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DIRECT INFERENCE OF TIDAL CONSTITUENTS

RESEARCH NOTE

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ABSTRACT

This paper suggests that the traditional methods for adjusting the results of a tidal analysis to allow for the effects of inferred constituents not included in the initial analysis are relics from the days of laborious manual computation. It proposes that the assumed relationships between inferred and reference constituents be incorporated into the initial analysis as conditions on the set of normal equations used in the least-squares solution. Conceptual and practical advantages of the proposed method are discussed.

1. INTRODUCTION

The writing of this paper was in part stimulated by the discussion by A.S. FRANCO (1985) of his experience in programming a micro-computer to perform tidal prediction. I too had purchased a Sinclair ZX81 micro-computer with 16 Kb of available memory and set myself the task of programming it to do tidal analysis and prediction. The analysis program presented the greatest challenge, but I succeeded in writing one that would accept any length of water level or current record (continuous or gapped), resolve up to 13 constituents, and infer as many additional constituents as desired. The ZX81 is too small and too slow to be recommended as a production tool for tidal analysis, but it served admirably as an aid for program development and testing.

The programming experience reinforced in me the belief that many tidal programs still employ some restrictive computational simplifications that had been necessary to facilitate manual computation, but serve only to limit the capability

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of an electronic computer. A particular example of this is the widely used method for adjusting, or "cleaning up", the results of a tidal analysis to allow for the effect of constituents that could not be separated from their neighbours in the length of record available. These are the constituents whose values must be inferred from the analysed constituents, using assumed amplitude and phaselag relationships between them. The inadequacies of this method of clean-up and inference are discussed below, and a method with which to replace it is recommended.

2. TRADITIONAL METHOD OF "CLEAN-UP" AND INFERENCE

In any method of tidal analysis, the inference of the harmonic constants of an unresolved constituent from those of a resolved constituent requires a reasonably accurate knowledge of the relationships between the constants (amplitude ratio and phaselag difference) in the region of observation. The traditional treatment of constituents to be inferred has been to ignore them completely in the initial analysis and then, using the assumed regional relationships, to calculate and remove from each reference constituent the contamination estimated to be due to the inferred constituents. The regional relationships are then used again to derive the constants of the inferred constituents from the adjusted constants of the reference constituents. A precise description of one version of this method is given by FOREMAN (1977). The method is not rigorously accurate since it involves some mathematical approximation and does not allow for possible contamination of constituents other than the chosen reference constituent : it is, however, sufficiently accurate when the record is long enough to permit resolution of all the major frequency bands and thus minimize the possibility of cross-contamination. As is the case with many approximate solutions, it is difficult to anticipate precisely the size of the error introduced under various operating requirements.

The method of clean-up and inference was evolved in the days of manual computation, when tidal analysis included many tricks and short-cuts designed to minimize the labour, and it was not at that time feasible to incorporate the inference conditions directly into the initial analysis. Now, however, computers can quickly and accurately solve the $2N + 1$ simultaneous equations involved in the rigorous least-squares evaluation of Z_o and N tidal constituents from a water level record. They are equally capable of incorporating the inference conditions directly into the solution if they are programmed to do so, and failure to take this step would seem to be an anachronism carried over from the pre-computer days.

3. PROPOSED DIRECT INFERENCE METHOD

The principles of tidal harmonic analysis by least-squares adjustment may be

found in many books (e.g. FORRESTER, 1983). If N constituents are considered adequate to represent the tidal signal, X(t), then we may write

$$X(t) = Z_o + \sum_{i=1}^{N} f_i H_i \cos (V_i + u_i + w_i t - g_i) \quad (1)$$

where the V_i are the Greenwich phases of the constituents in the equilibrium tide at time $t = 0$; f_i and u_i are the nodal parameters (amplitude and phase) of the constituents, usually taken at the mid-point and assumed constant over the record; w_i are the angular frequencies; H_i are the amplitudes; and g_i are the phaselags of the constituents at the observation site. For economy of space, we may define

$$E_i(t) = V_i + u_i + w_i t \quad (2)$$

so that, with the help of a little trigonometry, (1) becomes

$$X(t) = Z_o + \sum_{i=1}^{N} [f_i H_i \cos g_i \cos E_i(t) + f_i H_i \sin g_i \sin E_i(t)] \quad (3)$$

If the kth constituent is to be inferred from the jth constituent using the regional relationships

$$H_k/H_j = r_{jk} \text{ and } g_j - g_k = a_{jk} \quad (4)$$

then the combined contributions of the two constituents to the tidal signal X(t) may be written as

$$f_j H_j \cos g_j [\cos E_j(t) + (f_k/f_j) r_{jk} \cos (E_k(t) + a_{jk})] + f_j H_j \sin g_j [\sin E_j(t) + (f_k/f_j) r_{jk} \sin (E_k(t) + a_{jk})] \quad (5)$$

From (5) and (3) it is seen that the contribution of the inferred constituent may be incorporated into the least-squares solution simply by adding another term to the coefficients of $f_j H_j \cos g_j$ and $f_j H_j \sin g_j$ in the normal equations. If additional constituents are to be inferred from the jth constituent, it is necessary only to include the corresponding additional terms in the jth coefficients. Any number of constituents may serve as reference for one or more inferred constituents in the same analysis, as long as their coefficients are correspondingly modified. FOREMAN (1977) gives expressions for evaluating the initial term in the coefficients when the data are equally spaced. Since the additional terms are of the same form as the initial term, their contributions to the coefficients may be calculated in the same manner.

No mathematical approximation has been invoked here, and, since the inference conditions now form an integral part of the least-squares solution, contamination from the inferred constituents is removed from all other constituents to within the accuracy permitted by the precision of the assumed regional relationships. The final values for the reference constituents come directly from the analysis, and the values for the inferred constituents are deduced from these and the regional relationships in the usual manner.

4. DISCUSSION AND RECOMMENDATION

Because output from standard computer programs is often accepted without a full appreciation of the limitations in certain situations, it is important to make these programs as straightforward and as rigorous as possible. As part of a recent project I was required to derive estimates of the phaselag of M_2 from several records of only one or two days' duration. To accomplish this it was necessary to infer all other major semidiurnal constituents on M_2. The standard main-frame computer routines to which I had access either refused to carry out this task, or, which is worse, produced erroneous results. The reason for this lay in the inadequacy of their inference packages, which work fine on long records, but fail on very short records. Fortunately, my Sinclair micro-computer was able to perform the task, having been programmed to incorporate the inference conditions directly into the least-squares analysis, as described above in section 3. It is strongly recommended that any tidal harmonic analysis program which does not employ this direct inference method be altered to do so.

REFERENCES

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