a modern EDMI and the Total Station can serve double duty as a very powerful geodetic survey instrument. However, if a Total Station is not available, an EDMI can be set up on one of the range marks and a similar calibration can be performed. The survey vessel navigates the range and the distance is measured by an EDMI set up on the range marker. The expected precison of this method is : $\sqrt{(2.5)^2 + (1)^2}$ or 2.7 meters.

On NOAA ship *Mt. Mitchell,* a program for the daily system check solution is written for the ship's programmable calculator and carried onboard the survey vessel. In this manner, the onboard computer can be left in the data collection configuration. Often two launches are breasted together such that their navigation antennas are alongside one another and can, therefore, be calibrated simultaneously using only one set of reflectors. This method is as fast as static methods and is the method least affected by weather, sea, or visibility conditions.

Intersecting ranges method

Another method that utilizes only two established control points is the intersecting range method. In this method, two ranges can be established in a method similar to that described in the range and cut-off angle method. The problem with this procedure is that it is often difficult to place two ranges that will intersect in the desired area of the working grounds. If it is possbile to mechanize such a pair of ranges, the calibration is extremely quick and several launches following one another can be calibrated at the same time. The most desirable procedure is to steer one range and cross the other range, then turn on to the second range and cross the first. Using binoculars, the hydrographer can observe the ranges to ensure that an accurate crossing occurs. The rates for the electronic navigation system are pre-computed for the range intersection and, therefore, no computations are required. The expected accuracy of this method (based on previous range accuracy discussion) is $\sqrt{(2.5)^2 + (2.5)^2}$ or 3.5 meters for ranges intersecting at 90 degrees. For other angles of intersection, the precision will vary as 3.5 meters is divided by the sine of the intersection angle. An intersection of less than 30 degrees or greater than 150 degrees should not be used for the same reasons as navigation rates which intersect at similarly small or large angles are not used. This method, although quick and efficient, is not as desirable as the previous methods due to the need to be on two ranges simultaneously and the difficulty in setting up the two ranges in the first place. Also the calibration takes place in a very limited area and is almost static in nature.

Sextant resection method

The next method involves obtaining a three-point fix using two sextants and the resection solution. In this case, at least four established stations are required. This method normally requires more target construction. All targets must be visible simultaneously from the point of the daily system check. As can be seen from the previous discussion on fundamental measurements, sextant angles are the least accurate of the fundamental measurements. This fact, combined with known geometry problems, requires a careful selection of control and calibration area. Considerable statistical research has been conducted on this problem and a complete discussion will be presented in the previously mentioned NOS Technical Report.

The three basic control configurations to consider are : all shore controls lie in a straight line, are concave, or convex as they appear from the survey vessel. The most acceptable configuration from the standpoints of accuracy, flexibility in calibration area and determining an acceptable precision estimate occurs when the established control lies in a straight line. It is an easily established fact, however, that the strongest (highest precision) sextant fix occurs when the control forms a triangle and the vessel lies within the triangle. Unfortunately, such a configuration is difficult to obtain and the sextant angles are often too large to measure. When the control is convex or concave, the area in the survey grounds where the precision is high is normally very limited. It can be shown though that the error estimate for the first case remains fairly constant and below 5 meters when the control is equally spaced and the calibration site lies within an area whose greatest extent offshore is no greater than the distance between the center station and the nearer of the other two stations. Therefore, if the stations are spaced approximately 2 000 meters apart, the vessel being calibrated should lie within 2 000 meters of the center station. The error grows rapidly as the distance increases between the center station and the survey vessel. As mentioned earlier in this discussion, four established control stations are necessary for the system check. The fourth station is used to obtain a check fix. The check fix uses one of the fix angles plus one new angle measured to the opposite side of the center station. The check fix angle is observed simultaneously with the fix angles. The location of the survey vessel must therefore be close enough to the center station so that the fix and check fix will be of adequate precision.

Commonly, the measure of precision for the fix and check fix is the size of the inverse distance between the two fixes. It can be shown, however, that an analysis of both the inverse distance and the statistical relationship of the error ellipses for the fix and check fix is necessary. In other words, neither the inverse distance nor the error ellipse can be a true representative of the quality of the daily system check. This situation is particularly true when the survey vessel lies near the limit of acceptable precision for one of the fixes but not the other.

Another problem which can result in large positional errors is the "two-person" eccentricity condition. Since the fix requires a minimum of three persons to obtain the measured angles (all of whom are separated by at least two feet), an unavoidable eccentricity exists between the individual measured angles. If the intersection of the circles of position is slight (less than 30 degrees) and the eccentricity is 2 feet, the error can be as great as 30 meters. Another source of error with a similar magnitude results when the angles are not obtained simultaneously. A vessel speed of five knots and a time difference in observations of 1 second could result in positional errors of up to 30 meters. Sextant resection must therefore be used only after a careful examination of the position circle intersection defined by the observed angles. The method of constructing position circles is given in many publications and will, therefore, not be presented at this time. Other drawbacks to this method of daily system check include : number of required control stations, the geometric configuration of the control stations, excessive computations which require the use of an onboard computer, and the difficulty of obtaining multiple good measurements from a moving platform during rough seas or reduced visibility conditions. Another consideration is the cost for the necessary equipment. For instance, an EDMI which can be mounted on a theodolite costs little more than three sextants and can be used for geodetic control work as well as for daily system checks. Of all the methods described so far, sextant resection is the least accurate, most dependent on the vessel's position, and labor intensive.

Theodolite intersection

The last method to be discussed is the two theodolite intersection procedure. In the method, a minimum of two intervisible established control stations is necessary. A theodolite at each station simultaneously determines a geodetic azimuth to the same point on the survey vessel by initialling on the other control station. The accuracy of this method is better than the resection method because the angle measurements are of a very high precison and observed from a stable platform. The procedure has the same major geometric limitation as in the case of two intersecting ranges, that is the two lines of position must be restricted to between 30 and 150 degrees. In this case, however, the error at 2 000 meters from the theodolite stations should not exceed 2 meters. Although theodolite intersection is very accurate, it is the most labor intensive, relatively slow, has a high equipment cost and the mathematical solution must be solved using an onboard computer. A variation of this method utilizing one theodolite station and one range becomes a very accurate method. In this hybrid method, the survey vessel would navigate down a range and be intersected by the theodolite station at predetermined angles. The navigation system rates could then be compared against its precomputed values. A more complete statistical analysis of the intersection method will be covered in the previously mentioned NOS Technical Publication.

SUMMARY

All the methods presented (summarized in Table 2) will yield a visual fix precision of 5 meters (1 sigma) or better if used within the established guidelines. The precisions presented for the fundamental measurements will allow the user to compute the precisions of other hybrid systems. In every case, one must remember that the ultimate goal is to establish a daily system check which is efficient, has an estimable precision adequate for the scale of the survey and is not overly labor intensive. A careful review of the survey area must be conducted as part of the presurvey planning in order to determine which method of daily system check should be used. The proper choice will ultimately influence the accuracy of the survey soundings.

TABLE 2

Method	Static	Range/ Angle	Range/ Azimuth	Inter. ranges	Resection	Intersection
Accuracy (m)	2.5	3.5	1.2	3.5	5.0	2.0
Required control	1	2	2	2-4	4	2-4
Additional computations	none	none	minimal	none	excessive	excessive
Time	rapid	rapid	rapid	rapid	slow	slow
Area constraints	limited	none	none	limited	limited	none
Additional equipment	none	sextant	EDMI	none	3 sextants	2 theodolites
Additional personnel	none	none	1	none	none	2
Weather constraints	possible	slight	slight	possible	probable	probable

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