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CALCULATION OF EXACT POSITION USING INTERSECTION, RESECTION AND DISTANCES WITH LEAST SQUARES ADJUSTMENT

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ABSTRACT

This paper describes the algorithm of a computer program to obtain the compensated coordinates of a station using intersection, resection and distances, all together or independently.

The algorithm uses the variation of coordinates method to state the weighted observation equations and the solution is obtained by least squares adjustment. It takes an approximate point whose coordinates are computed by the program itself, with several hypotheses.

Finally, it also calculates the standard circular error.

1. INTRODUCTION

In triangulation problems, the surveyor is often confronted with problems due to ground features, and sometimes difficult access to certain stations, when determining coordinates of a point.

It is therefore very useful to utilize a system which can integrate together intersections, resections and distance measurements. Thus it is possible to obtain a high number of observation equations when only a limited number of known points are available.

Of course, the surveyor may also work with resections only, or with intersections and distances, distances only, or intersections only.

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2. VARIATION OF THE COORDINATES METHOD

This method renders it possible to obtain the coordinates of an unknown station geometrically and in a homogeneous way by using the maximum number of observations.

Based on knowledge of the approximate coordinates of the point, the system leads to the observation equation, whatever its nature :

$$\text{Approximate value} + \text{Variations} - \text{Observed value} = \text{Residual} \qquad (1)$$

The objective will be to minimize the residual; this is achieved by solution of the least squares which provides the corrections to be made to the approximate values used to obtain the definitive point.

Although the problem per se is simple, it remains arduous; yet, by using a computer, the task becomes relatively easy and quick.

In order to define the observation equations, we retain the variations of the measurements due to the coordinate variations, either by observation of directions or distances.

On each of the fixed points used one may have an intersection, a bearing and a distance measurement from it, which provides a maximum of three observation equations for each fixed point used for the observations.

3. THE PROGRAM

The program is prepared for interactive use on a console and computes step by step, the first of which is to obtain the approximate coordinates of the unknown point.

Then, it establishes the observation equations. These will always be redundant and thus greater in number than the number of corrections to be determined (unknowns).

Then comes the normalization of the equations by the least squares method, continuing with calculation of the corrections to be made to the approximate position and that of the standard circular error.

3.1 Approximate point

The program has been conceived to obtain the approximate coordinates in various alternative ways including by computation, in which case priority is given to using an intersection and a distance. In the second place it uses three distances instead of two, in order to eliminate ambiguity; this is valid in all cases. Two intersections follow and, if needed, three bearings. In the latter case, the provisional reference bearing is also computed.

The practical cases are thus always covered, otherwise the program considers that the observations are not sufficient to determine the exact position.

3.2 Observation Equations

The equations for directions (bearings) and those for distances do not have the same degree of usefulness and must be carefully determined.

The observations also allow for arc to chord corrections (dV or t–T) and the equations are established with a weight inversely proportional to the square of the standard error, considered, here, as a function of the distance.

3.3 Normalization of the equations

The equations always being redundant and having residuals, the system can be solved only by minimizing the sum of their square value, which is the purpose of the least squares method.

One thus gets a system of equations which can be put in matrix form. The solution will be easier since the matrix of the coefficients of the unknowns is symmetrical in relation to its diagonal.

3.4 Results

By solving the system of equations we obtain the corrections to be made to the approximate coordinates in order to get the coordinates of the point required.

In cases where there are bearings, one also gets the correction to the original provisional reference bearing, to obtain the exact bearing.

The calculation of the standard circular error is based on determination of the variance of the weight unit and that of the unknowns. Instead of considering the error ellipse we prefer to use the circular error defined by MIKHAIL (1976) which is more valid than the mean quadratic error. One thus obtains a measurement of an imprecise radius.

3.5 Checking the calculation

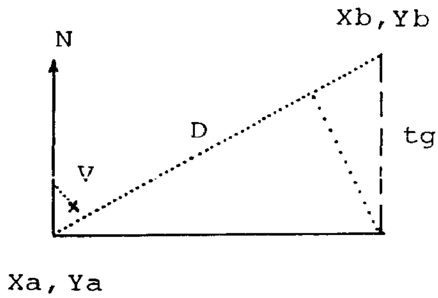
A verification can be made by determining the distance from the approximate position each sight passes. The program has been designed to consider a maximum of 50 cm, but this value may be changed by the operator on his console.

If an observed bearing passes more than 50 cm away from the approximate position, the program rejects it and re-calculates, but it always performs at least two iterations so as to be sure of the solution obtained.

Numerous tests have shown more iterations were not necessary, which proves that the method has only one solution.

4. OBSERVATION EQUATIONS

The variations have to be worked out from expression (1). They are obtained by the differential of bearing dV obtained from the expression which gives bearing V (Fig. 1) :



$$\mathrm{tg}\ V = \frac{Xb - Xa}{Yb - Ya} = \frac{\Delta X}{\Delta Y}$$

FIG. 1

and thus, after simplification :

$$dV = -\frac{\Delta Y}{D^2} \cdot dXa + \frac{\Delta X}{D^2} \cdot dYa + \frac{\Delta Y}{D^2} \cdot dXb - \frac{\Delta X}{D^2} \cdot dYb \qquad (2)$$

They may also be obtained by the differential of distance dD from the expression which gives distance D (Fig. 1) :

$$D = (Xb - Xa).\sin V + (Yb - Ya).\cos V$$

and thus, after simplification :

$$dD = -\frac{\Delta X}{D} \cdot dXa - \frac{\Delta Y}{D} \cdot dYa + \frac{\Delta X}{D} \cdot dXb + \frac{\Delta Y}{D} \cdot dYb \qquad (3)$$

In the case of resections, one deals with an original provisional reference bearing, Vo, the observed value of which must be corrected by a quantity, dVo, which has to be added, in expression (1), to the observed value, thus giving :

Variations + App. Value – (Obs. Value + dVo) = Residual.

dVo being another unknown, in that case, the expression will thus become :

Variations – dVo + (App. Value – Obs. Value) = Residual (4)

The coordinates of one of the two points (a) or (b) are known, which means that instead of four unknowns in expression (2) for the variations, we only have two. When we have resections, we have the unknown dVo as well. The maximum number of unknowns will thus be three and the equations will be of the following type :

$$Ai.X + Bi.Y + Ci.Z + Ki = Ri. \tag{5}$$

Such a system of equations has no solution as far as residuals are concerned, but the least squares method provides a solution by minimizing the sum of the squares of values of the residuals that is obtained by retaining the derivatives in relation to the unknowns which must be equal to zero, i.e. :

$$\frac{\partial \Sigma \eta^2}{\partial x} = 0 \qquad \frac{\partial \Sigma \eta^2}{\partial y} = 0 \qquad \frac{\partial \Sigma \eta^2}{\partial z} = 0 \tag{6}$$

The solution of equations (6) consists of a normalization of (5) and will result in a set of three equations with three unknowns, the solution of which will be :

$$\begin{aligned} &\Sigma Ai \times Ai \,.\, X + \Sigma Ai \times Bi \,.\, Y + \Sigma Ai \times Ci \,.\, Z + \Sigma Ai \times Ki = 0 \\ &\Sigma Ai \times Bi \,.\, X + \Sigma Bi \times Bi \,.\, Y + \Sigma Bi \times Ci \,.\, Z + \Sigma Bi \times Ki = 0 \\ &\Sigma Ai \times Ci \,.\, X + \Sigma Bi \times Ci \,.\, Y + \Sigma Ci \times Ci \,.\, Z + \Sigma Ci \times Ki = 0 \end{aligned} \tag{7}$$

Then, X and Y will be the corrections to apply to the approximate coordinates in order to obtain the exact position and Z will be the dVo correction to correct the provisional reference bearing at origin.

5. ACCURACY

In the program, the accuracy is represented by the circular error obtained from the standard errors in X and Y, (Sigma x and Sigma y). These come from the weightage and the determination of the unknowns which are obtained from the matrix calculation as given in expression (7).

6. INTEGRATION OF THE OBSERVATIONS

The joint use of intersections, resections and distances is made by using expression (1), the variations of which are provided by expressions (2) or (3); the approximate values are based on the calculation of approximate coordinates of the unknown point by bearings and distances respectively.

Regarding the resection, we get a 'reverse' bearing by adding the reading to the provisional reference bearing at origin.

The equation units must be respected : the programme uses circular units expressed in radians. The distance equations are suitably balanced in order to have a homogeneous system since the solution must be unique and, at the same time, assure its 'geometricity'.

7. CONCLUSIONS

The program is somewhat lengthy to take into account every hypothesis for the choice and calculation of the approximate position on the one hand and, on the other hand, to solve the system by matrix calculation while retaining the data for obtaining accuracy values.

As end product, we then have the coordinates of the exact position, the standard circular error and, finally, the exact original bearing in cases where bearings are used.

The program lists the input and the output data on a printer and, if one so wishes, all the intermediary calculations as well.

Being interactive, it is very easy to use and provides a very rapid and accurate solution.

8. ACKNOWLEDGEMENTS

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9. REFERENCES

MIKHAIL, E.M. (1976) : Observations and Least Squares. New York, Harper and Row.
FIADEIRO, P. (1983) : Complementos de Geodesia. Lisboa, Instituto Hidrográfico.

EXAMPLE

COORDINATES OF THE POINT — INSTITUTO HIDROGRÁFICO

STATIONS	COORDINATES	OBSERVATIONS	
POINT— 1 — AEROMAR	X = 120383.500 Y = 192315.290	Z = 279.415950 L = 0.000000	D = 9641.795
POINT— 2 — CHAMINE CUF	X = 118642.430 Y = 189545.730	L = 19.484100	
POINT— 3 — D.A.CORPO MAR.	X = 112278.210 Y = 188084.770	Z = 346.335200	D = 6019.802
POINT— 4 — CRISTO REI	X = 109654.540 Y = 190640.940	L = 100.401900	
POINT— 5 — RAPOSO	X = 108065.980 Y = 190044.860	Z = 35.503400 L = 116.083600	D = 4804.793
POINT— 6 — ESTRELLA	X = 110632.930 Y = 194455.370	L = 234.442200	

APPROXIMATE POINT

XA = 110879.545
YA = 193939.807

APPROXIMATE AZIMUTH

FROM POINT 1 TO THE ORIGIN = 99 DEGREES 41 MINUTES 59.50 SECONDS

ADJUSTED POINT

XC = 110879.522
YC = 193939.793
RMI = 0.013

ADJUSTED AZIMUTH TO ORIGIN = 99 DEGREES 41 MINUTES 58.36 SECONDS