INTRODUCTION

Most of the fixed production platforms in the North Sea and other oceans have been emplaced without any deterministic basis for monitoring the absolute height of deck datum above the seabed, nor for monitoring the relative air gaps between the underside of the deck structure and mean sea level. As a consequence, it proves difficult to provide detailed evidence of any historical changes in the vertical displacement of the platform since its original emplacement. Historical tilt measurements from inclinometers, which imply evidence of differential leg settlement, are insufficient to draw conclusions regarding the relative vertical settlement of the platform structure within the local seabed region.

By monitoring variations of the mean sea level using high precision tide gauges attached to subsea leg members, a method is readily available to establish the vertical control needed to monitor historical changes in vertical movement. The methodology is simple and inexpensive, but its relative accuracy and value is subject to the level of rigour exercised in the measurement, analysis, and interpretation of the tidal data. This paper outlines and explains the detailed stages of the method and derives a simple expression which might be used to standardize the practice.

OFFSHORE DATUMS

The position of an offshore structure in the global sense, in terms of easting and northing co-ordinates, can be established to a relatively high precision by satellite translocation (TRANSIT, GPS) within any of a number of geodetic frames of reference. The vertical co-ordinate, however, can only be defined in a very coarse way since it references the arbitrary centre of the appropriate geoid frame of
co-ordinates used. Consequently, it is necessary to define the vertical co-ordinate in terms of some local benchmark which is identifiably fixed in a temporal and spacial sense for that platform location. Presently, the most convenient benchmark is the mean sea level surface. Ideally, the best benchmark would consist of a network of seabed trig points providing the means for transferring horizontal and vertical control across extended seafloor regions interconnected to a land datum reference. This concept may be futuristic, but in the wake of hydroacoustic progress it will probably be achieved within the next decade.

**MEAN SEA LEVEL**

Mean sea level (MSL) is most commonly defined as the average height of the sea level surface measured with reference to some fixed datum. The period over which the average is taken varies from one month to one year, and the averaging is typically based on the arithmetic mean of hourly levels or of high and low water levels. This is an extremely coarse definition and inappropriate for describing the true nature of MSL.

MSL is a point measure of the mean sea level surface defined as the residual height of the sea levels after all oscillations of period greater than 1 cycle per day have been removed. These oscillations account for variations of the sea surface due to astronomical tides, waves and storm surges. As such, MSL contains the signature of seasonal, interseasonal and secular variations of the meteorological climate. It also contains low amplitude variations of quasi-tidal origin with periodicities ranging from a few weeks to many years. The term quasi-tidal is used because the astronomical tidal harmonics defined theoretically within the range of less than 1 cycle per day to 1 cycle per year fall within the direct influence of seasonal and interseasonal variations in meteorological conditions.

In the sense described here, the mean sea level surface should be viewed as an elastic surface which distorts slowly in all directions, the distortions being highly coherent within horizontal scale lengths of up to 100 km (or more) across open waters. The latter statement is an educated guess, since to date most researches concerning mean sea level variations have been based on coastal measurements, a view now being modified to incorporate deep sea offshore measurements to support the development of global ocean models.

The standard measure of MSL is based on the calendar monthly value of MSL, the annual value being the arithmetic mean of the 12-monthly values. These data are collated for worldwide coastal stations by the Permanent Service for Mean Sea Level (PSMSL), an international service under the auspices of FAGS and UNESCO, presently based at the Bidston Observatory, UK.

MSL can be expressed in the time series form:

$$m(t) = n_o + n(t) + c(t) + s(t) + r(t)$$

- $n_o$: average MSL height above some datum at some epoch $t = t_o$
- $n$: linear or non linear secular trends (typically of mm order per year)
- $c$: cyclic interseasonal variations (monthly and fortnightly)
s: seasonal variations (annual and semi-annual)
r: residual variations.

The series can be conveniently resolved by applying a suitable tidal filter to tide gauge measurements of the hourly sea surface elevation. In hourly form the series \( m(t) \) shows the progressive variation of MSL on a daily basis and its predominant response to mean wind and air pressure variations which would be synoptic over large open sea regions.

The variation of MSL within a 12-month period is dominated by two harmonic components of tidal origin, one with a period of 6 months (SSa) and one of 12 months (Sa). In the North Sea, Sa has an amplitude of the order 7-10 cm, the maximum being obtained around November, and SSa has an amplitude of the order 1-4 cm, the maxima being obtained around May and November. In European waters the theoretical equilibrium tide accounts for only about 10% of the observed values for Sa and SSa. Sa and SSa are synoptic in a regional sense and are either resolved by harmonic tidal analysis of one year of tide gauge records, or by regression analysis of several years of monthly MSL values. Because these components are fundamentally governed by seasonal meteorological effects, they will vary on an interannual basis and cannot be precisely defined in the same fashion as primary tidal harmonic components such as \( M_2 \) or \( S_2 \).

The variation of MSL within a 30-day period is affected by a number of harmonic components of tidal origin; the principal ones have a period of one month (Mm) and two weeks (MSf, Mf). These terms are generally far less significant than Sa and SSa but, because they are sensitive to interseasonal variations in meteorological effects, the terms cannot be strictly defined on the basis of an individual monthly analysis.

By reducing the hourly MSL series into calendar monthly mean values \( m_i \) (\( i = 1 - 12 \)) January to December, the daily variations are removed, allowing a simpler approximation of the form:

\[
m_i = n_0 + C_i + S_i
\]

\( C_i \): average of the monthly interseasonal variation
\( S_i \): monthly component of seasonal variation.

In the North Sea \( m_i \) will typically vary within a range of 20-30 cm over a 12-month period and the annual average will typically vary within a range of 1-5 cm on an interannual basis. If we assume platform settlement to take the simple linear form of \( \Delta \) mm per month, then for a gauge attached to a subsea member at the beginning of month \( i \), the mean sea level expression for month \( i \) becomes:

\[
m_i = n_0 + \Delta + C_i + S_i.
\]

The problem posed is to resolve the settlement trend \( \Delta \) from tide gauge records and to assess the amount of settlement which has accumulated since the platform was emplaced. The exact parallel problem is posed in connection with determining a control measure of platform deck air gap.
TIDAL ANALYSIS OF SEA LEVEL DATA

It is helpful if some remarks are made here to distinguish several important points regarding the analysis and interpretation of tidal data. Some of these points are overlooked in the growing practice of using the harmonic method of tidal analysis as a cookbook recipe for resolving a set of harmonic tidal constituents which are assumed to explicitly define the predictable variation and hence implicitly define the residual non-tidal variations.

The harmonic method of tidal analysis is designed to resolve a prescribed set of harmonic tidal constituents representative of the continuous tidal energy spectrum contained within the set of sea level measurements available. The prescribed constituents are based on theoretical tidal considerations and range in periodicity from one cycle per year to one cycle per hour. The long period end of the tidal spectrum is dominated by meteorological effects, and the short period end is dominated by shallow water non-linear effects. The number of harmonic constituents resolvable in a data set is determined by the length of the data set. In shallow coastal waters a typical set of 60-100 terms is required to adequately describe the tidal variations, whereas in deep offshore waters a set of less than 30 primary terms is often adequate. In the former case, a period of one year of data is required, and in the latter case, only 30 days. In both cases the harmonic analysis method, used in standard form, will automatically resolve a subsidiary set of quasi-tidal harmonic terms with periodicity slower than 1 cycle per day, and it will also compute the average value \( \Delta_0 \) of the residual differences remaining after minimising the least squares fit between the observed data and the synthesised harmonic data. \( \Delta_0 \) represents the mean vertical offset between the tide gauge measurement zero (e.g. pressure sensor datum) and the sea level surface, from which has been subtracted all variations induced by the full set of prescribed tidal harmonic terms resolved in the analysis. \( \Delta_0 \) is popularly assumed to mean MSL, and in terms of calendar month analysis, \( \Delta_0 \) is assumed to be the same as \( m_0 \).

In fact, this is not strictly correct, as suggested in the above discussion of MSL, and a less liberal approach must be followed to establish a consistency in the terminology relating to \( \Delta_0 \) and \( m_0 \), before the question of trying to isolate \( \Delta \) is tackled.

Recapping, sea levels \( h(t) \) can be expressed as:

\[
h(t) = m(t) + T(t) + \text{surge residuals}
\]

where \( m(t) \) is defined as before and \( T(t) \) is the tidal component consisting of harmonic terms starting within the diurnal energy band (1 cycle per day). Omitting secular trends and residual terms we can rewrite the expression as:

\[
h(t) = n_0 + c(t) + s(t) + T(t).
\]

In this form the long period quasi-tidal harmonic terms \( c(t) \) and \( s(t) \) are distinguished from the strictly tidal harmonic terms \( T(t) \). Direct harmonic tidal analysis of one year of \( h(t) \) will resolve the constant \( n_0 (= \Delta_0) \) and the full set of prescribed harmonic terms describing \( c(t) \), \( s(t) \) and \( T(t) \). It is popular practice to
presume that the \( c(t) \) and \( s(t) \) harmonics are explicitly part of the set of \( T(t) \) harmonics. For direct analysis of a calendar month \( i \) of \( h(t) \), \( A_0 \) data becomes equivalent to the compound value \( n_0 + S_i \) since the data length is insufficient to resolve the true harmonic terms \( S_a \) and \( S_{Sa} \) contributing to \( S_i \). Conversely, \( m_i \) is equivalent to \( A_0 \) plus the average value of repredicted heights based on using the few harmonic quasi-tidal terms describing \( c(t) \) for that month. This value, which is equivalent to \( C_i \), may or may not be significant in amplitude.

Harmonic analysis of the prefiltered data in the time series form \( h(t) - m(t) \) is simply equivalent to resolving a more precise description of the tidal harmonic terms representing \( T(t) \); e.g. the terms \( n_0 \), \( c(t) \) and \( s(t) \) are reduced to zero because they have been uncoupled from \( h(t) \) in the prefiltering process to isolate the MSL series \( m(t) \). Conversely, harmonic analysis of \( m(t) \) will recover the terms representing \( n_0 \), \( c(t) \) and \( s(t) \), and all tidal terms representing \( T(t) \) will be reduced to zero.

**DETERMINING VERTICAL CONTROL**

**FOR MONITORING SETTLEMENT AND AIR GAPS**

The above discussion serves to clarify the principle behind a relatively simple procedure which can now be established to offer a systematic method of maintaining vertical control for fixed offshore structures in the North Sea (and indeed elsewhere).

i) Assume a reference sea level surface defined as the height \( M_{n*} \) of annual mean sea level, for a historical year \( n \), above a fixed datum at a stable tide gauge location \( * \). \( M_{n*} \) could equally be taken as the average of a series of annual MSL values.

ii) For any year \( k \) close to year \( n \) we can harmonically analyse calendar months \( i \) of tide gauge data to resolve values of \( A_{n*,i} \) which can be expressed as:

\[
A_{n*,i} = M_{n*} + S_{ki}
\]

We make the reasonable assumption that annual MSL at a location represents a physically stable mean reference surface for that location over time scales of the order of 1-10 years. For example, any interannual variations of \( M_k \) with respect to \( M_n \) will be attributed to interannual anomalies in meteorological forcing influencing the monthly \( S \) terms.

iii) For an offshore structure \( o \), subject to a settlement rate equivalent to \( \Delta \) per month, we have in year \( k \)

\[
A_{o*,i} = M_{o*} + j \Delta + S_{o*,ki}
\]

where \( M_{o*} \) is the equivalent (to \( M_{n*} \)) sea surface reference for the location related to a structure datum at some time \( t_o \), and \( j \Delta \) is the accumulated vertical displacement of the structure datum (with respect to the \( M_{o*} \) sea reference plane) over a period of \( j \) months since \( t_o \). For convenience, we can take \( j = 1 \) for any first month of measurements made at location \( o \) and assume that \( M_{o*} \) is the appropriate sea reference plane at that time.
iv) If observations are initially made for some calendar month \( i \) in year \( k \) at locations * and o we have at the end of that month:

\[
A^*_k,i = M^*_n + S^*_k,i \\
A^o_k,i = M^o_n + \Delta + S^o_k,i
\]

By regional inference we can assume that \( S^*_k,i = S^o_k,i \) which gives the simplification:

\[
A^o_k,i = M^o_n + \Delta + (A^*_k,i - M^*_n)
\]

If we consider measurements made over a month, \( N \) months later we have:

\[
A^o_{k,i+N} = M^o_n + (N+1)\Delta + (A^*_k,i+N - M^*_n)
\]

Solving the two equations for \( \Delta \) and removing the suffix \( k \) we obtain:

\[
\Delta = \frac{1}{N} [(A^o_{i+N} - A^o_i) - (A^*_i+N - A^*_i)]
\]

This simple expression defines a systematic way of addressing the problem of vertical control at offshore structures. It also provides a means of detecting gross vertical settlement, and monitoring its change, over historical periods.

The expression assumes that the monthly MSL anomaly \( S_i \) is the same for both locations which is a reasonable assumption over offshore length scales of 10-50 km, but becomes less valid over longer length scales. A more rigorous attempt to scale the transfer of \( S_i \) over long distances results in a far more complex set of expressions which defeat the prime objective of simplicity and repeatability of the method.

Suitable data exist at UK east coast ports and at certain offshore structures to provide the primary information required to test the validity of the expression.

**CONCLUSION**

By giving careful consideration to the physical characteristic of MSL and by considering the essential aspects of the harmonic composition of tides, it is possible to derive a simple procedure which can be used to establish a method for the vertical control of offshore structures. The method and the associated expression for \( \Delta \) are based on two primary assumptions: (a) that there is little variation in the monthly MSL anomaly over a broad ocean region, and (b) that the reference gauge datum must be stable with respect to some selected annual MSL average. It is assumed that the repositioning of tide gauges on offshore structures is done with suitable precision, and that the calibration of gauges and analysis of data are undertaken to appropriate standards.

In the first instance, the closest coastal Standard Port, A Class tide gauge station, may suffice to establish the application of the method. However, a more appropriate and more correct application of the method can be achieved by establishing strategic reference stations directly on selected offshore structures known to be historically stable. Relative stability of an offshore structure, without
reference to a secondary station, can be determined to a high precision from analysis of one year of tide gauge measurements recorded at the structure.

The discussions above naturally lead one to recommend the establishment of primary gauge reference stations on fixed offshore structures. These may then be used to establish a standard and acceptable method for monitoring the vertical control of outlying structures. As well as meeting an important and immediate engineering information need, the historical accumulation of these data will be of immense value in advancing progress in our understanding of the true motion of the sea surface.

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