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ON THE CONDITIONS FOR CLASSIFICATION OF TIDES

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SUMMARY

This investigation is carried out to examine the conditions which are commonly used to classify a tidal regime as diurnal or semi-diurnal. It is observed that the application of COURTIER'S (1938) criterion in tidal predictions is not always appropriate. It is shown that during some neap tides most of the major tidal harmonics of the semi-diurnal band may conspire to oppose the M_2 tide. Consequently, at some ports the semi-diurnal component may degenerate into a dodge tide. Under these circumstances, though a tide may be classified as semi-diurnal according to Courtier's criterion, the tidal profile is determined by the constituents of the other species. A diurnal tide can occasionally become semi-diurnal for similar reasons. A simple mathematical explanation of these conditions and when they are likely to occur is given. For practical purposes, new conditions that must be satisfied for a tide to be diurnal or semi-diurnal during the full spring-neap cycle are suggested.

INTRODUCTION

This problem emerged from the recent computation of tidal predictions of Port Hedland (Western Australia). In computation of times and heights of high and low water, the gradient of tidal level is evaluated at predetermined time intervals 'Δt' which is selected according to the type of tide. Once the gradient changes its sign, to find the exact time a binary search is started within that interval to find the time and height of a high or low water. An optimal choice of time interval is essential because it should not be too long to lose a tide and it should not be too

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short to do unwanted computing. In this respect, the classification of tides suggested by COURTIER (1938) :

$$F = \frac{O_1 + K_1}{M_2 + S_2} \quad \begin{array}{ll} \geqslant 3.0 & \text{diurnal} \\ < 3.0,\ > 0.25 & \text{mixed} \\ \leqslant 0.25 & \text{semi-diurnal} \end{array} \qquad (1)$$

is widely used to determine the 'Δt'. F is called the form number. DIETRICH *et al.* (1966, p. 427) commented, "It indicates the form of the tide for one day. A strict classification based on form number cannot be made, for example, in the sense that exclusively a semi-diurnal tide exists in one area, and only diurnal tides in another. This limitation is explained by the fact that the form number is valid only for one instant during the spring-neap cycle, i.e. for the spring time". Figure 1, however,

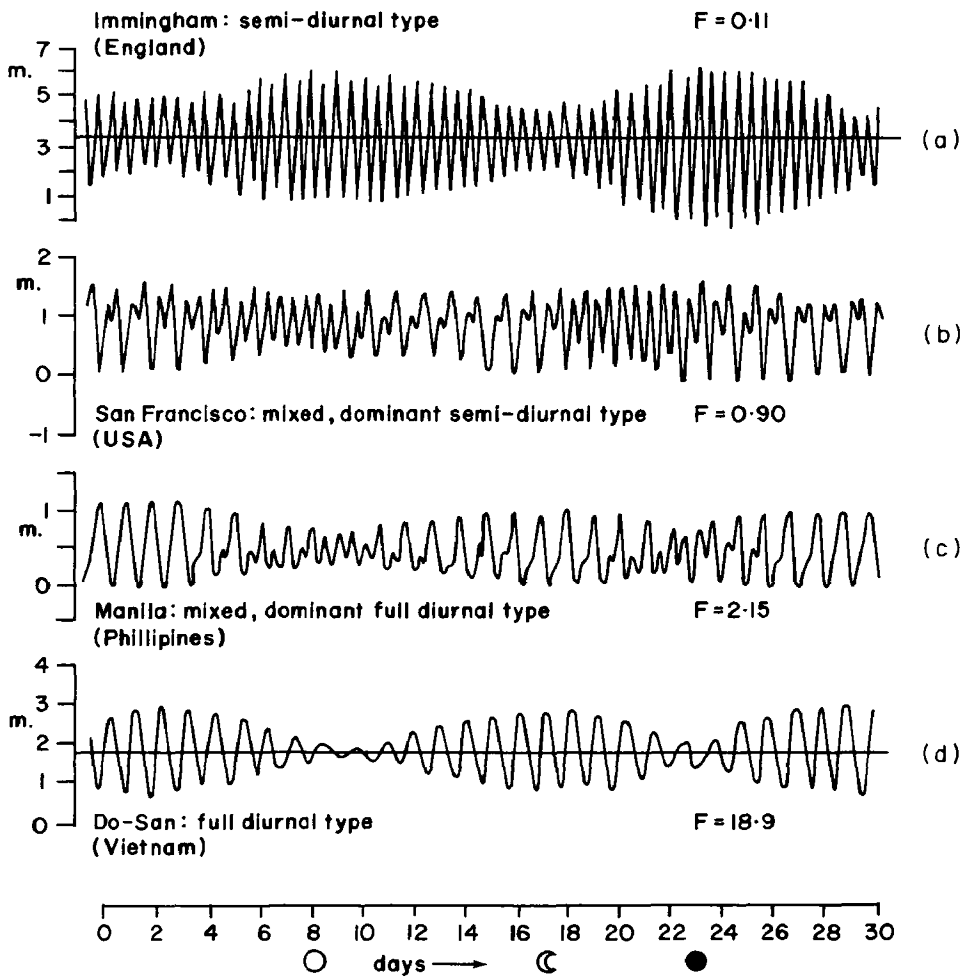


FIG. 1. — An example of different types of tides from DIETRICH *et al.* (1966).

gives a different picture in that it suggests that a semi-diurnal tide has two high and two low waters occurring each day throughout the spring-neap cycle.

It is noted that the form number is often used and quoted without any reference to its limitations. The application of this definition in the tidal predictions may result in confusion in some cases. It is the purpose of this research to clarify that a tide classified as a semi-diurnal tide may not have a semi-diurnal component at all on certain occasions. In those circumstances, the tidal profile will depend on constituents of the other species.

SEMI-DIURNAL TIDES

The condition of equation (1) is fully justified for spring tides, but it is narrow in the sense that it is limited to the spring tides only. To search for a condition which must be valid for the neap tides as well as spring tides, the neap tides of the smallest range must be investigated. By simple consideration, those neap tides occur at times when all the major constituents of the semi-diurnal band conspire to oppose the M_2 tide. In many respects, the problem of searching for the neaps of the smallest range is similar to the problem of finding the highest astronomical tides (CARTWRIGHT, 1974), or extreme high levels in real tides (AMIN, 1979) when nearly all constituents are in phase.

According to the tide-generating potential, the phases of the harmonic terms can be written as :

$$V = \mathbf{r} \cdot \boldsymbol{\theta} + \varphi \qquad (2)$$

where **r** is a vector of small integer (CARTWRIGHT & TAYLER, 1971), φ is a multiple of $\pi/2$ and

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} \qquad (3)$$

where :

θ_1 = local mean lunar time reduced to an angle;
θ_2 = the mean longitude of the Moon;
θ_3 = the mean longitude of the Sun;
θ_4 = the mean longitude of the Moon's perigee;
θ_5 = the negative of the mean longitude of the ascending node of the Moon,

and

θ_6 = the longitude of the Sun's perigee.

The longitude of the Sun's perigee varies only slightly over a period of a century and only affects the phase of constituents T_2 and R_2 which are very small; therefore its influence will not be considered.

Consider the harmonic terms M_2, S_2, K_2 and N_2, and the nodal terms ('satellites') of M_2 and K_2. In the equilibrium tide, their phase must satisfy the following conditions to oppose the M_2 constituent :

$$2\theta_1 = 0 \quad \text{for } M_2 \quad (4)$$
$$2\theta_1 - \theta_5 + 180 = 180 \quad M_{2,-\theta_5} \quad (5)$$
$$2\theta_1 + 2\theta_2 - 2\theta_3 = 180 \quad S_2 \quad (6)$$
$$2\theta_1 + 2\theta_2 = 180 \quad K_2 \quad (7)$$
$$2\theta_1 + 2\theta_2 + \theta_5 = 180 \quad K_{2,\theta_5} \quad (8)$$
$$2\theta_1 - \theta_2 + \theta_4 = 180 \quad N_2 \quad (9)$$

The second subscript with the name of term indicates that it is a nodal term associated with the principal term represented by the first subscript. These conditions can be satisfied by any one of the four values $\boldsymbol{\theta}$ in table 1.

TABLE 1

Values of orbital elements (degrees) which can satisfy conditions in equations (4-9).

	θ_1	θ_2	θ_3	θ_4	θ_5
A	0	90	0	90	0
B	0	90	180	90	0
C	180	90	0	90	0
D	180	90	180	90	0

When these conditions are satisfied the remaining principal constituents of the semi-diurnal species will have mixed contributions; μ_2 and λ_2 will oppose M_2, and $2N_2$, υ_2 and L_2 will be in phase with M_2.

In the diurnal species, main constituents which contribute to the tide are K_1, O_1, P_1, Q_1 and nodal terms of K_1 and O_1 and their phases are :

$$\theta_1 + \theta_2 + 90 = V_{K_1} \quad \text{for } K_1 \quad (10)$$
$$\theta_1 + \theta_2 + \theta_5 + 90 = V_{K_{1,\theta_5}} \quad K_{1,\theta_5} \quad (11)$$
$$\theta_1 - \theta_2 + 270 = V_{O_1} \quad O_1 \quad (12)$$
$$\theta_1 - \theta_2 - \theta_5 + 270 = V_{O_{1,-\theta_5}} \quad O_{1,-\theta_5} \quad (13)$$
$$\theta_1 + \theta_2 - 2\theta_3 + 270 = V_{P_1} \quad P_1 \quad (14)$$
$$\theta_1 - 2\theta_2 + \theta_4 + 270 = V_{Q_1} \quad Q_1 \quad (15)$$

By substituting the value of $\boldsymbol{\theta}$ in equations (10-15) it can be shown that constituents O_1 and K_1 combine together as they will be in phase, but P_1 and Q_1 oppose them. However, the opposing effect of Q_1 and P_1 will almost be compensated by nodal terms of O_1 and K_1 and the diurnal tide can be approximated by :

$$A_1 \sim O_1 + K_1 \quad (16)$$

To summarise, the neap tides of the smallest range occur when the following conditions are simultaneously satisfied :

(a) the Earth is at one of the equinoxes;

(b) the longitude of the Moon is 90° away from the perigee, and

(c) the longitude of the Moon's ascending node is near the spring equinox.

It is worth noting here that the conditions required for the diurnal tide to be near its peak include the condition that the Earth should be near perihelion. This contradicts condition *(a)* in centuries around the present time. It is difficult for all these elements to satisfy the conditions in table 1 simultaneously. The dates when they approach closest to these values in this century are listed in table 2. For exact value of θ_1, some adjustments in other elements will also be essential. Therefore some tolerance, say ε, is allowed to relax these conditions.

TABLE 2

The dates when θ approaches nearest to the values in table 1.

	θ_2	θ_3	θ_4	θ_5
1969 March 22	90.0	2.86	271.29	0.21
1986 September 25	90.0	184.43	263.50	21.63
1987 September 16	90.0	174.43	303.16	2.83

In the real tide, the phase of any tidal constituent lags with respect to its phase in the equilibrium tide. However, **θ** can be computed for a particular place by introducing the phase lags of constituents in equations (4-9) as :

$$2\theta_1 - g_{M_2} = 0 \quad \text{for } M_2 \tag{17}$$
$$2\theta_1 - \theta_5 + 180 - g_{M_{2,-\theta_5}} = 180 \quad M_{2,-\theta_5} \tag{18}$$
$$2\theta_1 + 2\theta_2 - 2\theta_3 - g_{S_2} = 180 \quad S_2 \tag{19}$$
$$2\theta_1 + 2\theta_2 - g_{K_2} = 180 \quad K_2 \tag{20}$$
$$2\theta_1 + 2\theta_2 + \theta_5 - g_{K_{2,\theta_5}} = 180 \quad K_{2,\theta_5} \tag{21}$$
$$2\theta_1 - \theta_2 + \theta_4 - g_{N_2} = 180 \quad N_2 \tag{22}$$

Substituting the values of g's in equations (17-22), and solving for **θ**, times can be computed. When these conditions are satisfied the amplitude of semi-diurnal tide can be approximated as :

$$A_2 \sim 0.93\ M_2 - (S_2 + K_2 + N_2) \tag{23}$$

The factor 0.93 is to account for the nodal tides of M_2 and K_2.

The reduction in the size of the semi-diurnal component depends on the amplitudes of S_2, N_2 and K_2. The relationship between the phases of S_2 and K_2 in the observed tide is nearly the same as in the equilibrium tide, and relationships between M_2 and its nodal term, K_2 and its nodal term in the observed tide are nearly the same as in the equilibrium tide. It is suggested that conditions *(a)* and *(c)* for the equilibrium semi-diurnal tide of the lowest range are expected to remain valid for the observed tide.

A change in the solution of **θ** (or times) will occur due to the phase lags of constituents in equations (17-22) for the observed tide as compared with that computed from equations (4-10) for the equilibrium tide. The diurnal tides O_1 and K_1 may not be in phase at that time, and the amplitude of diurnal component may be much smaller than anticipated by equation (23).

Example

Consider the tidal regime of Port Hedland (Australia), the tidal constituents of which are listed in table 3. Using the criterion of Courtier, the form number

($F = 0.145$) is well within the limit to classify the tide as semi-diurnal. Substituting the values of g in equations (17-22), the solution for $\boldsymbol{\theta}$ is as given in table 4.

TABLE 3

Tidal constituents of Port Hedland (W. Australia).

Name	Amplitude (m)	Phase lag (deg)
Q_1	0.033	265.12
O_1	0.150	272.71
P_1	0.072	288.10
K_1	0.244	299.39
$2N_2$	0.028	215.56
μ_2	0.047	294.72
N_2	0.276	276.16
ν_2	0.062	272.05
M_2	1.681	304.86
λ_2	0.026	307.34
L_2	0.064	323.73
S_2	1.021	14.04
K_2	0.282	13.28

TABLE 4

Values of orbital elements (degrees) which can satisfy conditions in equations (11-16) for the tide of Port Hedland.

	θ_1	θ_2	θ_3	θ_4	θ_5
A	152	125	0	277	0
B	152	125	180	277	0
C	332	125	0	277	0
D	332	125	180	277	0

For conjugate values of θ_2 and θ_4, four additional values of $\boldsymbol{\theta}$, which can satisfy equations (17-22), are obtained. The years of this century when θ_2, θ_3, θ_4 and θ_5 will be nearest to the values given in table 4 are 1969, 1986 and 1987. θ_1 can be adjusted to the required value at the cost of some small perturbations in the other elements.

The amplitude of the semi-diurnal component, A_2, when the above conditions are satisfied, will become :

$$A_2 \sim 0.93\ M_2 - (S_2 + N_2 + K_2) \sim 0 \qquad (24)$$

This shows that the semi-diurnal component can virtually disappear and the tidal profile will be determined by the constituents of the other species. The predicted tides, shown in figure 2, confirm that the procedure based on Courtier's criterion for classification of a tide as 'semi-diurnal' can break down. The actual dates when the tide became diurnal, computed using the full set of constituents, are 1969 February 27 and 1986 August 30; estimated dates, using equations (17-22), are 1969 March 28 and 1986 September 28 respectively. These displacements are due to non-linear tides which become important in the determination of such events which are very close otherwise.

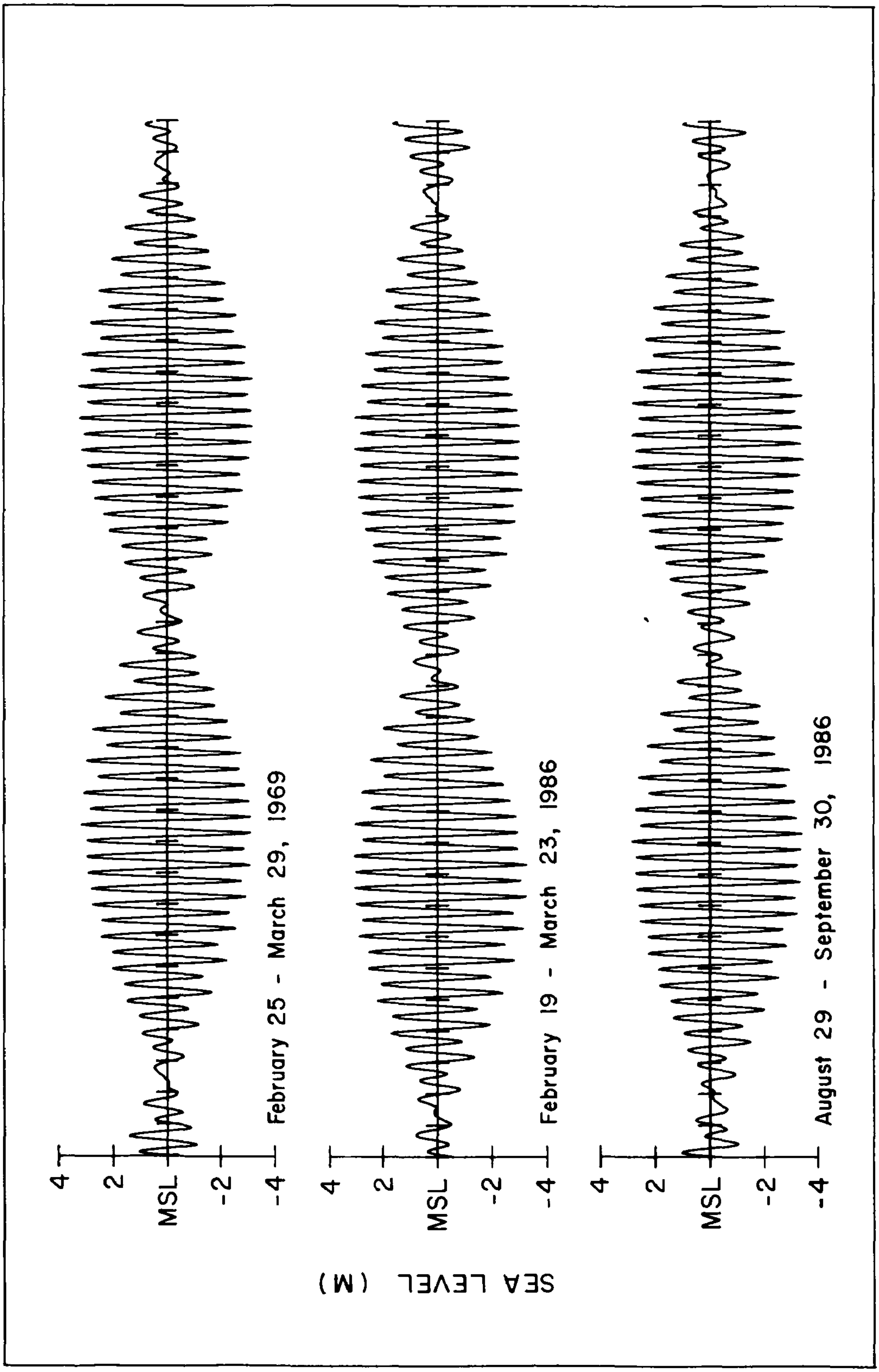


FIG. 2. — Predicted tides of Port Hedland (W. Australia).

TABLE 5

The estimated dates when θ is nearest to the values in table 4.

	θ_2	θ_3	θ_4	θ_5
1969 March 28	125.0	5.48	271.59	0.05
1986 September 28	125.0	186.97	263.82	21.72
1987 September 18	125.0	177.05	303.39	2.72

It is important to note that relaxation of tolerance ε_i in an element θ_i depends on the amplitude of constituents whose phase depends on it. For example, θ_5 is involved in the phase of nodal terms. Since their amplitudes are small, a large tolerance is possible for it.

It is shown that orbital elements do come very close to values which can substantially reduce the semi-diurnal component of tide. By representing diurnal and semi-diurnal components by the nominal speeds of M_1 and M_2 constituents respectively, it can be shown that that the tidal profile will be semi-diurnal if $A_2 > A_1/4$ (DOODSON & WARBURG, 1941, p. 221). However, the tide will be similar to the mixed dominant semi-diurnal type shown in figure 1(b). For a tide to be semi-diurnal on such occasions so that all low waters are below mean sea level and all high waters are above mean sea level, it must satisfy :

$$A_2 > A_1 \qquad (25)$$

It can be shown, by substituting the values of **θ** from table 4 in equations (10, 12) for the observed tide, that phases of constituents K_1 and O_1 differ by 30° when the semi-diurnal component of Port Hedland is minimum. The amplitude of the diurnal component should be computed by vector sum of constituents O_1 and K_1 and it will be smaller than that given by equation (16). The phase lags of the observed tides tend to relax condition (25) because A_1 decreases. In marginal cases it will be essential to calculate **θ** from equations (17-22) and phases of constituents O_1 and K_1 to compute the exact amplitude of the diurnal components.

DIURNAL TIDES

In the diurnal species, major constituents which determine the profile of the tide are Q_1, O_1, P_1 and K_1. Assuming that K_1 is the dominant constituent of the species, the diurnal component will be near its minimum when it is opposed by O_1, P_1 and Q_1. This would imply that phases of these constituents should satisfy the following conditions :

$$\theta_1 + \theta_2 + 90 - g_{K_1} = V_{K_1} = 0 \qquad (26)$$

$$\theta_1 - \theta_2 + 270 - g_{O_1} = V_{O_1} = 180 \qquad (27)$$

$$\theta_1 + \theta_2 - 2\theta_3 + 270 - g_{P_1} = V_{P_1} = 180 \qquad (28)$$

$$\theta_1 - 2\theta_2 + \theta_4 + 270 - g_{Q_1} = V_{Q_1} = 180 \qquad (29)$$

For the equilibrium tide, g = 0, the possible solutions of system of equations (26-29) are given in table 6.

TABLE 6

Values of orbital elements (degrees) which satisfy conditions in equation (26-29) for diurnal component to be minimum.

θ_1	θ_2	θ_3	θ_4
270	0	180	0
270	0	180	0
90	180	0	0
90	180	180	180

The longitude of the Moon's node is not important because nodal terms of constituents O_1 and K_1 will cancel each other. On those occasions the amplitude of the diurnal component can be approximated as :

$$A'_1 \sim K_1 - (O_1 + P_1 + Q_1) \qquad (30)$$

Substituting the values of $\boldsymbol{\theta}$ from table 6 in equations (4-9), it can be shown that constituents M_2, S_2, N_2 and K_2 are in phase and the semi-diurnal component reaches near to its maximum value :

$$A'_2 \sim M_2 + S_2 + K_2 + N_2 \qquad (31)$$

This is in accordance with the fact that the semi-diurnal component maximises at the expense of the diurnal component (CARTWRIGHT, 1975). For a tide to be diurnal all the time it must satisfy the condition (DOODSON & WARBURG, 1941) :

$$A'_1 > 4A'_2 \qquad (32)$$

However, in the real tide the semi-diurnal component may not be near its maximum value when the diurnal component is minimum. Thus the condition (32) can be relaxed to some extent.

Example

The form numbers of tides of Do-Son (Vietnam) and Poeloe Langkotas (Sumatra) calculated from their constituents in table 7 are 19 and 25, respectively; therefore, according to Courtier's criteria, these tides should be classified as diurnal. Do-Son tide is classified as diurnal by DIETRICH (1966), see figure 1. The values of $\boldsymbol{\theta}$, when K_1 is opposed by other constituents, are given in table 8 and dates when $\boldsymbol{\theta}$ approaches these values are listed in table 9. The diurnal component can easily vanish on these dates and the tidal profile will be determined by the constituents of the semi-diurnal species as is confirmed by predictions of 18 October 1948 in figure 3.

In the case of Poeloe Langkotas, condition (32) is not satisfied if A_2 is evaluated simply by equation (31). When the diurnal tide approaches near its minimum, i.e. $\boldsymbol{\theta}$ satisfies equations (26-29), constituents M_2 and S_2 differ in phase by 116°. For the observed tide A_2 should be calculated by vector sum of constituents M_2, S_2, ... instead of by a simple sum as in equation (31) which is applicable for in phase constituents of the equilibrium tide. A_2 computed in this way is smaller than that given by equation (31) and it satisfies condition (32) and tidal profile remains diurnal as shown in figure 4. Clearly, phase lags of constituents play an important role in the determination of profile of the tide.

TABLE 7

Constituents of Do-Son, Vietnam (Long. 106°47′ E, Lat. 20°43′ N) and Poeloe Langkotas, Sumatra (Long. 107°37′ E, Lat. 2°32′ S). Time zones are − 7, − 8 hours respectively.

Name	Do-Son		Poeloe Langkotas	
	Amplitude (m)	Phase lag (deg)	Amplitude (m)	Phase lag (deg)
Q_1	0.136	354.0	—	—
O_1	0.700	26.0	0.390	76.0
P_1	0.240	89.0	0.130	132.0
K_1	0.720	89.0	0.620	139.0
N_2	0.008	85.0	—	—
M_2	0.044	102.0	0.020	225.0
S_2	0.030	136.0	0.030	11.0
K_2	0.010	137.0	—	—

TABLE 8

Values of orbital elements (deg.) when diurnal component becomes minimum.

Do-Son				Poeloe Langkotas			
θ_1	θ_2	θ_3	θ_4	θ_1	θ_2	θ_3	θ_4
90	212	0	153	90	211	0	
90	32	180	153	270	31	180	

TABLE 9

Dates when orbital elements approach nearest to values in table 8 diurnal tides close to their minimum.

	θ_2	θ_3	θ_4
Do-Son			
22 September 1948	32	180	157
23 March 1966	32	1	149
Poeloe Langkotas			
23 September 1979	211	182	338
24 March 1985	31	1	202

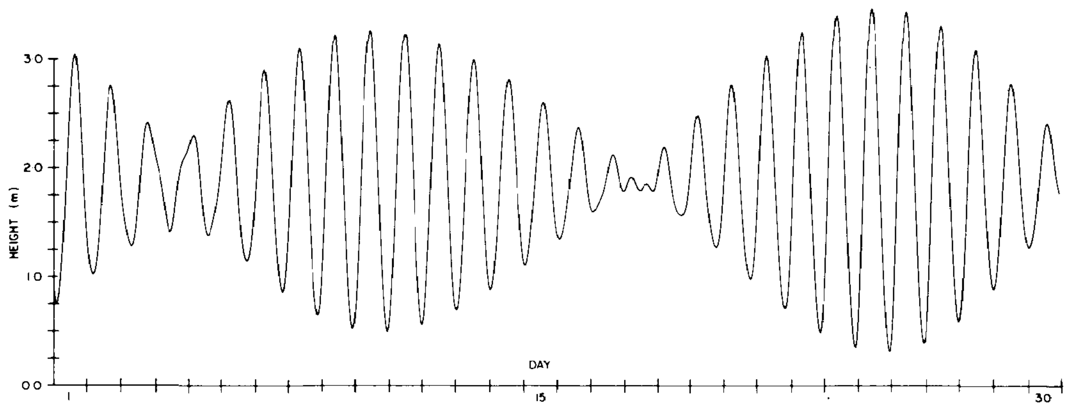


FIG. 3. — Tidal predictions of Do-Son (October 1-30, 1948) when diurnal component becomes minimum.

CONCLUSIONS

It is shown that there are times when most of the major tidal harmonics of the semi-diurnal species can conspire to oppose the M_2 tide. Similarly, there are times when the main constituents of the diurnal species oppose the K_1 tide.

The procedure of classification of tides, based on Courtier's criterion, is valid for the limited duration of spring tides. Moreover, the two assumptions, i.e. $M_2 = 3S_2$ and $O_1 = K_1$, are apparently inferred from typical tides of European waters. Therefore, its application to the other parts of the world must be treated with caution. It is overstretched when it is extended to a full spring-neap cycle. The semi-diurnal component of the tide is substantially reduced when the M_2 tide is opposed by the other major semi-diurnal constants. It is shown mathematically and by illustrating with an example that such events can occur. On those days the tidal profile is determined by the constituents of the other species, and a tide classified as semi-diurnal may behave as a mixed tide or diurnal tide.

Similarly a tide classified as diurnal under Courtier's criterion is shown to become semi-diurnal or mixed.

In view of the analysis presented in this paper, tides can be divided into the following categories :

(i) diurnal always (Poeloe Langkotas), if condition (31) is satisfied;

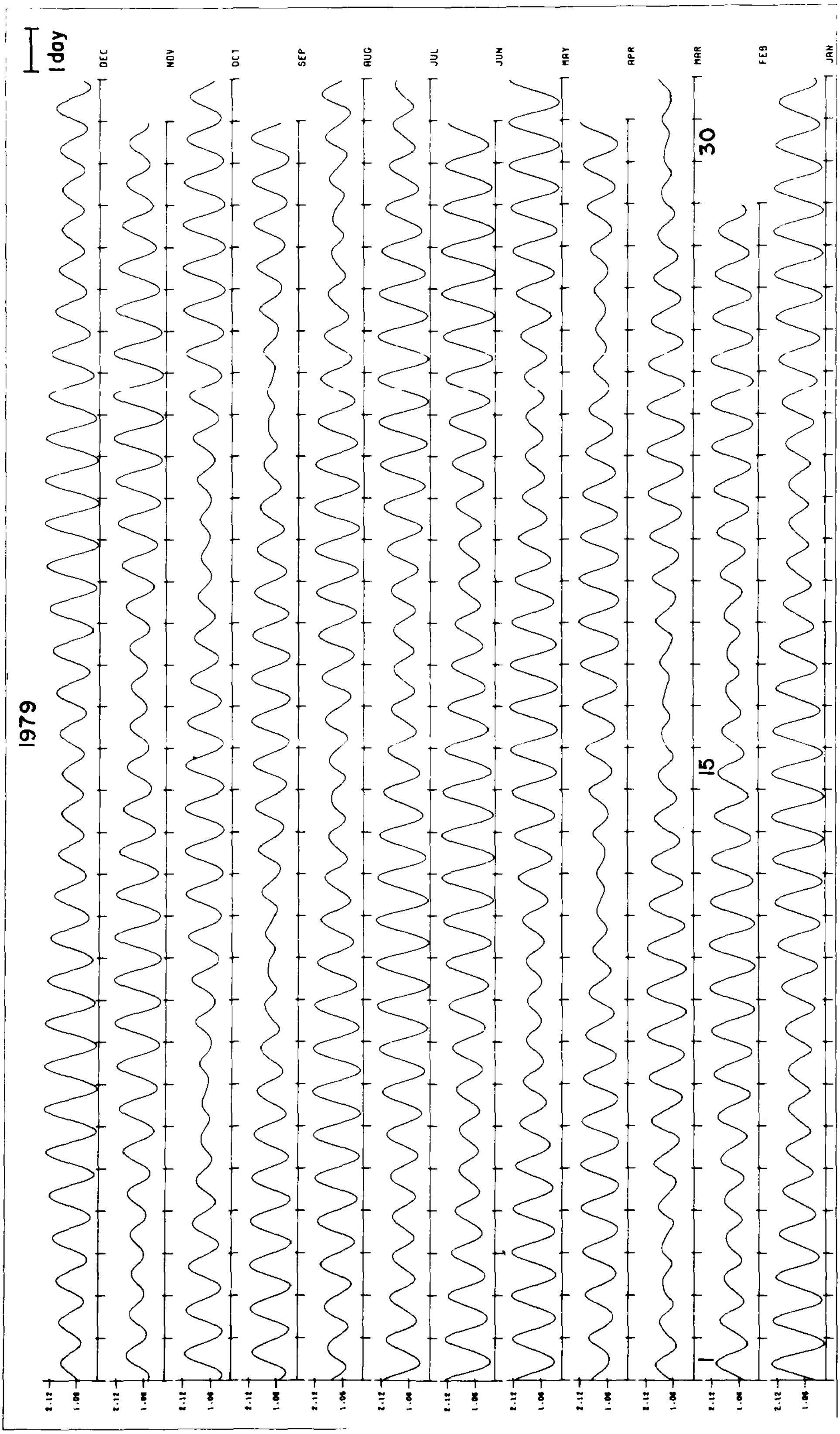
(ii) diurnal usually, with occasional semi-diurnal wave (Do-Son), if $F \geqslant 3$;

(iii) mixed (San Francisco and Manilla), if $3 > F > 0.25$;

(iv) semi-diurnal usually with occasional diurnal wave (Port Hedland) if $F \leqslant 0.25$, and

(v) semi-diurnal always, (Immingham), if condition (31) is satisfied.

Criteria for tides to be always diurnal or semi-diurnal are very restricted but can be safely applied to a full spring-neap cycle.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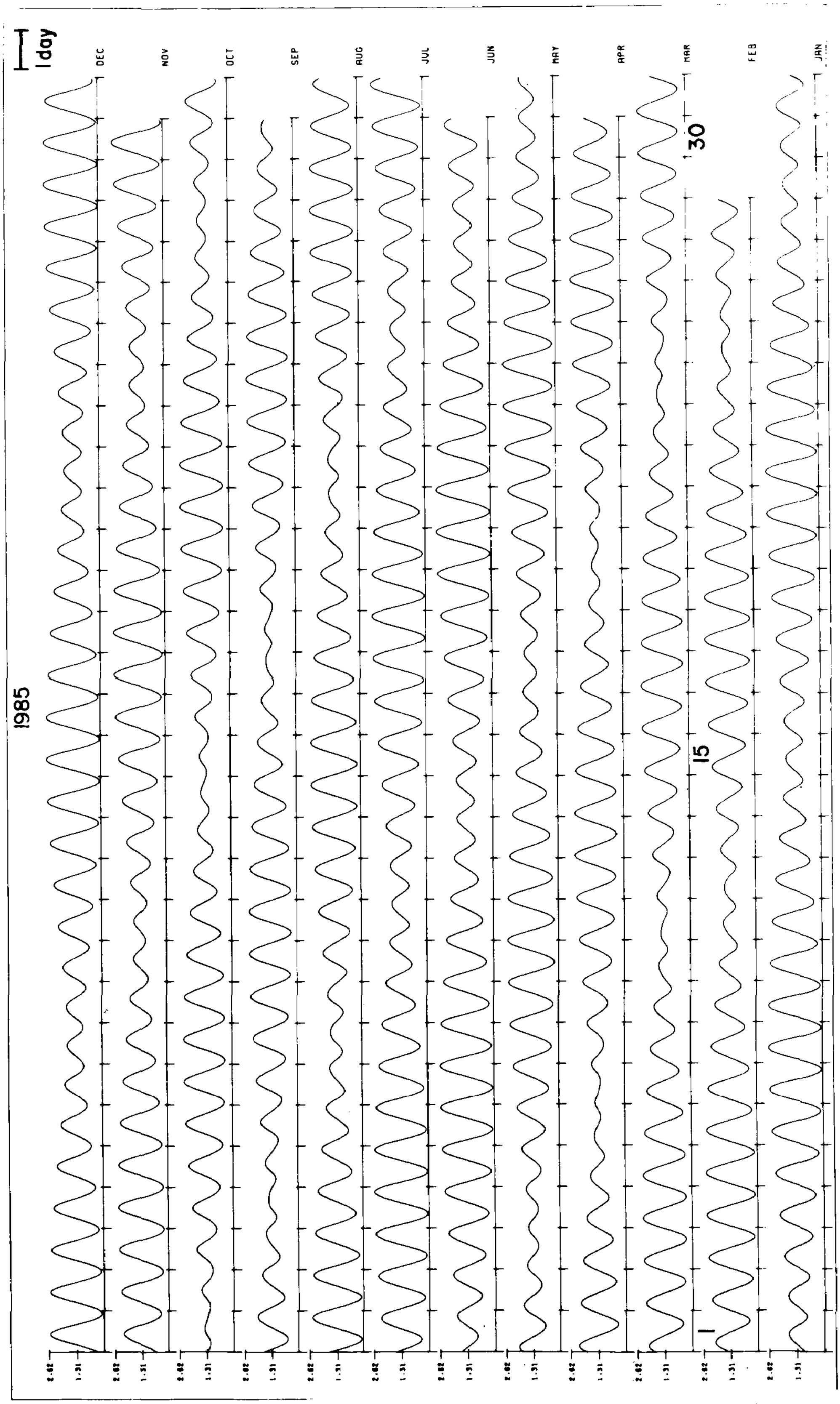

FIG. 4. — Predicted tides of Poeloe Langkotas (1979, 1985) for years when diurnal component becomes minimum.

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