

**NATIONAL OCEANIC
AND ATMOSPHERIC ADMINISTRATION
SEA BEAM SYSTEM 'PATCH TEST'**

by Lt. Daniel R. HERLIHY, NOAA (*), Lt. Bruce F. HILLARD, NOAA (*),
and Lt. Timothy D. RULON, NOAA (*)

Abstract

A procedure, commonly referred to as a 'Patch Test', has been developed by the National Oceanic and Atmospheric Administration (NOAA) National Ocean Service (NOS) to obtain correctors for Sea Beam system pointing errors and to verify system performance. The procedures described in this paper measure the biases associated with the fore and aft steering of the acoustic projector beam (pitch bias), the athwartship alignment of the received beams (roll bias), and the misalignment of the gyrocompass relative to the projector and receiver arrays (swath alignment bias). In addition, the repeatability of selected individual beams and the overall system is determined.

Verifying system performance before commencing survey operations is especially important with multi-beam sonar systems. Because of the depths in which they are operated, pointing and alignment biases can introduce significant systematic errors in both depth and position of multi-beam soundings.

Development of this procedure is a combined effort between NOS's Office of Marine Operations and Ocean Mapping Section (OMS). The procedure was developed for General Instrument Corporation (GIC) Sea Beam swath sonar systems configured to integrate sonar, navigation, and gyrocompass data into the data acquisition system and produce single-swath contour plots from the onboard data processing system. The general procedure is also applicable to the new NOS Intermediate Depth Swath Survey System currently under development, and other swath sonar systems capable of creating single-swath contour plots.

(*) NOAA/National Ocean Service, Office of Charting and Geodetic Services, Ocean Mapping Section, 6001 Executive Blvd., Rockville, Maryland 20852, USA.

I. INTRODUCTION

A procedure, commonly referred to as a 'Patch Test', has been developed by the National Oceanic and Atmospheric Administration (NOAA) National Ocean Service (NOS) to compute correctors for Sea Beam system pointing errors and to verify system performance. The procedures described in this paper measure the biases associated with the fore and aft steering of the acoustic projector beam (pitch bias), the athwartship alignment of the received beams (roll bias), and the misalignment of the gyrocompass relative to the projector and receiver arrays (swath alignment bias). The repeatability of selected individual beams and the overall system is also determined.

Patch Test procedures have been under development by the NOS since 1986 (WHEATON, 1988). A recent review of Patch Test data submitted to the Ocean Mapping Section revealed the need to refine, correct and standardize these procedures before bias corrections could be applied to survey data. In addition, computations of Patch Test parameters have been automated using spreadsheet software. This automation ensures the correct computation of parameters and provides a coherent history of system performance for all field units.

The Patch Test procedures assume that basic system performance characteristics such as projector power levels and receiver gain are operating within specifications. If not, these characteristics can generate similar error effects. NOAA bathymetric survey vessels assigned to map the United States Exclusive Economic Zone (EEZ) are required to run Patch Tests to confirm the correct operation of their Sea Beam systems prior to beginning each project, whenever hardware repairs or adjustments are made to a system, or anytime the correct operation of a system is in doubt.

II. TEST CONDITIONS

The following set of test conditions has been established by NOS to standardize Patch Test data acquisition and processing:

1. **TEST AREA:** approximately 5,000 metres square.
2. **BATHYMETRY:** a uniform slope in the range 15% to 25%, such that depth contour lines are relatively straight for a distance of at least two swath widths.
3. **DEPTH:** the deepest available in the test area consistent with the slope requirements; projected and received beam pointing errors are angular errors, the magnitude of the horizontal and vertical displacement of which are proportional to depth.
4. **TEST PATTERN:** consists of three sets of reciprocal lines, run in any convenient order (Fig. 1). Lines A and C are run perpendicular and upslope to the depth contours; lines B and D are run perpendicular and downslope to the depth contours; and crosslines E and F are run perpendicular to and bisect the

other four lines (Fig. 1).

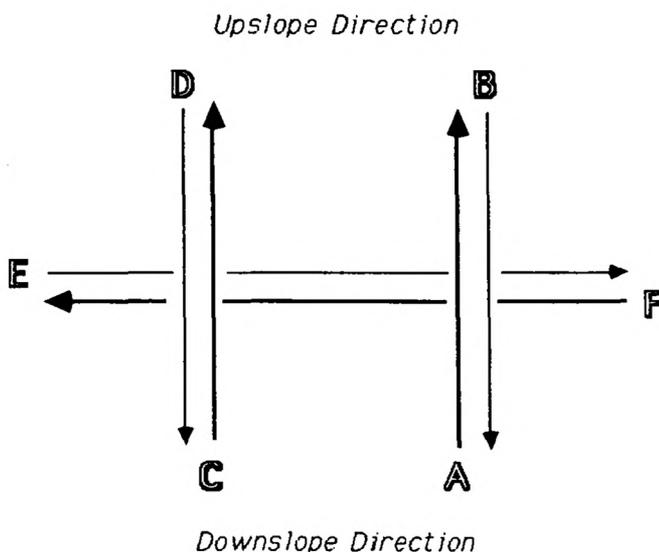


FIG. 1. — Patch Test line pattern.

5. **LINE LENGTH:** 4,000 metres \pm 100 metres.
6. **LINE SPACING:** arranged so that the outer beams (Beam 7) of the parallel survey lines (A/C & B/D) just overlap at the shallow end of the lines.
7. **SHIP'S SPEED:** the appropriate survey speed for the existing sea state.
8. **SEA STATE:** the minimum practical so as to produce the least sonar errors and the most consistent results among the various beams.
9. **POSITIONING:** radio navigation system with a required short term repeatability of \pm 15 metres.

III. DATA ACQUISITION AND PROCESSING

NOS currently operates two different Sea Beam data acquisition and processing systems: its original Digital Equipment Corporation (DEC) PDP-11 computer-based system and the newly developed DEC MicroVAX computer-based system. Although data acquisition and processing procedures vary between the two systems, the overall objective is the same, to create single-swath contour plots for comparison purposes:

1. Prior to data acquisition, sound velocity correctors are computed from CTD cast data and the primary navigation system is calibrated using a second shore-based navigation system or the Global Positioning System.

2. Data are acquired in the standard survey mode, producing a survey summary file and raw data magnetic tape (PDP-11 systems) or disk file

(MicroVAX systems).

3. For the PDP-11-based system, the raw data tape, which contains depths and navigation rates, and survey summary file are processed using a modified version of the Scripps Institute of Oceanography single-swath contouring package — programs PARGEN, DATGEN, and CONTUR. Sonar and navigation information from the raw data tape are combined and reformatted through program DATGEN and outputted as hard disk data files. Program PARGEN creates parameter files required by program CONTUR to plot single-swath contour plots from the DATGEN data files. The raw data are processed through DATGEN twice, once using three-ping averaging and once with no averaging.

4. For the MicroVAX-based system, the raw merged data file, which contains depths and positions, and survey summary file are first processed through program VAXCOP, to correct for sound velocity, navigation partial correctors, draft, and antenna/sonar array offset. The resulting corrected merged data file is run through a separate averaging program to create a three-ping averaged data set. Both the averaged and unaveraged data sets are plotted using a modified version of the CONTUR program.

5. Two sets of four contour plots are prepared from the processed data at a scale of 1:5,000. For each set, a separate plot is made of the upslope lines A and C, the downslope lines B and D, and for the crosslines E and F.

6. The first set of four plots, created from three-ping averaged data files, is used for estimating the beam and swath alignment biases alpha, beta, and gamma. Beam averaging provides a more legible contour plot for estimating systematic errors.

7. The second set of four plots, created from unaveraged data files, is used for estimating the percent root-mean-square (%RMS) repeatability of selected beams and for the overall system.

8. A standard set of plot parameters is used for all contour plots: 1:5,000 scale, 10-metre contour interval, pen color changes every one hundred metres, and darkened contour lines every even hundred metres.

IV. DATA ANALYSIS

The analysis of the Patch Test contour plots is performed in the following stages:

1. For this procedure, the beam pointing angles are defined as ALPHA for the pitch axis and BETA for the roll axis. The swath alignment angle is GAMMA. When the Sea Beam system is properly adjusted, ALPHA is set to zero degrees, BETA for the vertical beam of the swath (Beam 0) is set to zero degrees, and GAMMA is set to position the swath at a right angle to the ship's heading.

2. The lower case letters alpha (α), beta (β), and gamma (γ) represent pitch bias, roll bias, and swath alignment bias.

3. Alpha, beta, and gamma are estimated by overlaying contour plots of reciprocal lines, measuring the displacement of several sets of corresponding contours, and entering the displacement values into the respective bias equations.

4. Due to the complexity of the bias equations and the number of comparisons required, standardized LOTUS 1-2-3 spreadsheets are used to calculate average values for alpha, beta, and gamma. The spreadsheets also compute sample standard deviations as a measure of data quality and 90% confidence intervals as a measure of sample uncertainty (*).

5. %RMS repeatability estimates for selected beams and the overall system are computed as described above.

6. Computed values for alpha, beta, gamma, and %RMS are compared to the following NOS tolerances:

maximum alpha:	± 0.25 degrees
maximum beta:	± 0.25 degrees
maximum gamma:	± 0.75 degrees
%RMS (all beams):	0.6% of depth
%RMS (5 inner beams):	0.3% of depth

7. Bias correctors are determined for alpha, beta, and gamma and applied to survey data during post-processing.

V. PITCH BIAS (alpha)

Pitch bias (alpha) can occur due to physical or electronic misalignment of the vertical reference system (heave-roll-pitch sensor), which causes projected sound waves to be pointed forward or aft of vertical. The projected beam is electronically steered to vertical (ALPHA = 0 degrees) with a resolution of 0.1 degrees. A displacement of the transmitted beam due to pitch bias causes both depth and position errors (Fig. 2).

The pitch bias is estimated by overlaying the plots of the upslope lines A and C and downslope lines B and D, fixing the grids, and measuring the displacement between several sets of corresponding depth contour lines along the trackline. The vertical beam of the swath is assumed to be free of yaw and all but free of roll error.

(*) All of the estimates made herein are fundamentally and statistically uncertain to some degree. The interval estimate for the sample standard deviation is well known (WALPOLE and MYERS, 1972). A corresponding interval estimate could also be given for the %RMS error; however, these further refinements are considered not necessary and appropriate for this paper.

The estimation of averages (or mean values), variances, standard deviations, confidence intervals, etc., requires that the population be randomly sampled. For statistical purity, this requires that the sample be obtained by 'independent' experiments under essentially 'identical conditions' (WALPOLE and MYERS, 1972). The authors do not profess to have realized that purity here, but they do feel that their compromises are not unreasonable considering the huge investments necessary to run these tests in an ocean environment.

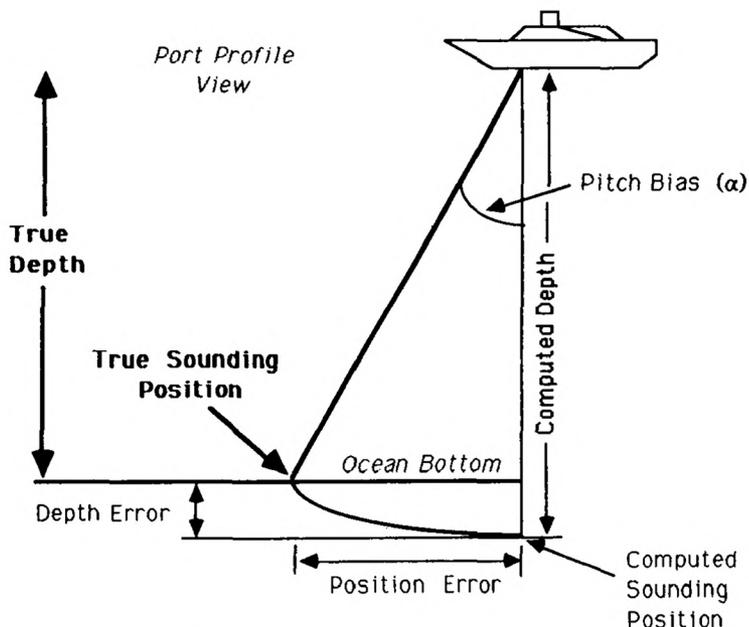


FIG. 2.— Forward pointing angle showing positive pitch bias.

Alpha is determined from the displacement 'x' in millimetres between corresponding depth contours, the scale factor of the plot, and the comparison depth

$$\alpha = \arctan [(x * \text{scale factor}) / (2 * \text{depth})] \quad (1)$$

The scale factor, equal to the plot scale divided by 1,000 in millimetres/metre, relates real-world distances in metres to plotting units in millimetres.

A more detailed explanation of the contour plot comparison procedure used to determine alpha is included (Appendix A).

VI. ROLL BIAS (beta)

Roll bias (beta) can occur during the reception of sonar echoes when there is a misalignment of the Sea Beam receiver array or the system's heave-roll-pitch sensor is in error. The receiver array offset is surveyed during installation and a roll constant is computed to adjust BETA to 0 degrees with a resolution of 0.05 degrees. As with pitch bias, roll bias can result in both depth and position errors (Fig. 3).

Roll bias is estimated by overlaying the plots of the upslope lines A and C and the downslope lines B and D. Shifting the plots to obtain a best match of

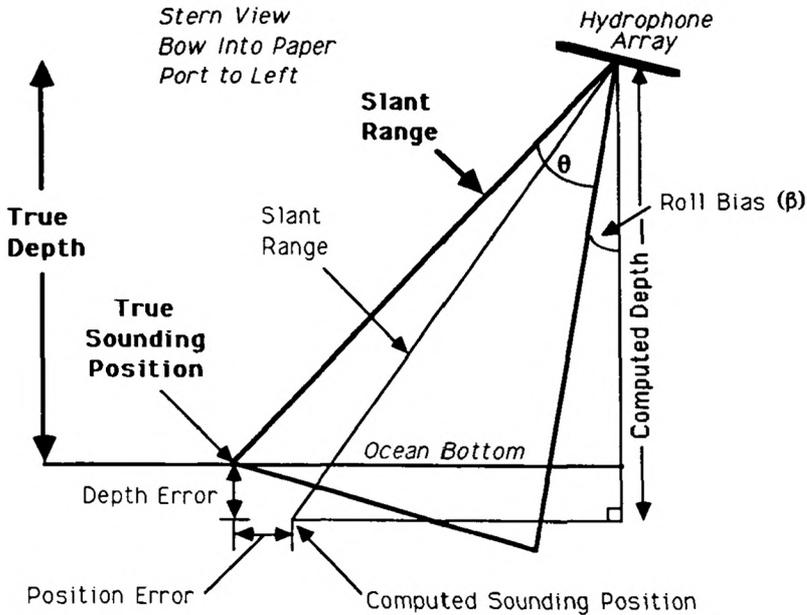


FIG. 3.— Port pointing angle showing a positive roll bias.

corresponding depth contours and keeping the grid lines parallel, the differences between several pairs of corresponding depth contours are determined at the outermost continuous beam.

Beta appears as a relative rotation of the contour lines about the intersection with the trackline. The overlaid contour pairs are free of gyro bias in that a constant gyro bias does not contribute to the relative rotation of the contour lines about the trackline for a reciprocal pair of lines. The effect of pitch is eliminated by moving the plots so that a match is obtained for corresponding depth contours at the vertical beam of the swath (along the trackline).

Beta is determined from the depth ratio in metres between corresponding depth contours (depth D_a /depth D_b) and the beam angle θ (beam $^{\circ} 2 \frac{2}{3}$ degrees for Sea Beam) of the outermost continuous beam where the depths are compared

$$\text{beta} = \arctan \left\{ \frac{1 - (D_a/D_b)}{[\tan \theta * (1 + (D_a/D_b))]} \right\} \quad (2)$$

Unlike the previously published equation for beta (WHEATON, 1988), the above equation is correct for all values of roll bias. A diagram (Fig. B3) and derivation of the new beta equation is included (Appendix B).

This test provides an indicator, not an accurate estimator, of roll bias. The sensitivity of the test is low; the depth error is only 6 metres per degree of roll for each 1,000 metres of depth. Multiple contour line comparisons are necessary to average the normal depth variations due to noise levels of the returned signals. If a roll bias is indicated by this procedure, a more precise GIC Sea Beam roll bias test is used to determine a new constant for the echo processor software. The GIC

test requires a flat test area, uses longer lines, and applies a statistical approach to minimize the effect of noise.

A more detailed explanation of the contour plot comparison procedure used to determine beta is included (Appendix B).

VII. SWATH ALIGNMENT BIAS (γ)

The swath alignment bias (γ) is the systematic bias in the misalignment of the gyrocompass relative to the projector and receiver arrays. Although swath alignment biases do not directly cause depth errors, the depths acquired correspond to incorrect positions (Fig. 4).

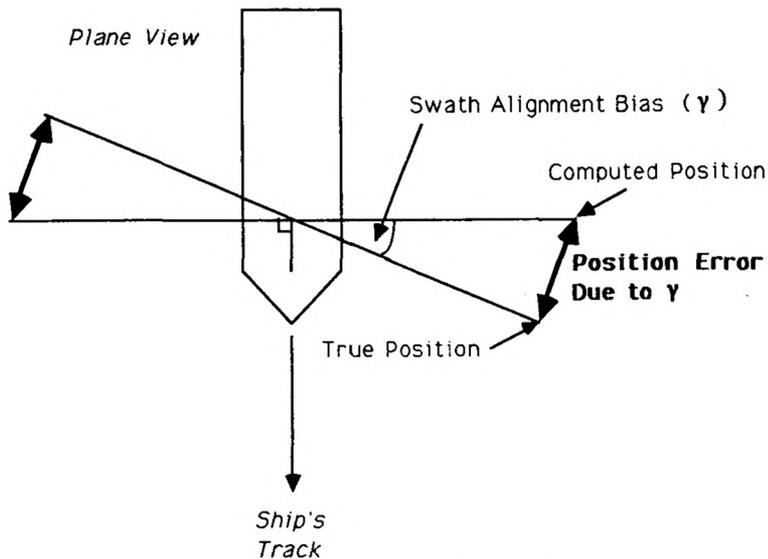


FIG. 4.— Positive swath alignment bias.

Gamma is estimated using each of the separate contour plots of parallel lines A/C and B/D, from measurements in the area between tracklines where corresponding depth contours should overlay. The horizontal displacement x in the direction parallel to the trackline of corresponding points (beams) on the contour lines and the crosstrack distance y from the points to the trackline are scaled to calculate gamma

$$\text{gamma} = \arctan [x / (2 * y)] \quad (3)$$

The effect of gamma is to rotate the contour lines about their intersection with the trackline. Where gamma exists alone, the same rotation appears in all four lines A, B, C, & D. Both pitch bias and roll bias distort the presentation of depth contours from parallel tracklines. By comparing contour plots of adjacent

lines in the same direction, such as A and C, pitch error is not apparent, but the effect of roll error is additive, so that the total observed line rotation is the sum of swath alignment and roll bias. On contour plots of lines run in opposite directions, such as A and D, pitch bias displaces contours along the trackline, but roll bias subtracts from the total observed line rotation.

A more detailed explanation of the x- and y-value measurement procedure used to determine gamma is included (Appendix C).

VIII. RMS REPEATABILITY ESTIMATES

Individual RMS repeatability estimates for selected beams and an overall RMS repeatability estimate are calculated to assess system performance. RMS repeatability estimates are calculated from depth discrepancies between corresponding depth contours, which are expressed as fractional errors.

The first estimate is formed from the contour plots of lines A and C and B and D, along selected lines parallel to the tracklines. The plots are overlaid so that the grids are fixed and eight depth comparison reference lines are drawn across the swaths (Appendix D — Fig. D1). These reference lines are to include the outermost and intermediate beams on the starboard side of line A, the center beams of both lines A and C, three evenly-spaced beams including the overlapped beams between the tracklines, and the outermost beam on the port side of line C. Twenty-five sets of corresponding contour lines are selected and compared along each of the eight reference lines for a total of 200 depth comparisons. The comparison procedure is the same as described for roll bias.

A second set of error estimates is made from the contour plots of lines A and C (or B and D) and the plot of crossline E (or F). The plots are overlaid so that the grids are fixed. Five depth comparison reference lines corresponding to the five inner beams of line A are drawn across the swath and parallel to the trackline of A (Appendix D — Fig. D3). Twelve sets of corresponding contour lines are selected and compared along each of the five reference lines for a total of 60 depth comparisons.

The %RMS error is computed for each of the 13 reference lines

$$\%RMS\ error = \sqrt{[\text{sum of squares} / (n - 1)]} \quad (4)$$

where the sum of the squares is equal to the sum of the %fractional errors squared, which is equal to the depth discrepancy between corresponding contours divided by 2, divided by the average depth and multiplied by 100, and n is the number of comparisons (*).

The NOS system repeatability tolerances are:

for the average of the first eight reference beams — 0.6% of depth;

for each of the five inner beams — 0.3% of depth.

(*) Dividing the depth discrepancy by two ($D_a - D_b/2$) is considered to provide the best estimate of the true depth. However, it could be argued that either depth D_a or D_b is in fact the true depth, in which case the % fractional error would be twice the magnitude stated in this paper.

A more detailed explanation of the comparison procedures used to determine %RMS errors is included (Appendix D).

IX. FUTURE CONSIDERATIONS

Patch Test procedures are not consistent with respect to the selection of comparison depths. Although the use of spreadsheet software removes computation errors, the scaling of distances and interpolation of depths from field plots introduce subjectivity to the procedures. Developmental work is underway to automate and improve the present methods so that time spent verifying system operation can be used to acquire more data.

A computer program is being developed to analyze digital Patch Test data. Overlaid depth values from reciprocal lines that lie within a given search radius of each other are compared and the results over all lines are statistically analyzed. An iterative process of fitting values for pitch bias, roll bias, and swath alignment then occurs until a best fit solution for all three values is determined.

Another procedure under development uses video-graphic techniques to graphically determine Patch Test parameters. Like the above method, digital Patch Test data are read by the computer and converted to x, y and z cartesian coordinates. Using a graphics terminal, video images of the lines are displayed on the screen. The user positions the images using commands or a joystick until the two are superimposed. When an optimum fit of the images is obtained, the values for alpha, beta, and gamma are displayed.

Although both of these methods will offer a significant improvement over the present procedures, the patch test methods described herein will be used as a control or check for new procedures.

X. References

- WALPOLE, R.E. and MYERS, R.H.: *Probability and statistics for engineers and scientists*, MacMillan Publishing Company, Inc., 1972.
- WHEATON, G.E., "Patch Test, A system check for multi-beam survey systems", Third Biennial U.S. Hydrographic Conference '88 Proceedings, pp. 85-90, April 1988.

APPENDIX A

Procedure for Determining Pitch Bias (α) Correctors

$$\alpha = \arctan [(x * \text{scale factor}) / (2 * \text{depth})]$$

where: x = alongtrack displacement between contours;

scale factor = plot scale / 1000 and

depth = the comparison depth.

1. Overlay the combined contour plot of lines A and C on top of the combined contour plot of lines B and D, keeping the grids fixed.
2. Choose 20 sets of corresponding contour lines to be compared between lines A and B.
3. Use a metric ruler to measure, to the nearest half-millimetre, the displacement between each set of corresponding contours along the overlaid tracklines of A and B (Fig. A1). The displacement is 'x' in the alpha equation.

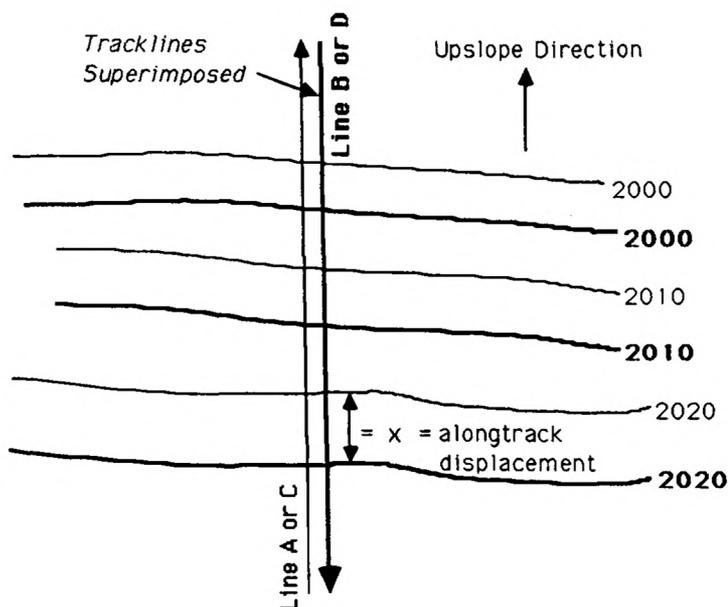


FIG. A1.— Overlaid contour plots for determining pitch bias, indicating a negative corrector.

4. Record each x displacement value for the comparison depth on a log sheet.
5. The sign of the displacement value x is determined by moving the plot of lines A and C over the plot of lines B and D until corresponding contours match. Moving the top plot in the direction of ship's travel represents a positive

x displacement and a forward pointing angle, requiring a positive alpha corrector. Moving the top plot in a direction opposite ship's travel represents a negative x displacement and an aft pointing angle, requiring a negative alpha corrector.

6. Enter the data for the 20 comparison depths into a LOTUS 1-2-3 spreadsheet to calculate the average value for alpha, the sample standard deviation, and the 90% confidence interval (Fig. A2).

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NATIONAL OCEAN SERVICE
NOAA SHIP DISCOVERER
SEA BEAM PATCH TEST
LINES A and B
SCALE 1: 5000
PITCH BIAS (alpha) DETERMINATION
SURVEY VERSION 10.0
DATE: 03-Feb-89
```

```
alpha = arctan [(x * scale factor) / (2 * depth)]
```

```
scale factor = plot scale/1000
x = alongtrack displacement between contours
depth = the comparison depth
```

POINT	depth (m)	x VALUE (mm)	alpha (degrees)
1	2000	2.5	0.179
2	2010	1.5	0.107
3	2020	3.5	0.248
4	2030	0.0	0.000
5	2040	2.0	0.140
6	2050	3.0	0.210
7	2060	3.5	0.243
8	2070	2.5	0.173
9	2080	4.0	0.275
10	2090	1.5	0.103
11	2100	2.0	0.136
12	2110	0.0	0.000
13	2120	3.5	0.236
14	2130	4.5	0.303
15	2140	1.5	0.100
16	2150	5.5	0.366
17	2160	5.0	0.332
18	2170	2.5	0.165
19	2180	1.0	0.066
20	2190	0.0	0.000

```
Average Line A/B alpha = 0.17 degrees
Sample Standard Deviation = 0.11 degrees
90% Confidence Interval = 0.17 +/- 0.04 degrees
```

FIG. A2.— Example of spreadsheet used to calculate pitch bias.

7. Repeat steps 2 through 6 for lines C and D.

8. Average the alpha values for each pair of survey lines to determine the overall pitch bias.

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* * *

APPENDIX B

Procedure for Determining Roll Bias (beta) Correctors

$$\text{beta} = \arctan \{ [1 - (D_a/D_b)] / [\tan \theta * (1 + (D_a/D_b))] \}$$

where: depth D_a = line A or D contour depth;
 depth D_b = line B or C contour depth and
 θ = beam[#] * 2 2/3 degrees.

1. Overlay the combined contour plot of lines A and C on top of the combined contour plot of lines B and D, to obtain a best match of corresponding contours along the trackline, keeping the grid lines of the two plots parallel.
2. Determine, by inspection, the outermost continuous beam (beam[#]) of lines A and B. This should be either the sixth or seventh beam from the vertical.
3. Use a straight edge to draw a reference line through the outermost continuous beam on the plot of line A, parallel to the trackline.
4. Choose 20 sets of corresponding contour lines to be compared between lines A and B.
5. Using a pair of ten point dividers, calculate the depth difference between each set of corresponding contours by holding line A (top) contours fixed and measuring the offset in the contours of line B (bottom) along the outermost beam reference line (Fig. B1).

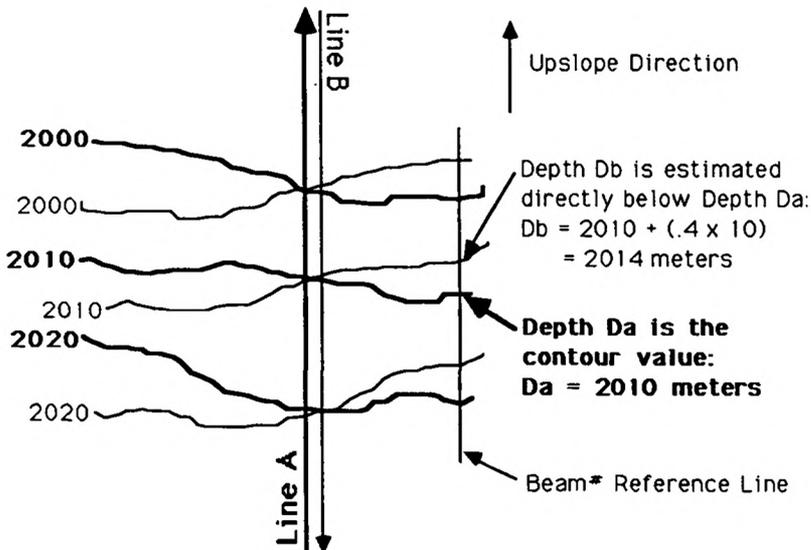


FIG. B1.— Overlaid contour plots for determining roll bias, indicating a positive corrector.

6. In the beta equation, line A contour depths correspond to D_a and line B contour depths correspond to D_b .

7. A positive beta represents a port pointing angle and a negative beta represents a starboard pointing angle.

8. Record each set of D_a and D_b values on a log sheet.

9. Enter the data for the 20 comparison depths into a LOTUS 1-2-3 spreadsheet to calculate the average value for beta, the sample standard deviation, and the 90% confidence interval (Fig. B2).

```

NATIONAL OCEAN SERVICE
NOAA SHIP DISCOVERER
SEA BEAM PATCH TEST
LINES A and B
ROLL BIAS (beta) DETERMINATION
SURVEY VERSION 10.0
DATE: 03-Feb-89
beam# = 6
theta = 16.00 Degrees

beta = arctan {[1 - (Da/Db)] / [tan(theta) * (1 + (Da/Db))]}

depth Da = line A or D contour depth
depth Db = line B or C contour depth
theta = beam# * 2 2/3 degrees

```

POINT	D_a	D_b	D_a/D_b	beta (degrees)
1	2000	2001	0.9995	0.050
2	2010	2012	0.9990	0.099
3	2020	2021	0.9995	0.049
4	2030	2034	0.9980	0.197
5	2040	2041	0.9995	0.049
6	2050	2053	0.9985	0.146
7	2060	2062	0.9990	0.097
8	2070	2075	0.9976	0.241
9	2080	2083	0.9986	0.144
10	2090	2093	0.9986	0.143
11	2100	2106	0.9972	0.285
12	2110	2115	0.9976	0.236
13	2120	2127	0.9967	0.329
14	2130	2133	0.9986	0.141
15	2140	2143	0.9986	0.140
16	2150	2152	0.9991	0.093
17	2160	2161	0.9995	0.046
18	2170	2175	0.9977	0.230
19	2180	2185	0.9977	0.229
20	2190	2194	0.9982	0.182

```

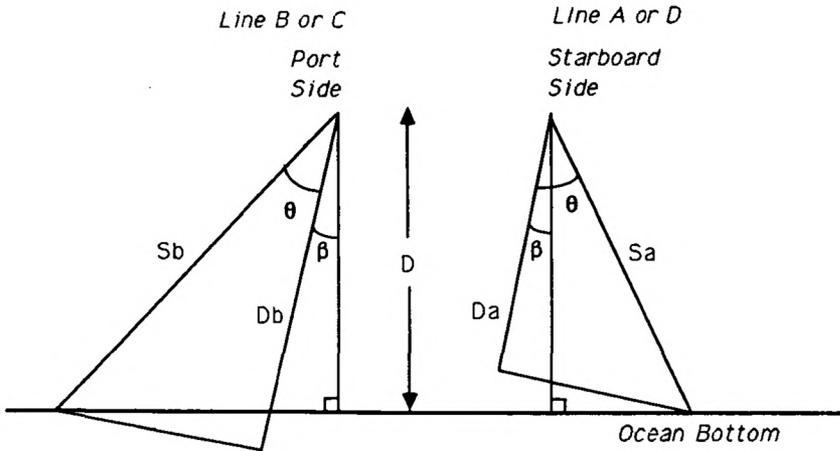
Average Line A/B beta = 0.16 degrees
Sample Standard Deviation = 0.08 degrees
90% Confidence Interval = 0.16 +/- 0.03 degrees

```

FIG. B2.— Example of spreadsheet used to calculate roll bias.

10. Repeat steps 2 through 9 for lines C and D. In the beta equation, line D contour depths correspond to D_a and line C contour depths correspond to D_b .

11. Average the beta value obtained for each pair of survey lines to determine the overall roll bias.



Legend:
 β = Roll Bias Angle (Port Side Up = $+\beta$)
 θ = Beam Angle
 D = True Depth
 D_a = Starboard Side Comparison Depth
 D_b = Port Side Comparison Depth
 S_a = Starboard Side Slant Range
 S_b = Port Side Slant Range
 $D_b - D$ = Port Side Depth Error
 $D_a - D$ = Starboard Side Depth Error

FIG. B3.— Diagram to derive roll bias equation showing an example of positive roll bias.

Roll Bias Formula Derivation

Assuming co-located comparison depths,

$$D = S_b * \cos(\theta + \beta) = S_a * \cos(\theta - \beta)$$

By substituting,

$$\begin{aligned} S_b &= D_b / \cos \theta \\ S_a &= D_a / \cos \theta \end{aligned}$$

$$(D_b / \cos \theta) * \cos(\theta + \beta) = (D_a / \cos \theta) * \cos(\theta - \beta)$$

Multiplying by $\cos \theta$ and cross multiplying,

$$\frac{D_a}{D_b} = \frac{\cos(\theta + \beta)}{\cos(\theta - \beta)}$$

Letting $x = D_a/D_b$ and using trigonometric identities,

$$x = \frac{(\cos \theta * \cos \beta) - (\sin \theta * \sin \beta)}{(\cos \theta * \cos \beta) + (\sin \theta * \sin \beta)}$$

Dividing numerator and denominator by cos β and cross multiplying,

$$x = \frac{\cos \theta - (\sin \theta * \tan \beta)}{\cos \theta + (\sin \theta * \tan \beta)}$$

$$\cos \theta - (\sin \theta * \tan \beta) = (x * \cos \theta) + (x * \sin \theta * \tan \beta)$$

Moving 'tan β' terms to left side of equation and factoring,

$$\tan \beta * [\sin \theta + (x * \sin \theta)] = \cos \theta - (x * \cos \theta)$$

Transposing terms and factoring,

$$\tan \beta = \frac{\cos \theta (1 - x)}{\sin \theta (1 + x)}$$

Resubstituting $x = Da/Db$ and solving for β,

$$\beta = \arctan \{ [1 - (Da/Db)] / [\tan \theta * (1 + (Da/Db))] \}$$

* * *

APPENDIX C

Procedure for Determining Swath Alignment (gamma) Correctors

$$\text{gamma} = \arctan [x / (2 * y)]$$

*where: x = alongtrack displacement between contours and
y = displacement perpendicular to the trackline.*

1. Use the combined contour plot of lines A and C.
2. Use a straight edge to draw a reference line through the center of the overlap between line A and C contour lines, parallel to the two tracklines.
3. The variable 'y' in the gamma equation is the distance in millimetres between the reference line and either of the two tracklines.
4. Choose 20 sets of corresponding contours to be compared between lines A and C.

5. Use a metric ruler to measure, to the nearest half-millimetre, the displacement between each set of corresponding contours along the reference line (Fig. C1). The displacement is 'x' in the gamma equation.

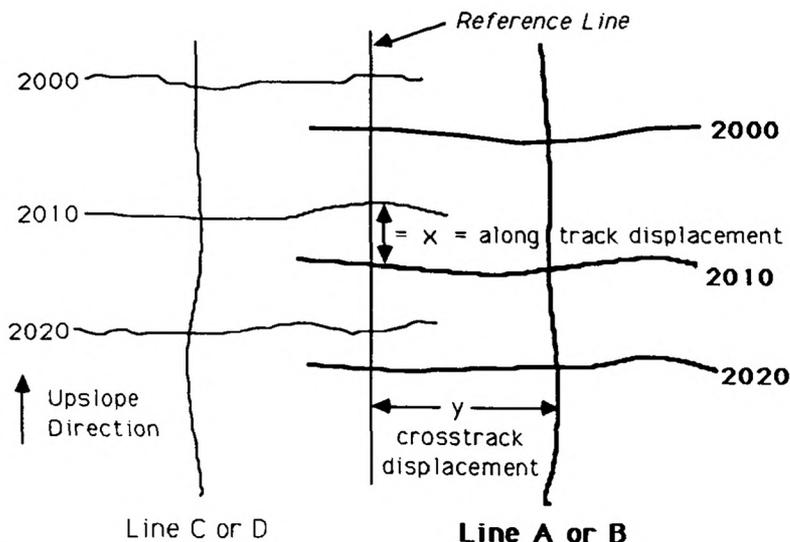


FIG. C1.— Contour plot for determining swath alignment bias, indicating a positive corrector.

6. The sign of the x is determined by the rotation of the contours required to bring corresponding contour lines together. For a clockwise rotation, x is positive, which represents a positive gamma corrector. For a counterclockwise rotation, x is negative, which represents a negative gamma corrector.

7. Record each displacement value for the comparison depth on a log sheet.

8. Enter the data for the 20 comparison depths into a LOTUS 1-2-3 spreadsheet to calculate the average value for gamma, the standard deviation, and the 90% confidence interval (Fig. C2).

9. Repeat steps 2 through 8 for lines B and D.

10. Average the gamma values obtained for each pair of survey lines to determine the overall swath alignment bias.

NATIONAL OCEAN SERVICE
 NOAA SHIP DISCOVERER
 SEA BEAM PATCH TEST
 LINES A and C
 SCALE 1: 5000
 SWATH ALIGNMENT ERROR (gamma) DETERMINATION
 SURVEY VERSION 10.0
 DATE: 21-Jan-89

$\text{gamma} = \arctan [x / (2 * y)]$

x = displacement parallel to the trackline
 y = displacement perpendicular to the trackline

POINT	DEPTH (m)	x VALUE (mm)	y VALUE (mm)	gamma (degrees)
1	2000	2.0	150.0	0.382
2	2010	1.0	150.0	0.191
3	2020	1.0	150.0	0.191
4	2030	3.0	150.0	0.573
5	2040	2.0	150.0	0.382
6	2050	1.5	150.0	0.286
7	2060	2.5	150.0	0.477
8	2070	1.5	150.0	0.286
9	2080	3.0	150.0	0.573
10	2090	2.0	150.0	0.382
11	2100	3.0	150.0	0.573
12	2110	2.5	150.0	0.477
13	2120	3.0	150.0	0.573
14	2130	2.0	150.0	0.382
15	2140	1.5	150.0	0.286
16	2150	1.0	150.0	0.191
17	2160	1.0	150.0	0.191
18	2170	3.0	150.0	0.573
19	2180	3.0	150.0	0.573
20	2190	2.0	150.0	0.382

Average Line A/C gamma = 0.40 degrees

Sample Standard Deviation = 0.15 degrees

90% Confidence Interval = 0.40 +/- 0.06

FIG. C2.— Example of spreadsheet used to calculate swath alignment bias.

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* *

APPENDIX D

Procedure for Determining System Repeatability (%RMS) Errors

$$\%RMS \text{ error} = \sqrt{[\text{sum of the squares} / (n - 1)]}$$

where: depth D_a = line A or C contour depth;

depth D_b = line B or D contour depth;

%fractional error = $[(D_a - D_b)/2 / (D_a + D_b)/2] * 100$;

sum of the squares = sum of the %fractional errors squared;

n = the number of comparisons.

1. For the first RMS error estimate, overlay the combined contour plot of lines A and C on top of the combined contour plot of lines B and D, keeping the grids fixed.

2. Use a straight edge to draw eight reference lines at selected beam intervals across the swaths and parallel to the two tracklines. The eight reference lines are to include the following: the outermost and intermediate beams on the starboard side of line A, the center beams of lines A and C, three evenly-spaced beams between the two tracklines, and the port side outermost beam of line C (Fig. D1).

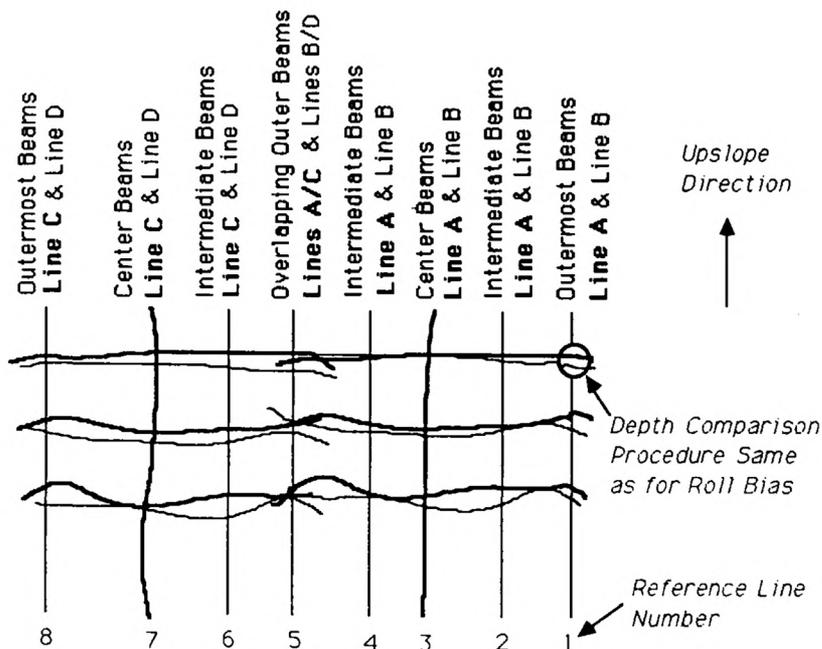


FIG. D1.— Overlaid plots for determining %RMS repeatability errors.

3. Choose 25 sets of corresponding contour lines to be compared for each of the eight reference lines (a total of 200 comparisons).

4. The procedure for determining the depth discrepancy at each comparison point is the same as for roll bias. In the Fractional Error equation, line A and C depths correspond to D_a and line B and D depths correspond to D_b .

5. Record each set of D_a and D_b values for each of the eight reference lines on blank log sheets.

6. Enter the data for the 25 comparison depths into a LOTUS 1-2-3 spreadsheet to calculate the average %RMS error for each of the eight selected beams (Fig. D2).

7. The overall %RMS error for the system is formed by averaging the data for all eight reference beams.

8. For the second set of RMS error estimates, overlay the combined contour plot of line E (or F) on top of the combined contour plot of lines A and C (or B and D), keeping the grids fixed.

NATIONAL OCEAN SERVICE
 NOAA SHIP DISCOVERER
 SEA BEAM PATCH TEST
 %RMS REPEATABILITY ERRORS
 SURVEY VERSION 10.0
 DATE: 21-Jan-89
 SCALE 1: 5000
 LINE A/C - LINE B/D COMPARISON
 REFERENCE LINE 1

%RMS error = SQRT [sum of the squares / (n-1)]
 depth Da = line A or C contour depth
 depth Db = line B or D contour depth
 %fractional error = [(Da - Db)/2 / (Da + Db)/2] * 100.
 sum of the squares = sum of the %fractional errors squared
 n = number of comparisons

	(1)	(2)	(3)	(4)	(5)	(6)
POINT	Da	Db	(Da - Db)/2	(Da + Db)/2	%fractional error (3)/(4)	(5)squared
1	2000	2002	-1	2001.0	-0.050	0.002
2	2010	2010	0	2010.0	0.000	0.000
3	2020	2022	-1	2021.0	-0.049	0.002
4	2030	2034	-2	2032.0	-0.098	0.010
5	2040	2042	-1	2041.0	-0.049	0.002
6	2050	2054	-2	2052.0	-0.097	0.009
7	2060	2065	-2.5	2062.5	-0.121	0.015
8	2070	2070	0	2070.0	0.000	0.000
9	2080	2082	-1	2081.0	-0.048	0.002
10	2090	2094	-2	2092.0	-0.096	0.009
11	2100	2102	-1	2101.0	-0.048	0.002
12	2110	2116	-3	2113.0	-0.142	0.020
13	2120	2123	-1.5	2121.5	-0.071	0.005
14	2130	2132	-1	2131.0	-0.047	0.002
15	2140	2142	-1	2141.0	-0.047	0.002
16	2150	2153	-1.5	2151.5	-0.070	0.005
17	2160	2162	-1	2161.0	-0.046	0.002
18	2170	2174	-2	2172.0	-0.092	0.008
19	2180	2183	-1.5	2181.5	-0.069	0.005
20	2190	2191	-0.5	2190.5	-0.023	0.001
21	2200	2202	-1	2201.0	-0.045	0.002
22	2210	2213	-1.5	2211.5	-0.068	0.005
23	2220	2220	0	2220.0	0.000	0.000
24	2230	2233	-1.5	2231.5	-0.067	0.005
25	2240	2241	-0.5	2240.5	-0.022	0.000
				sum of the squares =		0.117
				number of comparisons =		25
				%RMS error =		0.07

FIG. D2.— Example of spreadsheet used to calculate %RMS error of the eight selected beams.

9. Use a straight edge to draw five reference lines across the swath, corresponding to the five inner beams of line A (Fig. D3).

10. Choose 12 sets of corresponding contour lines to be compared between lines A and E for each of the five reference lines (a total of 60 comparisons).

11. Repeat steps 4 through 6 for the five reference lines. A LOTUS 1-2-3 spreadsheet is also used to calculate the average %RMS error for each of the five lines (Fig. D4).

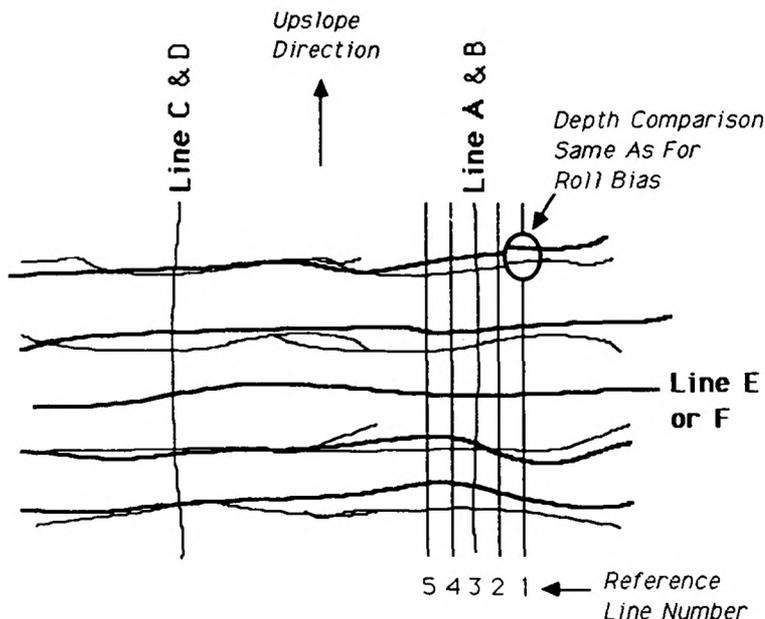


FIG. D3.— Overlaid plots for determining %RMS repeatability errors.

NATIONAL OCEAN SERVICE
 NOAA SHIP DISCOVERER
 SEA BEAM PATCH TEST
 SYSTEM REPEATABILITY ERROR ESTIMATES
 SURVEY VERSION 10.0
 DATE: 21-Jan-89
 SCALE 1: 5000
 LINE A/C - LINE E/F COMPARISON
 REFERENCE LINE 1

$$\%RMS\ error = \sqrt{\text{sum of the squares} / (n-1)}$$

depth Da = line A or C contour depth

depth Db = line B or D contour depth

$$\%fractional\ error = [(Da - Db)/2 / (Da + Db)/2] * 100.$$

sum of the squares = sum of the %fractional errors squared

n = number of comparisons

	(1)	(2)	(3)	(4)	(5)	(6)
POINT	Da	Db	(Da - Db)/2	(Da + Db)/2	%fractional error (3)/(4)	(5)squared
1	2000	2002	-1	2001.0	-0.050	0.002
2	2010	2010	0	2010.0	0.000	0.000
3	2020	2022	-1	2021.0	-0.049	0.002
4	2030	2033	-1.5	2031.5	-0.074	0.005
5	2040	2042	-1	2041.0	-0.049	0.002
6	2050	2053	-1.5	2051.5	-0.073	0.005
7	2060	2062	-1	2061.0	-0.049	0.002
8	2070	2070	0	2070.0	0.000	0.000
9	2080	2082	-1	2081.0	-0.048	0.002
10	2090	2094	-2	2092.0	-0.096	0.009
11	2100	2101	-0.5	2100.5	-0.024	0.001
12	2110	2112	-1	2111.0	-0.047	0.002

sum of the squares = 0.035
 number of comparisons = 12
 %RMS error = 0.06

FIG. D4.— Example of spreadsheet used to calculate %RMS error of the five inner beams.