

## ON THE PRACTICAL USE IN HYDROGRAPHY OF FILTERED DAILY VALUES OF MEAN SEA LEVEL

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### ABSTRACT

It is shown that the mean monthly sea level of a non-permanent station can be referred to the long-term sea level of a permanent one when tidal stations are influenced by similar meteorological and oceanographical effects. Practical methods are derived : one is based on the direct transference and the other based on the ratio of the variances of the daily mean sea level. Applications of the methods are made for the ports of Cananeia, Ubatuba, Santos and Ilha Grande on the southern coast of Brazil.

### 1. INTRODUCTION

Argument about the correct sounding datum plane is older than the I.H. Bureau itself (FRANCO, 1981). All the discussions during conferences sponsored by the International Hydrographic Bureau ended with the practical difficulty of appropriately defining the tidal datum being paramount. The old definition is really poor from the scientific point of view. There are some obvious limitations such as : what is the meaning of "... tide will but *seldom* fall below it" ? What is "seldom" in figures ? We believe that the practical interpretation given by the British Hydrographic Department and published in the *International Hydrographic Bulletin* of February 1963 is much better. This considers the lowest predicted tidal height in 19 years as the convenient datum. But there remains to be defined what is meant by "seldom".

It must be added that the accepted definition is limited to the (astronomical) tide, without any reference to the considerable variation of the daily mean sea level, which may affect the reduction of soundings.

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It can be said that the accuracy in the present state of the art of reducing soundings is below the accuracy of both position fixing and depth measurement operations.

If soundings are plotted on nautical charts, the navigator can easily and adequately judge the depths of the charted waters for his purposes. However, when engineering is involved, as, for instance, in dredging, the problem is completely different. An example will clarify the problem involved.

The need to dredge a channel in Paranaguá harbour obliged us to investigate into what the dredging depth should be, according to the tidal range at that place. The only available tidal data was a 32-day set of hourly heights measured by the Brazilian Navy. As there was no time to wait for a one-year data set to be obtained in that harbour, we filtered sets of 72 hours or hourly heights and constructed Figure 1a.

The filter was a 24-hour running mean, followed by a 25-hour running mean of the results from the first step and the average of the 25 final values. Then a step of 24 hours was adopted. This filtering removes all the contributions from high frequency (diurnal, semidiurnal, etc.) tide.

Figure 1a shows that differences between the daily filtered mean sea level values were sometimes too large to be ignored in the depth measurements. In fact, on 23 April the mean sea level was 43 cm below the average monthly mean sea level and on 1 May the difference was + 33 cm.

However, a doubt arose : how reliable was the average monthly mean sea level ? We tried to find the answer to that question by computing the daily filtered mean sea level at Cananeia, a nearby port, for the same period (Fig. 1b). Since Cananeia has a permanent tidal station, it was possible to compare the average of the filtered daily values with the mean sea level for 1975. That value was 6 cm below the average monthly mean sea level (Fig 1b).

Then another question arose : should the difference found be considered the same for Paranaguá as for Cananeia ? The answer will be given in the next section.

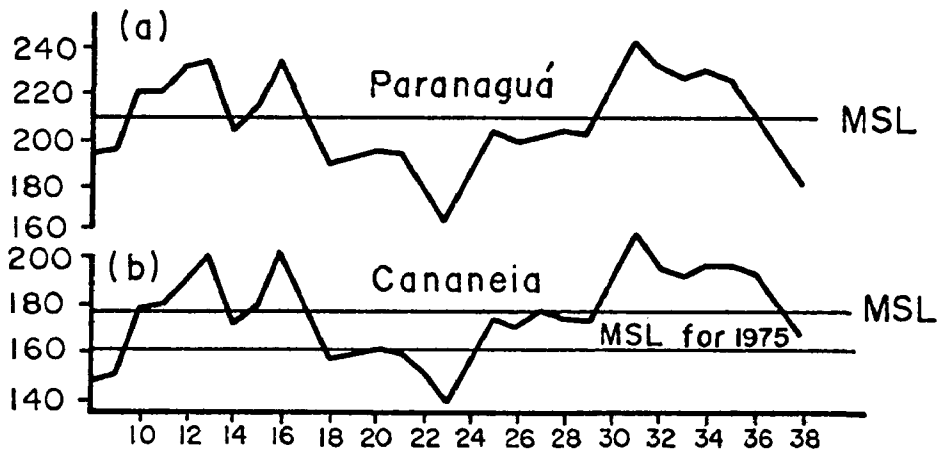


FIG. 1. — MSL at Paranaguá and Cananeia from 8/4 to 8/5/1975.

## 2. TRANSFER OF MSL VALUES

Let us suppose that, at two not-too-distant places, there are permanent tide gauges. After a one-year period of hourly tidal heights observations, the averages are worked out for the two places and the resulting mean sea levels, above the respective zeros, are known. Then conventional levelling gives the heights of mean sea levels below the respective nearby benchmarks. Precise levelling between the two above-mentioned benchmarks can show different heights of mean sea levels. Such a difference will be a constant and may be considered as the result of the dynamical equilibrium of the sea surface. In fact, an annual average is independent of tidal effect and of random meteorological variations.

However, the monthly variation of the mean sea level has a seasonal component which produces a low frequency oscillation so that, in one month, for example, the filtered mean sea level can be considerably above or below the annual mean (Fig. 1b). If this difference is valid for another place (Fig. 1a) where the filtered mean sea level is known, for the same period, then the problem may be solved.

It can be seen that the above-mentioned difference will be valid for two places if the variation of the *monthly* mean sea level is also valid for these places. A plotting of such monthly mean sea level values will show if their variations are about the same. A glance at Figure 2, drawn with the values of Table I, shows that this is the case for the four chosen tidal stations. The farthest stations (Cananeia and Piraquera) are about 400 km apart. Transfer of the mean sea level values from the main to the secondary stations is possible, if it is assumed that the monthly residuals of the main station are approximately the same as for the secondary ones. If, for example, Cananeia is taken as the main station, the annual MSL at the secondary station will be found by subtracting from the mean sea level at that station, for the month, the *residual* at Cananeia. If we have the mean sea level for May at the secondary station (Table 1), we must subtract 12.53 cm from that value to find the MSL for the year.

MSL			
	Transf.	Comp.	Dif.
Piraquera.....	107.58	103.92	3.66
Ubatuba.....	100.34	98.90	1.44
Santos .....	148.92	143.06	3.86

The largest difference is found at Santos for October 1978, where the transferred value is 138.85 and the one directly computed is 145.06. The difference is 6.21 cm, which is still acceptable from the hydrographic point of view.

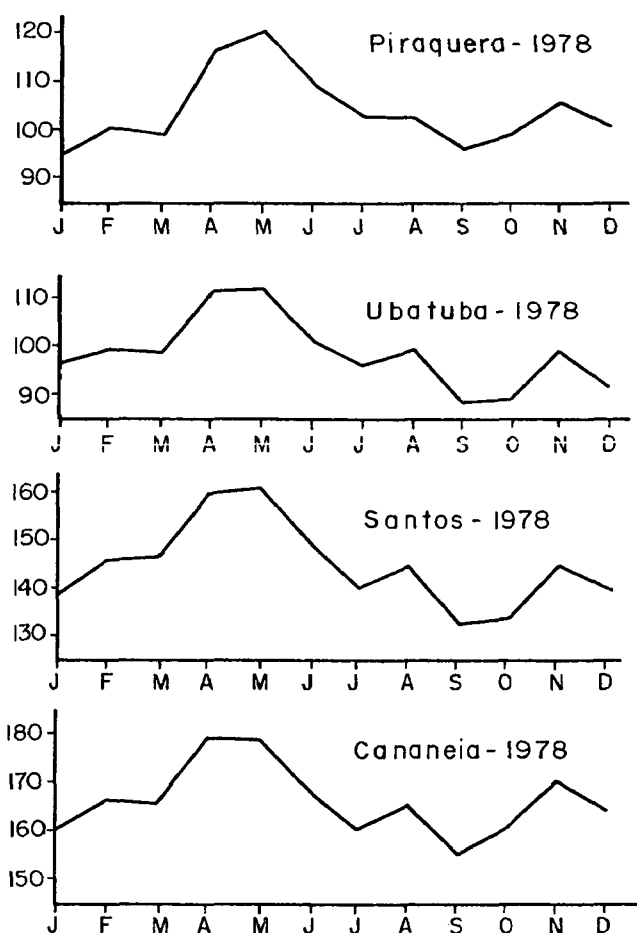


FIG. 2. — Monthly mean sea levels.

TABLE 1

Filtered monthly mean sea levels for 1978

	Piraquera		Ubatuba		Santos		Cananeia	
	MSL	Resid.	MSL	Resid.	MSL	Resid.	MSL	Resid.
January .....	95.18	- 8.74	96.19	- 2.71	138.29	- 6.77	160.10	- 6.11
February .....	100.51	- 3.41	99.29	0.39	145.86	0.80	166.71	0.50
March .....	98.57	- 5.35	98.81	- 0.09	146.65	1.59	165.23	- 0.98
April .....	116.77	12.86	111.23	12.33	160.10	15.04	178.70	12.49
May .....	120.11	16.20	112.87	13.97	161.45	16.39	178.74	12.53
June .....	108.71	4.80	102.20	3.30	150.57	5.51	168.10	1.89
July.....	103.64	- 0.28	96.61	- 2.29	140.97	- 4.09	160.94	- 5.27
August .....	102.03	1.89	99.35	0.45	144.87	- 0.19	165.16	- 1.05
September .....	95.86	- 8.06	88.30	- 10.60	132.70	- 12.36	155.30	- 10.91
October .....	98.34	- 5.58	89.55	- 9.35	133.74	- 11.32	161.10	- 5.11
November .....	105.84	1.93	98.80	- 0.10	145.47	0.41	170.13	3.92
December .....	101.42	- 2.50	93.55	- 5.35	140.06	- 5.00	164.26	- 1.95
Annual MSL	103.92	- 0.02	98.90	0.03	145.06	0.01	166.21	- 0.05

TABLE 2

Mean daily sea levels at Cananeia, Santos, Ubatuba and Piraquera,  
March 1978

Day	Cananeia (cm)	Santos (cm)	Ubatuba (cm)	Piraquera (cm)
1	136	113	74	76
2	157	130	81	82
3	174	152	102	100
4	174	153	108	105
5	181	160	112	107
6	181	164	117	112
7	184	169	115	110
8	198	181	123	116
9	192	178	123	120
10	170	157	103	106
11	156	138	85	89
12	148	130	84	84
13	161	141	91	89
14	167	155	104	102
15	159	150	105	103
16	143	130	88	91
17	139	122	74	78
18	151	133	80	82
19	159	134	91	92
20	168	148	100	99
21	168	150	104	101
22	160	144	99	99
23	154	137	92	95
24	156	134	90	92
25	157	137	91	94
26	164	142	94	98
27	165	152	102	103
28	167	153	105	103
29	173	154	104	103
30	171	151	103	105
31	172	154	107	108

### 3. THE VARIATION RATIO

Results of section (2) suggest that, given a set of N simultaneous observations  $\{Y_i\}$  of the permanent station and  $\{X_i\}$  of the non-permanent one, the ratio of the variation is :

$$C_1 = \frac{(X_i - C)^2}{(Y_i - D)^2}, \quad (1)$$

where :

$$C = \frac{1}{N} \sum_{i=1}^N X_i \quad \text{and} \quad D = \frac{1}{N} \sum_{i=1}^N Y_i, \quad i = 1, \dots, N.$$

It follows that for N values :

$$NC_1 = \frac{(X_1 - C)^2}{(Y_1 - D)^2} + \frac{(X_2 - C)^2}{(Y_2 - D)^2} + \dots + \frac{(X_N - C)^2}{(Y_N - D)^2} \quad (2)$$

where zeros of denominator should be avoided by an adequate truncation value.

Under these assumptions :

$$C_1 = \frac{1}{N} \sum_{i=1}^N \frac{(X_i - C)^2}{(Y_i - D)^2} \quad (3)$$

is bound to be an estimator of an invariant  $C_1$ .

It is clear that once  $C_1$  is calculated, the mean sea level  $C$  of the non-permanent station can also be estimated in terms of  $C_1$  from equation (1) extended to  $N$  observations :

$$C = \frac{1}{N} \sum_{i=1}^N X_i - \frac{1}{N} \sum_{i=1}^N (C_1)^{1/2} (Y_i - D) \quad (4)$$

where  $\{X_i\}$ ,  $\{Y_i\}$  and  $D$  are known.

### 3.1 Transference of mean monthly values

Expression (4) can be used to estimate

$$\delta = C - \frac{1}{N} \cdot \sum_{i=1}^N X_i \quad (5)$$

in the cases where  $\{X_i\}$  are unknown, but are simultaneous to known values of  $\{Y_i\}$  with mean  $D$ .

This possibility is used (Table 3) to determine the monthly residuals of Santos, Ubatuba and Piraquera (Ilha Grande Bay) from known daily values  $\{Y_i\}$  of Cananeia. The mean daily values of the sea levels of March were previously taken to determine  $C_1$ , in each case, as the mean of March is closer to the annual mean sea level (see Table 1) than the other m.m.s.l. With this hypothesis, results of

TABLE 3

Calculated values of the monthly residuals ( $\delta$ ) via expression (5) using known  $\{Y_i\}$  values of Cananeia. The  $C_1$  values were, respectively, 1.33 for the stations Cananeia and Santos; 1.17 for Cananeia and Ubatuba and 0.94 for Cananeia and Piraquera. The  $C_1$  values were calculated by using expression (3) and the March daily values given in Table 2. Columns Dev. indicate the difference between the residuals of Table 1 and the values of  $\delta$  as shown below.

Station	Santos		Ubatuba		Piraquera	
	Res. ( $\delta$ )	Dev.	Res. ( $\delta$ )	Dev.	Res. ( $\delta$ )	Dev.
January .....	- 6.82	0	- 6.4	3	- 5.7	- 3
February .....	0.28	0	0.27	0	0.24	- 3
March .....	- .37	- 1	- .34	0	- .31	- 5
April .....	14.5	0	13.59	1	12.24	0
May .....	14.03	- 2	13.73	0	12.36	+ 4
June .....	3.99	- 1	3.74	0	3.37	+ 1
July.....	- 4.96	- 1	- 4.65	2	- 4.19	4
August .....	- 2.32	- 2	- 2.18	2	- 1.95	0
September .....	- 12.53	0	- 11.75	1	- 10.53	2
October .....	- 5.63	- 6	- 5.28	- 4	- 4.73	0
November .....	4.68	3	4.39	4	3.94	- 2
December .....	- 1.87	3	- 1.76	- 4	- 1.57	1

expression (5) represent estimates of the deviations of mean  $(\frac{1}{N} \sum X_i)$  of the unknown  $\{X_i\}$  set of observations relative to the longer term mean of the permanent station,  $C \cong C_1$ .

In Table 3 the column Dev indicates the differences between the calculated and the actually measured monthly residuals shown in Table 1. As it can be seen, these deviations have a maximum value of  $-6$  cm, but most values are close to zero, as is desirable. The mean values of the deviations and the associated standard deviations ( $\delta$ ) increase from Santos to Piraquera, as is also expected to occur (Santos :  $-0.5 \pm 2.3$ ; Ubatuba :  $0.4 \pm 2.4$ , and Piraquera  $0.6 \pm 2.7$ ) as the stations are progressively more distant from Cananea.

#### 4. TRANSFERENCE OF DAILY MEAN VALUES

Another check was worked out for Santos and Cananea for January 1980. That period was chosen because there was an abnormal elevation of the sea level over the São Paulo coast (MESQUITA, 1983). Figure 3 shows that abnormal elevation on 1 January. The transferred value of the mean sea level from Cananea to Santos differed from the annual value for Santos by 3 cm.

Since hydrographers often do not have time to wait more than 15 days to choose a datum plane, we decided to try the procedure for a 15-day span. Then we computed averages of 15 filtered daily values of mean sea level at Cananea and

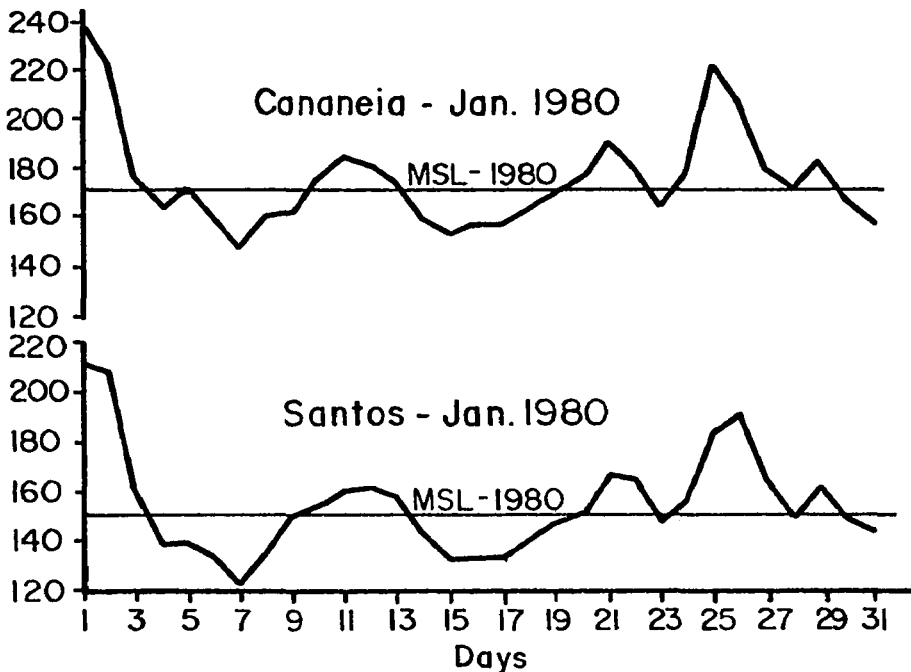


FIG. 2. — Filtered daily values of MSL for Cananea and Santos.

Santos from 1 December to 15 December and found 185.27 and 163.60, respectively. Since the MSL for 1980 at Cananeia was 172, the difference  $185 - 172 = 13$  was subtracted from 164. The result is 151 against 152, which is the MSL at Santos for 1980.

## 5. IMPROVING SOUNDING REDUCTION

Inspection of Figures 1 and 3 shows how the soundings can be influenced by the daily changes of the "mean" sea level. In fact, Figure 1, corresponding to Paranaguá, shows a mean sea level value, for 23 April 1975, 37 cm below the annual average of 203 cm and another one 39 cm above that average on 1 May. In the same way, Figure 2, corresponding to Santos, shows even larger differences on 1 January 1980. Consequently, if soundings are being collected at these times, corrections must include the mean sea level variations. Linear interpolation between consecutive daily values will be sufficient for all the work.

It is interesting to note that very often the daily changes in mean sea level are much larger than the monthly mean sea level displacement.

Since the reductions of the raw soundings are worked out in the office, it is very easy to remove the meteorological and oceanographical effects on the soundings.

In engineering work, especially in dredging, the removal of the meteorological effects is essential. In addition, the extreme low mean sea levels of another port can be transferred to the place of interest. Thus the lowest values of the MSL can be taken into account.

## 6. CONCLUSION

We believe that, if meteorological and oceanographical effects are not removed from soundings, the internationally adopted criterion to define the sounding datum plane has no meaning, since it takes into account the astronomical tide only. If the two phenomena are separated, any real time forecast of the meteorological tide will be much more meaningful.

It is interesting to note that the procedure is limited to a one-year period. In fact, the long term change of MSL is not taken into account. However, it is possible to assume that the change of MSL at the secondary station will follow the change at the main station.

From the above reasoning it is possible to show, for tidal stations influenced by similar meteorological and oceanographical effects, that the mean monthly sea level of a non-permanent tidal station can be referred to the long term mean sea level of a permanent tidal station, either by direct transference of the known monthly residuals, or by using the ratio of variation of daily mean sea level at the stations. Both methods led to maximum deviations  $\delta$  in the determination of the m.m.s.l. of 30% relative to the annual range, but an average absolute value of 0.5 cm was obtained for the deviation for the year.



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