International Hydrographic Review, Monaco, LXVII(1), January 1990

ACCURACY EVALUATION OF POLAR POSITIONING SYSTEMS TAKING POLARFIX (*) AS AN EXAMPLE

by Ingo HARRE (**)

1. INTRODUCTION

Positioning systems measuring in polar co-ordinates, with automatic target tracking capability have become increasingly popular in hydrographic surveying. In comparison to traditional positioning systems using range-range or hyperbolic intersection, they provide better accuracy, higher reliability of the measurement, the possibility of direct water level corrections in the area of the survey, and the operational advantage of a single shore station.

This paper briefly describes the ATLAS POLARFIX system, which was the first laser based automatic positioning system for hydrographic application when it was introduced into the market in March 1983.

POLARFIX has been in use for more than six years and is in operation worldwide. During this period of time, the equipment has been tested by various institutions with regard to its applicability and accuracy of measurement. The results of these tests are partly available with large volumes of measurement data, in the respective reports. In many tests, the positional errors have not been integrated into circular errors, thus different presentations of the results may easily lead to wrong interpretations. The theory concerning positional errors is, however, but little known. For this reason, it is summarized in this text. Using this theory, the various POLARFIX test results will be converted to defined accuracy standards, thus making them comparable. The author establishes an error budget, produces an error law from the test results and compares both with the manufacturer's specification.

(*) The ATLAS POLARFIX System has been replaced by the ATLAS POLARTRACK, the description of which is being included in the IHO publication SP-39, in preparation.

(**) Krupp Atlas Elektronik GmbH, Postfach 44 88 45, D 2800 Bremen 44, FRG.

2. POLARFIX

If we perceive the task of hydrographic surveying as the determination of the coordinates of a point on the ground, by means of a system for horizontal positioning and another system for measuring the depth, then it is desirable that there are similar degrees of accuracy and resolution for both dimensions. Especially in surveying in-shore and near-shore waters, the traditional positioning systems could, by far, not match the accuracy of echo sounding. The reliability of data supplied by positioning systems using electromagnetic waves is not only limited by the wavelength and pulselength of the parameters but also by the influence of geometry (intersection angles) and propagation conditions (e.g. multi-path effects). Recognizing these facts, POLARFIX has been developed to overcome these limitations or at least to provide improved performance.

The POLARFIX is a range-azimuth positioning system using a laser beam for range measurement and a shaft encoder for angle measurement. The angle is referenced to a base line established between the position of the POLARFIX shore station and the position of a visible reference target. The survey vessel is fitted with a ring of 12 reflecting prisms, the range and azimuth of which is measured, and transmsitted by a UHF radio link from the shore station to the POLARFIX survey processor aboard the vessel (see Fig. 1). Within this unit, the received polar coordinates of the vessel are converted into rectangular X/Y coordinates, and displayed. At the site of the POLARFIX shore station, the actual water level can be measured by the optional water level gauge. The level is then also transmitted to the vessel for reducing the measured depths by further processing.

A survey echo sounder of the type Atlas Deso 20 or Atlas Deso 25 can be connected to the POLARFIX survey processor. Each single depth sounding can be associated with a position by means of a predictive filter interpolating position between the fixes, which are received at a rate of two per second.

The POLARFIX Survey Processor contains software for track-line navigation with the capability of storing 99 fixed points. Two of these points suffice to define a basic profile line to which a multitude of parallels can be created. More points will be needed to create 'chain' profiles. Up to 25 maps can be stored in the processor, each defined by the lower left edge coordinates, grid turning angle and grid width. With the help of the Navigation Display Unit, which among other information, displays track offset information (left/right indication) and profile length information (distance-to-go), the helmsman can steer precisely along the predetermined profile line (see Fig. 2). If a plotter is connected, an on-line track plot, a depth figure plot (see Fig. 3) or a depth profile plot can be created. The POLARFIX with all its peripherals for hydrographic surveying is outlined in Figure 4.

According to its specifications, the POLARFIX has a specified range of 3000 m in IEC laser class I and 5000 m in class III a. Under normal visibility conditions, the ranges are actually much greater. The shore station normally operates from a built-in 12 V rechargable battery, the survey processor from a 24 VDC source.

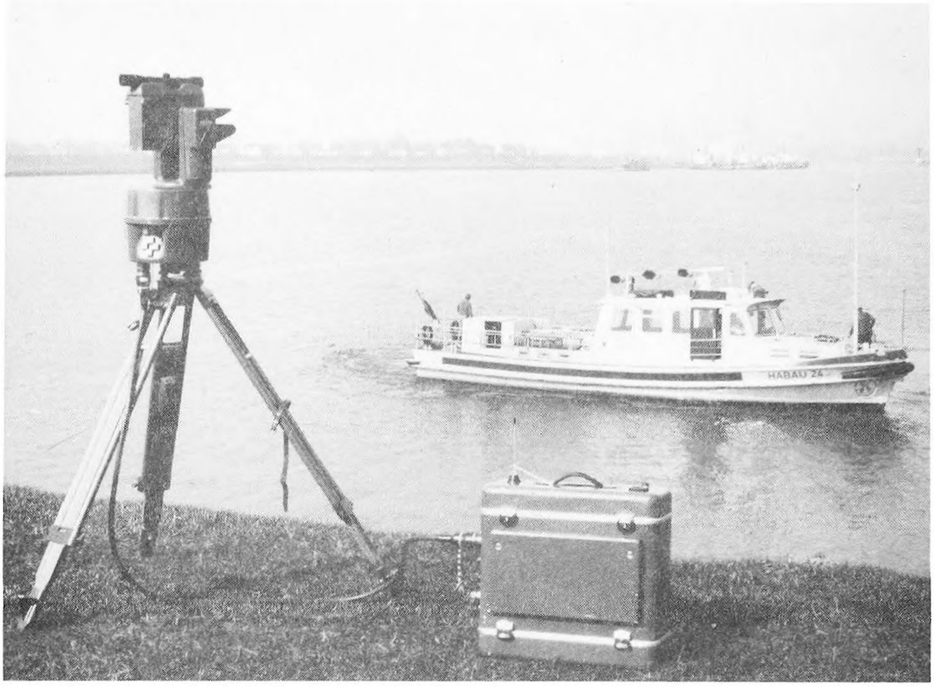


FIG. 1.— The POLARFIX polar positioning system.

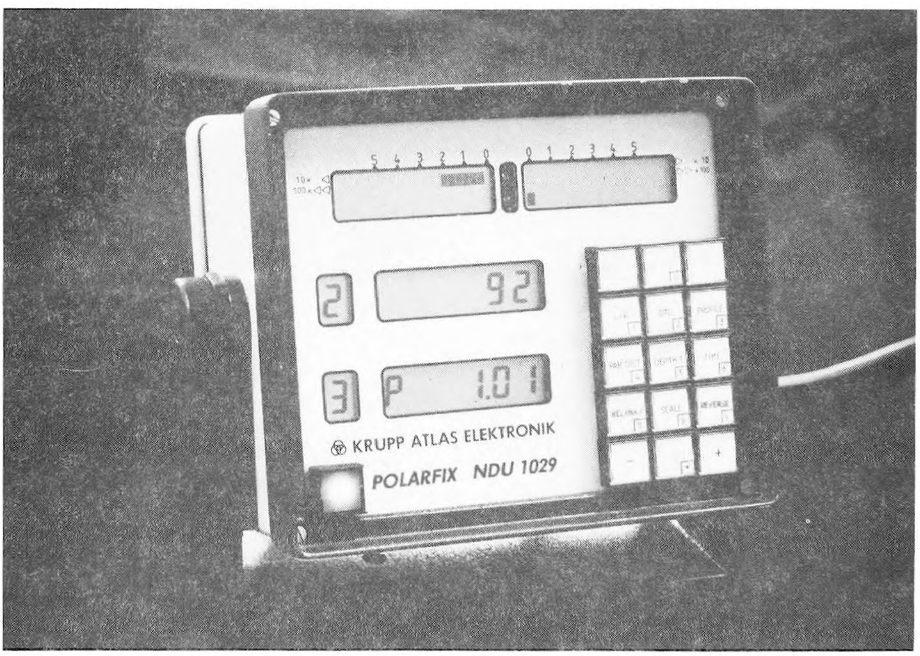


FIG. 2.— The Navigation Display Unit of the POLARFIX system.

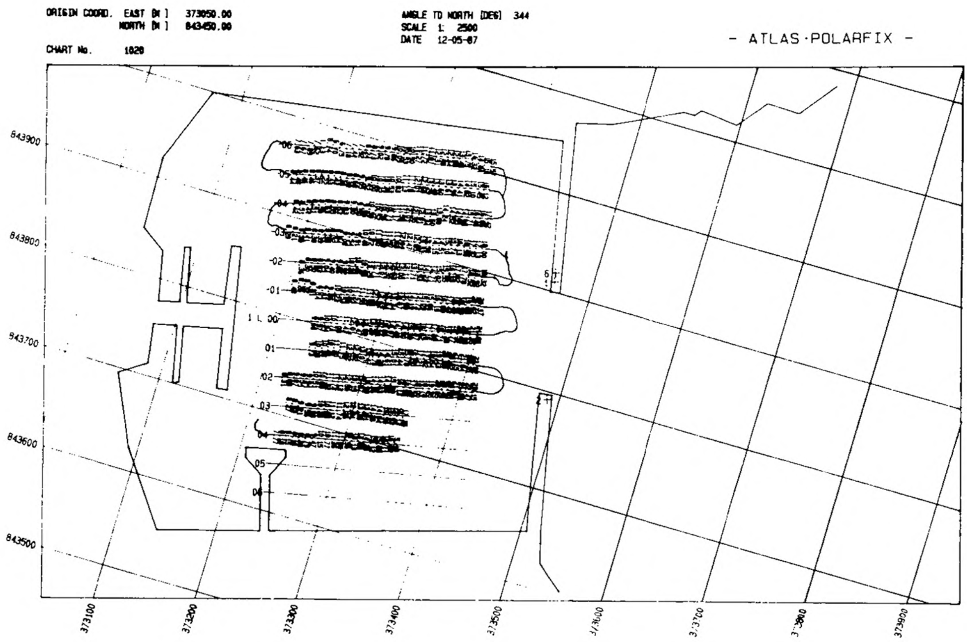


FIG. 3.— Survey of the port of Monte-Carlo, May 1987.

3. MEASURES OF ACCURACY

The accuracy analysis of position determinations on a plane is a two-dimensional statistical problem. The Gauss error theory can be applied initially to both primary measured quantities of polar position-finding, namely the range and the angle relative to the baseline. As a result of a large number of observations, one obtains, in this way, the mean value and the standard deviation for the two measured quantities. How can a measure of the *position* accuracy now be derived from these values? Most of the relevant source literature comes from the USA, partly from military research (see ref. [1] and [2]). In such literature, the accuracy of navigation methods is mostly described with the aid of the error circle dRMS or, less frequently, the error circle CEP (Circular Error Probabilities) (see ref. [3], [4]). Both types of error circle data are determined solely from the statistical error, characterized by the standard deviations; the systematic error, characterized by the arithmetic mean error, is not taken into account. This procedure is justified because the systematic error normally depends on the individual measuring equipment and the series of measurements concerned; theoretically, it can be reduced to zero by calibration of the measuring equipment. In practice, however, a residual systematic error is always present no matter how much effort is put into calibration. The theory of these error measurements, supplemented by some approximation formulae, has recently been presented

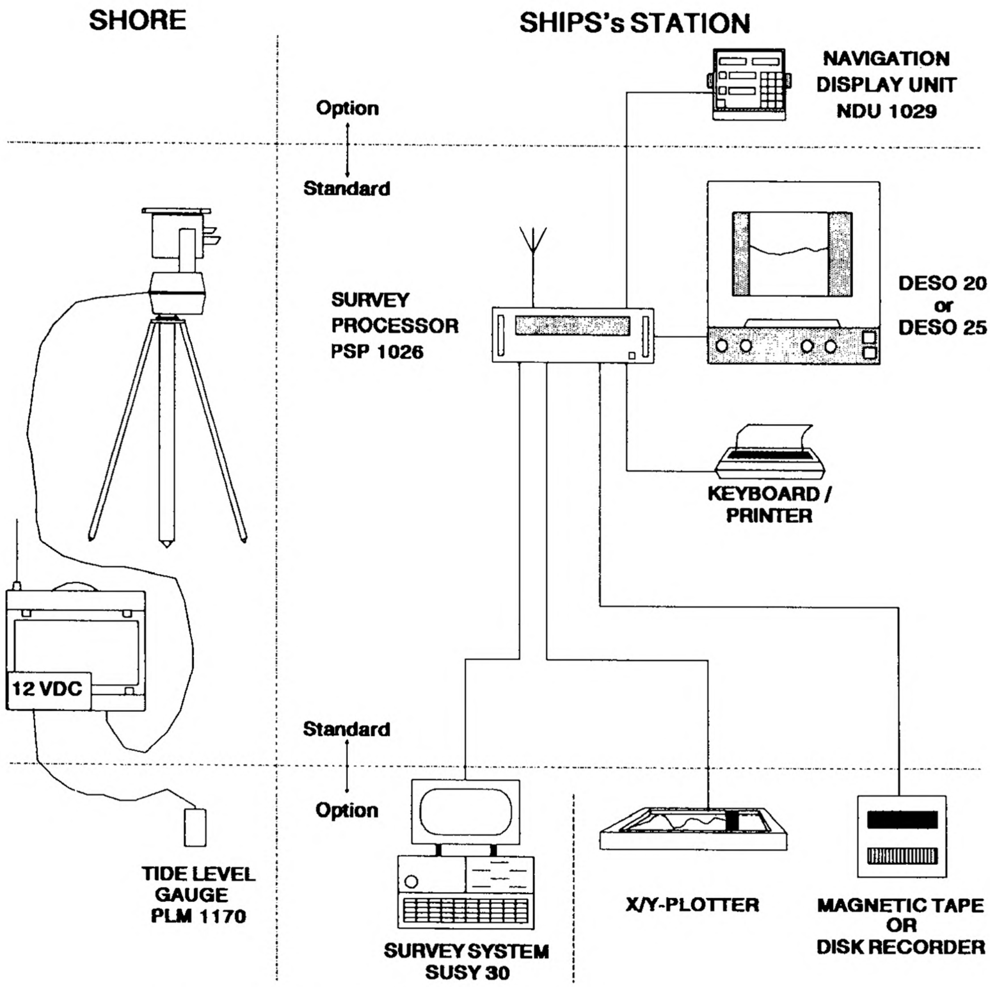


FIG. 4. — The POLARFIX with peripherals, forming a stand-alone survey system.

in a paper by the author (see ref. [5]), and will be described in the following only insofar as is necessary.

3.1 The Error Ellipse

The classical description of two-dimensional position errors is the error ellipse. In the case of polar position-finding, its semi-axes are defined by the standard deviations of the range measurement and azimuth measurement. At close range, with this kind of position-finding method, the variance of the range measurement is generally larger; at long range, the variance of the angular measurement is generally larger (Fig. 5). At a particular range, the two standard deviations are equally large, and at this point the error contour is a circle. If the probability density functions belonging to these standard deviations are plotted in

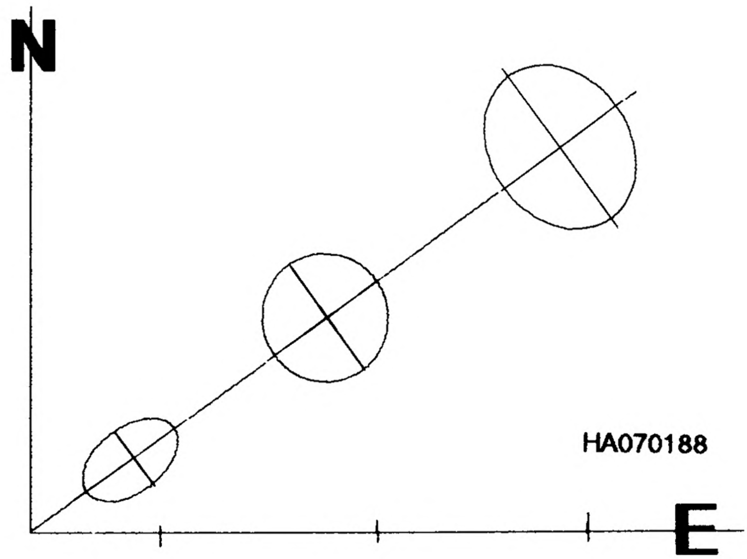


FIG. 5.— Shape of the error ellipses in the case of polar position finding.

a three-dimensional coordinate system, with the probability density represented by the vertical axis, a surface in the form of a mound is obtained. The probable position is situated vertically below the apex (Fig. 6). In the general case, all

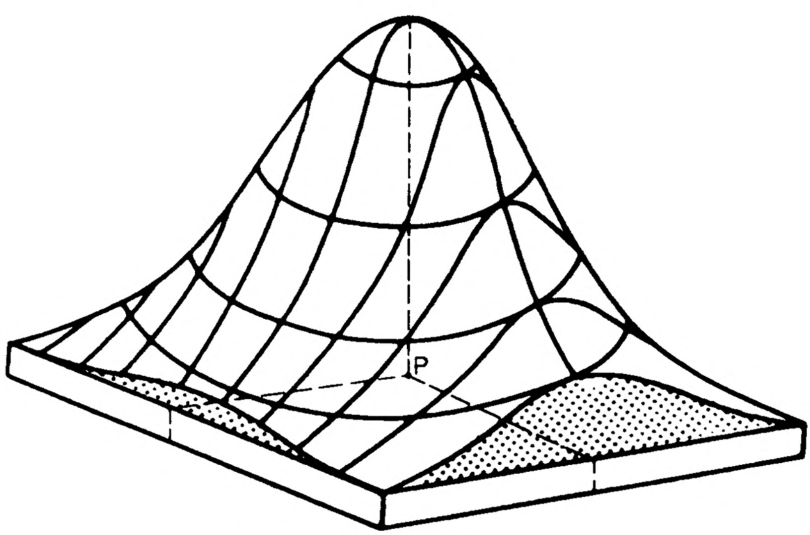


FIG. 6.— Surface of the two-dimensional Gauss distribution.

height contours are ellipses and represent points of equal probability density. The ellipse having semi-axes corresponding exactly to the standard deviations is called 'error ellipse'. The probability that the actual position is enclosed by the height contour ellipse is represented by the ratio of the volume of the column bounded by the ellipse to the volume of the entire mound.

For ellipses having semi-axes which are k times greater than the standard deviation, the following probabilities apply:

$$k = 1 => p = 39.9\%$$
$$k = 2 => p = 86.0\%$$
$$k = 3 => p = 98.9\%$$

In order to describe the size and orientation of the error ellipses, at least three parameters are required. An error contour which can be described with only one parameter, and which is therefore easier to apply in practice, is the circle.

3.2 Distance Root Mean Square (dRMS)

One approach to the definition of an error circle is based on the law of statistics which states that the variance of the sum of independent random quantities is equal to the sum of their variances. The following error circle radius is thus defined:

$$1 \text{ dRMS} = \sqrt{\sigma_u^2 + \sigma_v^2} \tag{1}$$

This error measure describes the radius of an error circle round the measured position; the radius is determined as follows from the set of measurement data obtained from the series of measurements: if the mean values and standard deviations occur, for example, in a rectangular x-y coordinate system, the measurement errors.

$$\Delta x_i = x_i - \overline{x}\ ,\ \Delta y_i = y_i - \overline{y} \tag{2}$$

are formed from the individual measurements. The standard deviations (variances) must be formed from mutually independent random quantities or measurement errors respectively. The measure of correlation of the measurement errors is expressed by the correlation coefficient Rho, which is calculated as follows:

$$\rho = \frac{\sum_{i=1}^{N} (\Delta x_i \cdot \Delta y_i)}{\sqrt{\sum_{i=1}^{N} (\Delta x_i)^2 \cdot \sum_{i=1}^{N} (\Delta y_i)^2}} \tag{3}$$

Uncorrelated data can be generated by rotating the coordinate system in the mathematical direction of rotation through an angle of

$$\Theta = \frac{1}{2} \text{arc tan} \frac{2\rho\sigma_x \cdot \sigma_y}{\sigma_x^2 - \sigma_y^2} \tag{4}$$

As a result of the following coordinate transformation, the systematic error portion (calibration error) simultaneously becomes zero, and the uncorrelated, random errors remain:

$$\begin{pmatrix} u_i \\ v_i \end{pmatrix} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} \Delta x_i \\ \Delta y_i \end{pmatrix} \tag{5}$$

From the new data, it is then possible to calculate the standard deviations for determination of the 1 dRMS error circle. If double the standard deviation is used for the calculation according to formula (1), the error radius 2 dRMS is obtained. The calculation of the dRMS error circles is comparatively simple. Unfortunately, in contrast to the one-dimensional case, it is not possible to assign a fixed probability to these error measures, as shown by BURT (see ref. [3]). Depending on the ratio of the smaller standard deviation to the larger one, the following probability intervals are applicable:

$$1\ \text{dRMS} => p = 68.3 \ldots 63.2\%$$
$$2\ \text{dRMS} => p = 95.4 \ldots 98.2\%$$

The values 63.3% and 98.2% apply to the special case in which both standard deviations are equally large. The error contour is then a circle; this error distribution is called a 'Rayleigh distribution'. Its probability content is easy to calculate.

3.3 Circular Error Probability (CEP)

The CEP error circles define the position-error with fixed probabilities. The theoretical approach is based on the two-dimensional probability density function (which describes the surface of the mound). In the same way as in the one-dimensional case, its (double) integral describes the cumulative probability. If a circle is defined as the error contour, the volume proportion situated inside the circularly bounded column in the mound corresponds to the cumulative probability. If a particular probability level is required, this leads to the radius of the circle. Unfortunately, there is no analytical solution for the double integral. HARTER (see ref. [4]) has integrated and tabulated the integral numerically over circular contours. Error circles with the probabilities 50, 66, 75, 90 and 95% can be calculated by means of the following auxiliary functions:

$$C = \sigma_{small} / \sigma_{large} \tag{6}$$

$$CEP = K \cdot \sigma_{large} \tag{7}$$

For this purpose, use is made of the approximation polynomials $K = f(C)$ which are shown in Figure 8 and were determined by the author from HARTER's tables (Fig. 7)(see ref. [4]). The error circles CEP calculated in this way are written with the predefined probability as a suffix. For example, the error radius CEP75 signifies that if a sufficiently large number of position-findings are perfor-

	P \ c	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
*	.5000	0.67449	0.68199	0.70585	0.74993	0.80785	0.87042	0.93365	0.99621	1.05769	1.11807	1.17741
	.7500	1.15035	1.15473	1.16825	1.19246	1.23100	1.28534	1.35143	1.42471	1.50231	1.58271	1.66511
	.9000	1.64485	1.64791	1.65731	1.67383	1.69918	1.73708	1.79152	1.86253	1.94761	2.04236	2.14597
	.9500	1.95996	1.96253	1.97041	1.98420	2.00514	2.03586	2.08130	2.14598	2.23029	2.33180	2.44775
	.9750	2.24140	2.24365	2.25053	2.26255	2.28073	2.30707	2.34581	2.40356	2.48494	2.58999	2.71620
	.9900	2.57583	2.57778	2.58377	2.59421	2.60995	2.63257	2.66533	2.71515	2.79069	2.89743	3.03485
	.9950	2.80703	2.80883	2.81432	2.83289	2.83830	2.85894	2.88859	2.93347	3.00431	3.11073	3.25525
	.9975	3.02334	2.02500	3.03010	3.03898	3.05234	3.07144	3.09871	3.13969	3.20586	3.31099	3.46164
	.9990	3.29053	3.29206	3.29673	3.30489	3.31715	3.33464	3.35949	3.39647	3.45698	3.55939	3.71692
**	.6667	0.967	0.973	0.989	1.018	1.065	1.124	1.192	1.263	1.335	1.408	1.482

FIG. 7.— The K-factor for calculation of the CEP error circles (*HARTER [4] **HARRE [5]).

CEP 50: $K = 0.671958 + 0.01137C + 1.013785C^2 - 0.523503\ C^3$ max. deviation: 0.67%
CEP 66: $K = 0.969594 - 0.103347C + 1.037990C^2 - 0.422166C^3$ max. deviation: 0.35%
CEP 75: $K = 1.153897 - 0.121090C + 0.925773C^2 - 0.291798C^3$ max. deviation: 0.34%
CEP 90: $K = 1.647780 - 0.063608C + 0.439992C^2 + 0.125765C^3$ max. deviation: 0.27%
CEP 95: $K = 1.960787 + 0.004121C + 0.114151C^2 + 0.371707C^3$ max. deviation: 0.17%

FIG. 8.— Table of the approximation polynomials for the K-factor (HARRE, 12/86).

med, only a quarter of all measurement results will lie outside the error circle.

4. POLARFIX ACCURACY SPECIFICATION

Accuracy specifications as shown in the POLARFIX literature are based on tests performed with units directly off the production line on test areas at the manufacturer's premises. These measurements are taken for each piece of equipment to be delivered using the following method:

First, the equipment is aimed at a point with a distance of exactly 2,740.65 m. Then a statistic of 1,000 position measurements is established resulting in errors suggesting gaussian distributions over distance and azimuth. Typical standard deviations for the range measurement are 0.1 m and for the angular measurement 0.01° (Fig. 9).

Together with distance-dependent error portions known from the literature to be caused by deviations in the velocity of light and by refraction as well as consideration of the width of the prism of 7.2 cm and possible differences in the tolerances of the individual product, the resultant specification for the statistical measurement is:

Sigma (range) = 0.1 m + 0.1 m / km
Sigma (azimuth) = 1/d × 5.73 deg. × m or 0.015 deg. (whichever is greater)
d = measured range [m]

In the dynamic case error components are added due to the tracking of a moving target. At close range, the angular error is influenced by the horizontal extension of the prism ring, it is possible that the tracking device may lock onto one of two outer prisms rather than on the central one.

Yet the dynamic accuracy is stated as follows:

Sigma (range) = 0.1 m + 0.1 m / km
Sigma (azimuth) = 1/d × 28.65 deg. × m or 0.015 deg. (whichever is greater).

The 1 dRMS position error can be computed as follows: a u-v coordinate system is established in such a way that the u-axis is pointing in the direction of the range measurement axis, the v-axis is perpendicular to the u-axis. This results in:

Sigma (u) = Sigma (range)
Sigma (v) = d × tan (sigma (azimuth)).

The 1 dRMS positional error can be predicted using formula (1).

Figure 19 shows this positional error calculated in accordance with the manufacturer's specification as a function of range (top graph).

The tests evaluated below will show that, in reality, the results are far better.

ACTUAL DISTANCE 2,740.65
ACTUAL ANGLE 02

NUMBER OF MEASUREMENTS TAKEN: 1,000
DISTANCE MEASUREMENTS OUT OF BOUNDS: 0
ANGLE MEASUREMENTS OUT OF BOUNDS: 0

NUMBER/CLASS			
585		765	*
572		748	*
559	*	731	*
546	*	714	*
533	*	697	*
520	*	680	*
509	*	663	*
494	*	646	*
481	*	629	*
468	*	612	*
455	*	595	*
442	*	578	*
429	*	561	*
416	*	544	*
403	*	527	*
390	*	510	*
377	*	493	*
364	*	476	*
351	*	459	*
338	*	442	*
325	*	425	*
312	**	408	*
299	**	391	*
286	**	374	*
273	**	357	*
260	**	340	*
247	**	323	*
234	**	306	*
221	**	289	*
208	**	272	*
195	**	255	*
182	**	238	*
169	**	221	*
156	**	204	**
143	**	187	**
130	**	170	**
117	**	153	**
104	**	136	**
91	***	119	**
78	***	102	**
65	***	85	**
52	***	68	**
39	****	51	**
26	****	34	**
13	*******	17	******
	098765432101234567890 – Diff. in (dm) + distance		098765432101234567890 – 1/100 (degr.) + angle

FIG. 9.— Histograms of the POLARTRACK range and azimuth measurements.

5 THE AUTHOR'S ERROR BUDGET

For the purpose of demonstrating how a simple error budget can be worked out, assumptions on error components shall be made, these components shall be integrated to formulate the 1 dRMS position error, and finally the so found error shall be compared with the test results.

Error components	Magnitude	Assumed distribution	Sigma
1. Range:			
Bias	0.1 m (1σ)	gaussian	0.1 m
Dist. depend.	$0.0001 \times D$ (1σ)	gaussian	$0.0001 \times D$
2. Azimuth:			
Width or prism	0.072 m ($=3\sigma$)	gaussian	0.024 m
Resolution	0.01°	rectangular	0.0029°

These values have been used to calculate distance-dependent error values within the measuring range of the POLARFIX. These data have been plotted in the diagram, (Fig. 19, middle graph). A comparison with the graph of the official POLARFIX specification shows that the budget's error values are lower by the factor of 3, approximately. An evaluation of the various test results will show the magnitude of error which are obtainable in practice.

6. ACCURACY TESTS OF INDEPENDENT INSTITUTIONS

When Krupp Atlas Electronik first introduced POLARFIX in 1983, no comparable equipment for polar positioning was available on the hydrographic instrument's market utilizing laser technology and tracking the target automatically. Consequently potential customers first wished to thoroughly test the equipment before deciding upon procurement.

6.1 Test by the Applied Physics Laboratory of John Hopkins University (APL)

Under a contract by interested U.S. Government Agencies, POLARFIX was tested on an unused airfield in Beltsville, Maryland. This test, because of the methodologies applied, was probably one of the most accurate ones performed to date. The POLARFIX reflector was mounted on a small truck. On the test range itself, spots were marked exactly every 10 m. Every time, the vehicle passed over one of the marks an optical switch, installed vertically under the POLARFIX reflector, triggered the POLARFIX measurement and the position was recorded.

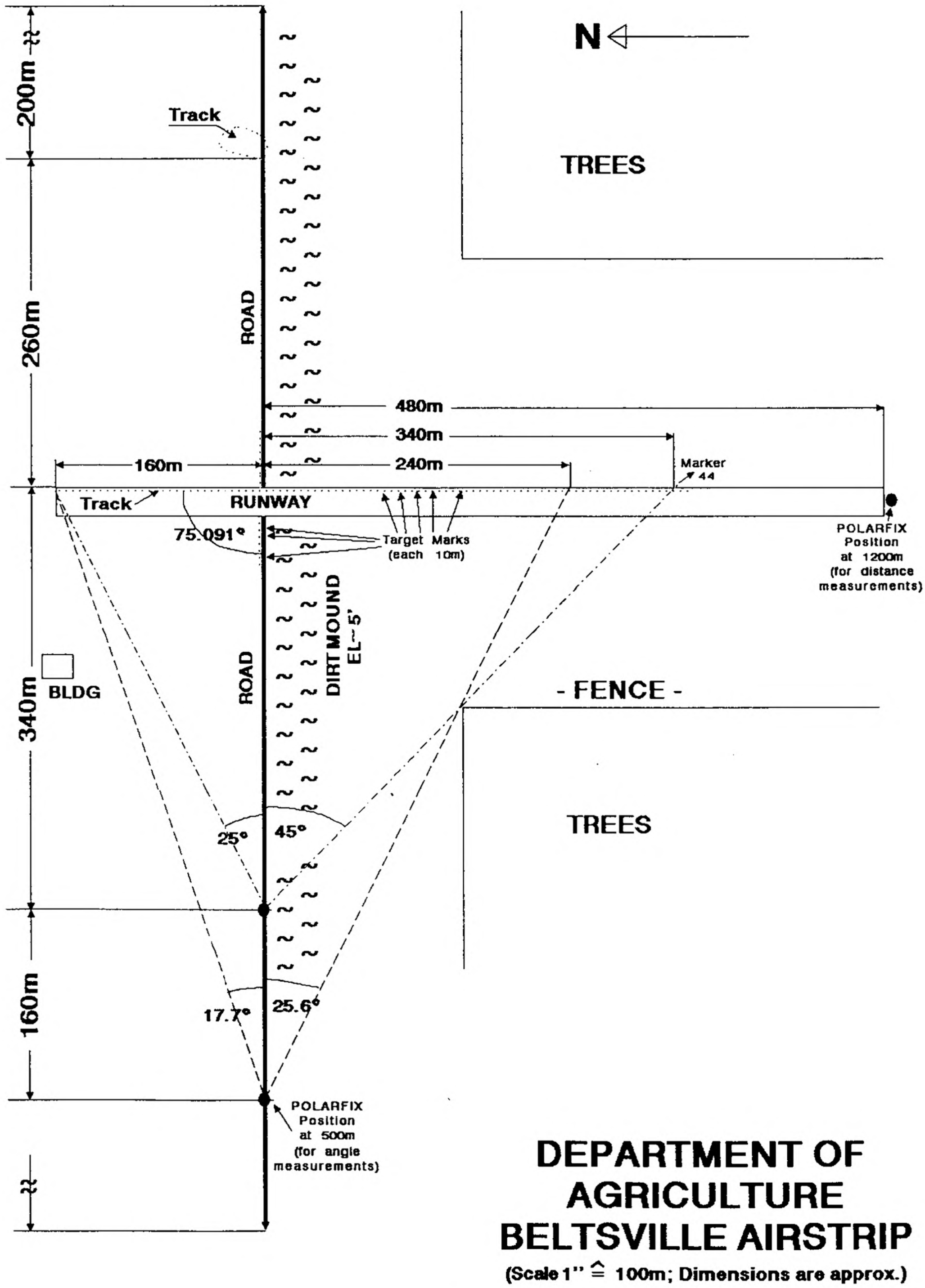
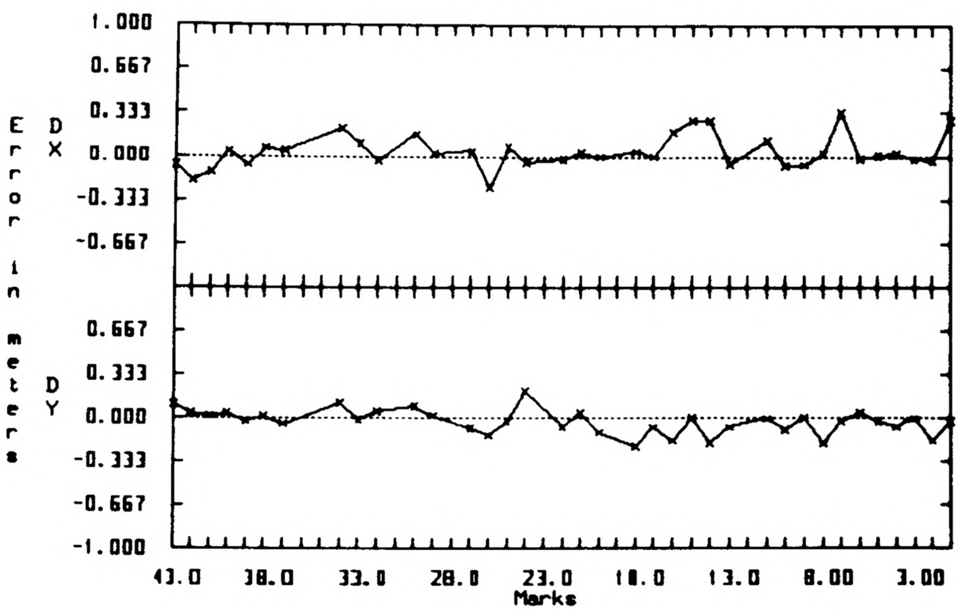


FIG. 10.— Set up of testing area of APL in Beltsville, MD, USA.

A total of approximately 100 test runs were performed of which about one quarter were so called 'short range' tests at a distance of around 500 m and the remainder were 'long range' tests at around 1200 m (Fig. 10). The tracks ran tangential and radial to the location of the POLARFIX with the minimum distances mentioned above. During both types of test runs, the speed of the vehicle was varied between 1 m/s and 5.5 m/s. Several runs were performed at each selected speed. Figure 11 shows a typical test protocol from which the

KRUPP ATLAS EVALUATION ... POINT 2 SOUTH TO NORTH
Run 157 3-Apr-85 11:44:36
Laser position:—128.64, —483.17
Target speed: miles/hr = 6, meters/sec = 2.68, knots = 5.20



	Along track (meters)	Across track (meters)
Mean deviation:	0.035	-0.029
RMS deviation:	0.131	0.096
Std. deviation of deviations:	0.128	0.093
Std. deviation of mean deviation:	0.021	0.015
Maximum deviation (absolute):	0.339	0.214
occuring at markers:	7	18

Number of markers encountered: 44 Number of 'good hits': 36
Calculated target speed: 6.60 (MPH) 2.95 (Meters/sec) 5.43 (knots)

FIG. 11.— Typical measurement protocol produced by APL

deviations of the measurement values both in the X and Y axes can be seen. The number of data contained in the individual protocols available was reduced by the author into accuracy statements using the accuracy standards described above. This was done for both, the short range and the long range tests. The analysis shows that POLARFIX is in fact considerably more accurate than the manufacturer's specification indicates. This is even true when the calibration error is included to form a root sum square error (RSS) (see Fig. 12, 13). Mean values

POLARFIX TEST BY JOHN HOPKINS UNIVERSITY, U.S.A.,
25.03. – 05.04.85 (S)

NO.	MEAN DEVIATION DX	DY	STD. DEV. OF DEV. DX	DY	MAX. ABS. DEVIATI DX	DY	GOOD #	TOTAL #	SPEED M/S
183	0,130	-0,150	0,089	0,109	0,366	0,366	41	44	1,11
210	-0,118	-0,081	0,062	0,081	0,229	0,257	39	44	1,49
211	-0,113	-0,090	0,070	0,092	0,252	0,358	40	44	1,49
207	-0,069	-0,072	0,072	0,082	0,236	0,248	38	44	1,50
209	-0,112	-0,068	0,069	0,070	0,271	0,213	44	44	1,50
208	-0,105	-0,063	0,073	0,074	0,351	0,221	43	44	1,51
206	-0,097	-0,063	0,070	0,090	0,258	0,243	43	44	1,55
157	0,035	-0,029	0,128	0,093	0,339	0,214	36	44	2,95
155	0,012	-0,048	0,141	0,095	0,481	0,299	38	44	2,98
158	0,076	-0,029	0,150	0,084	0,403	0,234	33	44	2,98
160	0,047	-0,039	0,107	0,074	0,345	0,209	37	44	2,99
161	0,119	-0,028	0,139	0,096	0,494	0,243	39	44	2,99
203	-0,069	-0,106	0,133	0,061	0,253	0,221	36	44	4,85
198	-0,065	-0,080	0,118	0,088	0,270	0,218	35	44	4,97
193	-0,121	-0,049	0,175	0,072	0,372	0,182	31	44	4,97
191	-0,136	-0,058	0,166	0,074	0,354	0,240	38	44	5,02
186	-0,150	-0,135	0,207	0,068	0,381	0,285	21	44	5,52
22	-0,043	-0,072	0,116	0,080	0,312	0,242	809	968	3,41
COUNT	MEAN	MEAN	MEAN	MEAN	MEAN	MEAN	SUM	SUM	MEAN
	0,084 RMS		0,141 1dRMS						
		0,164 RSS							

FIG. 12.— The author's integration table of the 'short range test' by APL (clippings).

of the results of the individual test runs show the following circular errors:

Mean Range 500 m => 1 dRMS = 0.141 m
(22 test runs with 809 measurements evaluated)

Mean Range 1200 m => 1 dRMS = 0.209m
(56 test runs with 1075 measurements evaluated).

An analysis of the test runs with different speeds indicates a slight dependency of the 1 dRMS positional error from the velocity of the tracked target. The positional errors relative to the velocity are depicted in Figure 14. The plotted regression line computed using the method of least square fitting shows a velocity dependent error portion of approximately 4 cm per m/s travelling speed.

POLARFIX TEST BY JOHN HOPKINS UNIVERSITY, U.S.A.,
25.03. – 05.04.85 (L)

NO.	MEAN DEVIATION DX	MEAN DEVIATION DY	STD. DEV. OF DEV DX	STD. DEV. OF DEV DY	MAX. ABS. DEV. DX	MAX. ABS. DEV. DY	GOOD #	TOTAL #	SPEED M/S
74	-0,180	0,118	0,175	0,108	0,533	0,270	20	21	0,86
143	-0,001	-0,088	0,203	0,161	0,780	0,712	20	21	0,88
141	-0,051	-0,092	0,150	0,080	0,409	0,242	20	21	0,93
137	0,100	-0,098	0,122	0,073	0,416	0,239	19	21	0,94
144	0,045	-0,032	0,119	0,066	0,266	0,157	20	21	0,97
76	-0,084	0,201	0,096	0,098	0,249	0,333	18	21	1,47
90	-0,121	0,093	0,109	0,072	0,461	0,194	20	21	1,54
92	-0,071	0,058	0,114	0,071	0,359	0,241	20	21	1,54
79	-0,103	0,188	0,142	0,083	0,376	0,341	20	21	1,54
84	-0,140	0,056	0,174	0,070	0,467	0,226	20	21	1,55
82	-0,070	0,039	0,137	0,064	0,426	0,142	20	21	1,56
101	-0,143	-0,024	0,202	0,066	0,584	0,125	16	21	4,41
104	-0,069	-0,025	0,282	0,084	0,646	0,187	19	21	4,45
102	-0,179	-0,015	0,308	0,077	1,070	0,202	16	21	4,45
103	-0,178	-0,045	0,307	0,121	1,220	0,278	17	21	4,46
105	-0,056	-0,087	0,214	0,183	0,557	0,682	18	21	4,50
107	-0,111	-0,056	0,195	0,087	0,441	0,265	19	21	4,51
56	-0,070	0,031	0,167	0,099	0,477	0,294	1075	1176	2,38
COUNT	MEAN	MEAN	MEAN	MEAN	MEAN	MEAN	SUM	SUM	MEAN
	0,077 RMS		0,194 1dRMS						
		0,209 RSS							

FIG. 13.— The author's integration table of the 'long range test' by APL (clippings).

6.2 Test at the NATO-FORACS Range

POLARFIX was subjected to a further accuracy test in October 1985 on a NATO test range near Stavanger, Norway. The test comprised position measurements of the frigate *BERGEN* at ranges between 3194 m and 4373 m. The POLARFIX measurements were compared with the synchronized bearings taken by three theodolites.

In the overall evaluation of the author, the measurement errors were first decorrelated. Due to the low correlation coefficient, the 1 dRMS error circle changes only insignificantly (Fig. 15). Following error circle is the result:

Mean range 3800 m = > 1 dRMS = 0.61m
(1 run with 217 measurements)

Also in this case the positional error of POLARFIX remains well under the manufacturer's specification even when the calibration error is included (Fig. 15). Following the accuracy test, a range test was run, in which a maximum range of 9900 m was established using two prism rings with a total of 24 prisms.

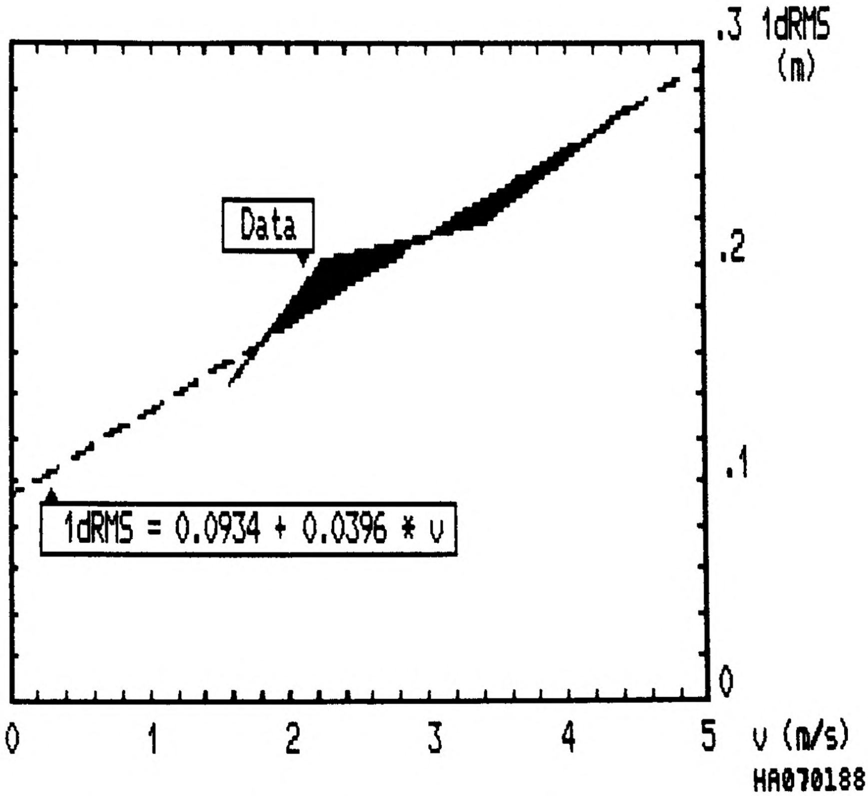


FIG. 14.— Speed dependency of the position error, as determined from the APL tests.

6.3 Test by Rijkswaterstaat (RWS), the Netherlands

At the end of November 1987, the POLARFIX was tested in the Netherlands together with three other competitive models. As reference for the range measurement, an AGA system and as a reference for the azimuth, a Minilir system were used. The combination of both systems is supposed to be one class better in accuracy, but on the other hand is two classes more expensive than POLARFIX. The following error circle was found in the author's evaluation:

Mean range 936 m = > 1 dRMS = 0.58 m
(1 run with 313 measurements).

This result is not as good as the one obtained by APL which can be attributed to the poor weather conditions during the tests. The measurements were partly impaired by rain and gale-force winds which were rocking the POLARFIX and the vessel. Even under these severe conditions, the accuracy achieved was within the specification.

The POLARFIX test was repeated under better conditions on March 28,

POLARFIX TEST AT FORACS RANGE, NORWAY
23.10.1985

MARK	XDIFF	YDIFF	XDEV	YDEV	XDEV*YDEV	XDEV*XDEV	YDEV*YDEV	XDEVK	YDEVK
13	0,54	-0,39	0,39	0,00	0,00	0,15	0,00	0,39	-0,05
14	0,29	-0,66	0,14	-0,27	-0,04	0,02	0,08	0,11	-0,29
15	0,56	-0,64	0,41	-0,25	-0,10	0,17	0,06	0,38	-0,30
16	0,13	-0,56	-0,02	-0,17	0,00	0,00	0,03	-0,04	-0,17
17	0,42	-0,26	0,27	0,13	0,03	0,07	0,02	0,28	0,09
18	0,30	-0,47	0,15	-0,08	-0,01	0,02	0,01	0,14	-0,10
19	0,80	-0,26	0,65	0,13	0,08	0,43	0,02	0,66	0,05
20	0,19	-0,21	0,04	0,18	0,01	0,00	0,03	0,06	0,17
21	0,36	-0,54	0,21	-0,15	-0,03	C,04	0,02	0,19	-0,18
22	0,28	-0,26	0,13	0,13	0,02	0,02	0,02	0,15	0,11
304	-0,85	0,22	-1,00	0,61	-0,60	1,00	0,37	-0,92	0,72
305	-0,15	-0,38	-0,30	0,01	0,00	0,09	0,00	-0,30	0,04
306	0,29	-0,44	0,14	-0,05	-0,01	0,02	0,00	0,13	-0,07
307	-0,58	-0,31	-0,73	0,08	-0,05	0,53	0,01	-0,71	0,16
308	-0,19	-0,67	-0,34	-0,28	0,10	0,11	0,08	-0,37	-0,24
309	-0,89	0,22	-1,04	0,61	-0,63	1,08	0,37	-0,96	0,72
310	0,05	0,02	-0,10	0,41	-0,04	0,01	0,16	-0,05	0,41
311	-0,45	-0,28	-0,60	0,11	-0,06	0,36	0,01	-0,58	0,17
312	-0,31	0,30	-0,46	0,69	-0,31	0,21	0,47	-0,37	0,73
313	0,36	-0,17	0,21	0,22	0,05	0,04	0,05	0,24	0,19
217	0,15	-0,39	0,49	0,37	-2,53	50,82	29,66	0,48	0,38
COUNT	MEAN	MEAN	STDV	STDV	SUM	SUM	SUM	STDV	STDV
	0,41 RMS		0,61 RMS					0,61 1dRMS	
		0,74 RSS						0,00 MEAN	0,00 MEAN

38,82 Nenner k
-0,07 Korrelationsgrad k
0,24 Tangens Theta
0,12 Theta (rad)
6,72 Theta (degr)

FIG. 15.— The author's integration table of the test at FORACS range (clippings).

1988, with favourable results. From the data lists of the official test report, the author extracted the following accuracy figures. During the tests, position measurements were taken every 0.5 seconds. The average values of the position deviations and the standard deviations were calculated for groups of 50 successive measurements in the report. The author has used 3 to 18 of these groups (150 ... 900 single fixes) to establish positional accuracies for various median distances between 158 and 1977 m (Fig. 16).

Test	Mean range	1 dRMS	Pattern
22/4	158 m	0.20 m	circles
20/5	188 m	0.14 m	radial
20/6	405 m	0.13 m	radial
20/5	472 m	0.14 m	radial
20/6	774 m	0.16 m	radial
21/1	776 m	0.18 m	tangential
22/2	1977 m	0.17 m	tang./circ.

# BEG.	# END	Ap beg	Ap end	MAp	MRp	SAp	SRp
22/4							
8	40	326,95	292,95	0,30	0,10	0,10	0,40
41	90	291,55	244,25	0,30	0,10	0,10	0,20
91	140	243,35	215,95	0,40	0,00	0,10	0,20
142	190	216,75	203,55	0,30	0,00	0,10	0,10
192	240	203,05	178,45	0,20	0,00	0,10	0,10
241	290	177,65	160,85	0,10	0,00	0,10	0,20
291	340	161,45	189,55	0,10	-0,10	0,10	0,10
341	390	189,45	160,85	0,20	-0,20	0,10	0,60
391	440	159,75	110,65	0,00	-0,10	0,10	0,10
441	491	109,25	81,55	0,00	-0,10	0,10	0,10
492	540	82,05	111,35	0,00	-0,10	0,10	0,10
541	590	111,05	121,15	0,00	-0,10	0,10	0,10
591	640	121,75	119,85	-0,10	0,00	0,10	0,10
641	690	118,95	69,05	-0,10	-0,10	0,10	0,10
691	740	67,75	67,05	0,00	0,00	0,10	0,10
741	790	68,05	81,85	-0,10	0,00	0,10	0,10
	782	165,55	150,56	0,10	-0,04	0,10	0,17
Mean Dist.:	158	RMS:		0,11	1dRMS:		0,20
20/6							
8	40	933	898	0,30	-0,40	0,10	0,10
41	90	897	838	0,30	-0,30	0,10	0,30
91	140	837	777	0,20	-0,30	0,10	0,10
141	190	776	716	0,20	-0,30	0,10	0,10
191	240	715	654	0,30	-0,30	0,10	0,10
241	290	653	592	0,30	-0,20	0,10	0,10
	282	802	746	0,27	-0,30	0,10	0,13
Mean Dist.:	774	RMS:		0,40	1dRMS:		0,17
22/2							
7	41	2117	2119	0,10	0,40	0,20	0,10
42	90	2118	2118	0,00	0,30	0,10	0,20
91	140	2118	2118	0,00	0,30	0,10	0,10
141	190	2118	2118	0,00	0,40	0,10	0,10
191	240	2118	2118	0,10	0,40	0,10	0,10
241	291	2118	2118	0,00	0,30	0,10	0,10
292	340	2118	2118	0,00	0,30	0,10	0,10
341	390	2118	2114	0,10	0,30	0,10	0,20
391	440	2114	2115	0,10	0,30	0,10	0,10
441	490	2105	2074	-0,10	0,40	0,10	0,10
493	540	2072	2016	-0,10	0,40	0,10	0,10
541	590	2015	1952	0,00	0,40	0,10	0,10
591	640	1951	1886	0,00	0,30	0,10	0,10
641	690	1884	1822	0,00	0,40	0,20	0,20
691	740	1821	1814	0,10	0,20	0,10	0,10
741	791	1816	1846	0,10	0,20	0,10	0,20
792	841	1845	1800	0,10	0,30	0,10	0,10
842	890	1799	1738	0,00	0,30	0,10	0,20
891	940	1737	1679	0,00	0,30	0,10	0,20
941	990	1678	1624	-0,10	0,30	0,10	0,10
	983	1989	1965	0,02	0,32	0,11	0,13
Mean Dist.:	1977	RMS:		0,32	1dRMS:		0,17

FIG. 16.— The author's integration table of the test by RWS in March 1988 (sample sections).

As additional information, the above table indicates the vessel's motion pattern relative to the POLARFIX shore station. There is no significant influence from the latter on the accuracy, not even when moving in circles (Fig. 17) (provided the system's predictive filter is switched off). The results confirm the high accuracy implied by the previous tests.

7. SUMMARY OF TESTS

The table in Figure 18, summarizes the author's evaluation of the data published in the reports of the above-mentioned tests. At the head of the table, the following abbreviations are used:

D	:	mean deviations of the data measured by POLARFIX from those of the reference system
S	:	standard deviations
SE	:	magnitude of the systematic error vector
1 dRMS	:	root mean square position error (probability p = 0.632 ... 0.683)
2 dRMS	:	root mean square position error (probability p = 0.954 ... 0.982)
CEPxx	:	circular error of fixed probability (probability p = 0.xx).

The tabular data show that the magnitude of the systematic error vector lies within the range 0.11 ... 0.45 m and is not significantly distance-dependent. This error can be kept small with a properly calibrated POLARFIX system which is thoroughly set up for operation.

The 1 dRMS position error values of the individual tests are shown as scattered points in the diagram, Figure 19 (the horizontal 'distance' axis as well as the vertical 'error' axis are divided in logarithmic scales). From these scatter points, the following error equation has been found, using least squares approximation:

$$1\ \text{dRMS} = 8.60e - 05 \times (\text{Range} \simeq 1.03) + 0.095$$

(Some of the 9 data points lie directly on the graph and are therefore hardly visible. The data point, shown as a box, is from the RWS test in 1987 and has not been used here).

The upper graph signifies the company specification. A comparison of the error curve with the empirically found graph of the test results shows that the inherent accuracy of POLARFIX is two to three times better than claimed by the manufacturer.

The third curve which is just above the test results' graph represents the author's error budget.

When considering the systematic and the statistical portions of the error, it

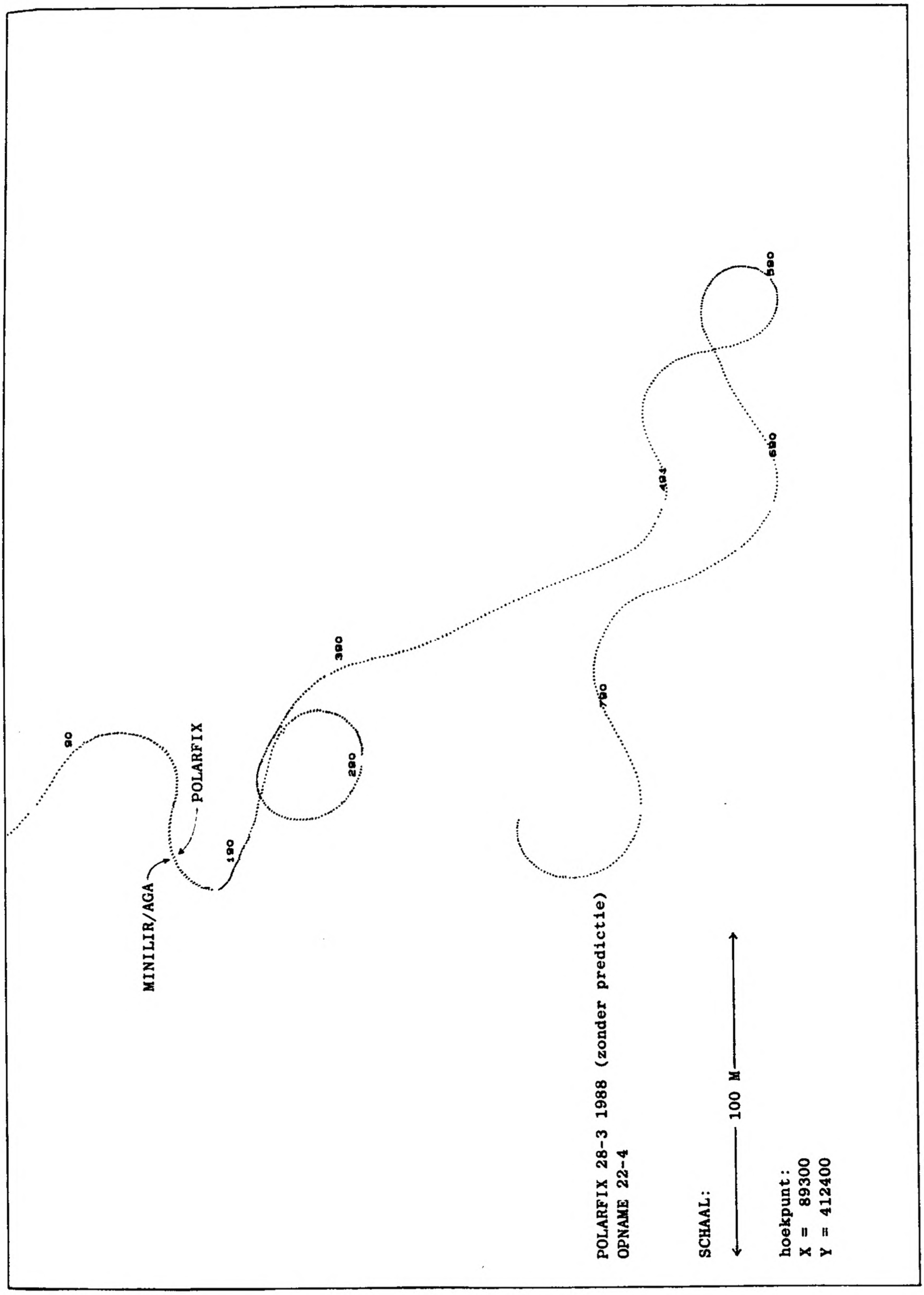


FIG. 17.— Plot of non-straight track patterns, performed within the RWS test.

TEST	RANGE (m)	Dx (m)	Dy (m)	Sx (m)	Sy (m)	SE (m)	1dRMS (m)	1dRMS (m/km)	2dRMS (m)	CEP50 (m)	CEP66 (m)	CEP75 (m)	CEP95 (m)
RWS	158	0.10	-0.04	0.10	0.17	0.11	0.20	1.25	0.39	0.16	0.20	0.23	0.35
RWS	188	0.40	-0.23	0.10	0.10	0.46	0.14	0.75	0.28	0.12	0.15	0.17	0.25
RWS	405	0.35	-0.20	0.10	0.08	0.40	0.13	0.32	0.26	0.11	0.13	0.15	0.22
RWS	472	0.38	-0.25	0.10	0.10	0.45	0.14	0.30	0.28	0.12	0.15	0.17	0.25
APL	500	-0.04	-0.07	0.12	0.08	0.08	0.14	0.28	0.28	0.11	0.15	0.16	0.25
RWS	774	0.27	-0.30	0.10	0.13	0.40	0.16	0.21	0.33	0.14	0.17	0.19	0.29
RWS	776	0.29	-0.31	0.13	0.12	0.42	0.18	0.23	0.35	0.15	0.19	0.21	0.31
APL	1200	-0.07	0.03	0.17	0.10	0.08	0.19	0.16	0.39	0.15	0.20	0.22	0.35
RWS	1977	0.02	0.32	0.11	0.13	0.32	0.17	0.09	0.34	0.14	0.18	0.20	0.30
FORACS	3800	0.15	-0.39	0.48	0.38	0.42	0.61	0.16	1.22	0.51	0.64	0.72	1.07

Ok
RUN

FIG. 18.— Integration table of the tests evaluated.

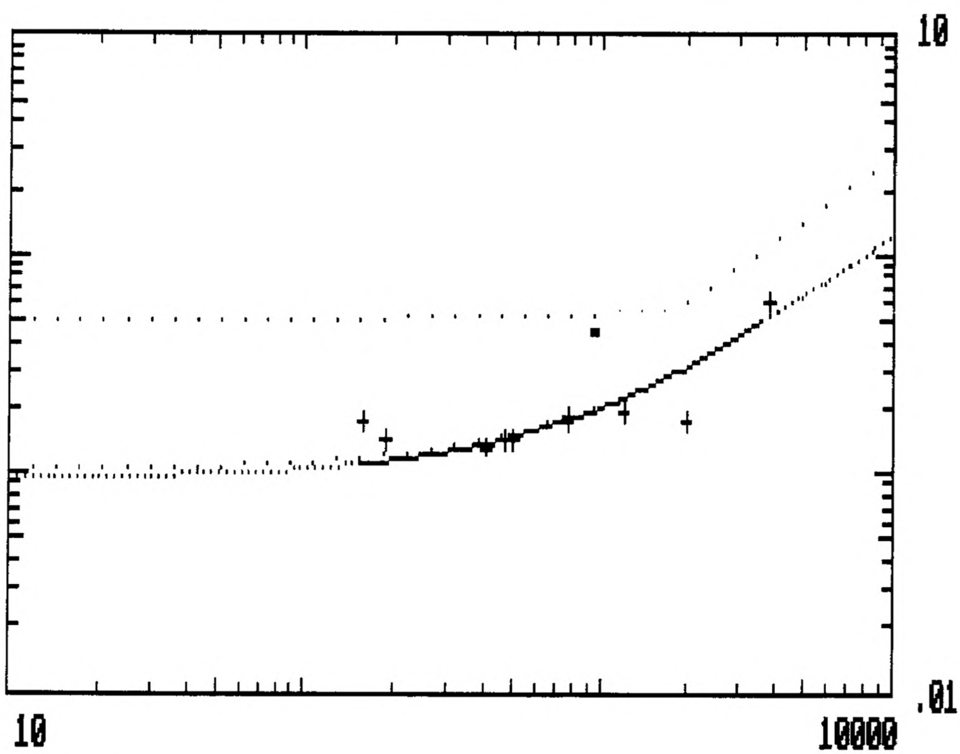


FIG. 19.— Graphical presentation of the Polarfix position error; from top to bottom: company specification, the author's error budget, empirical error curve, derived from tests evaluated with the individual test results indicated by crosses.

should be noted that both of them are containing residual errors from the reference measurement system which are inseparably connected to the errors of the equipment tested. Provided that the different POLARFIX units show only little difference in their individual accuracy, the test results also allow conclusions as to the accuracy of the reference system.

Error estimations have been given for the accuracies of the reference systems used by APL and RWS, both being in the order of 5 cm. At the FORACS test, the probable noise of the reference measurement is extremely small considering the method of three independent angle measurements and the vocal synchronization.

8. CONCLUSION

With the position fixing error found in the evaluated practical tests, POLARFIX is reaching a level of precision which is out of reach of conventional hydrographic positioning systems. Besides the accuracy, further criteria are important for practical use, such as the minimum and maximum range, the ability to track a target automatically, even at short range, the reaction in case of target loss and, last but not least, the maturity of a product documented by its reliability record. All factors, however, which have not been a subject of this evaluation.

By the example of the polar positioning system POLARFIX, it was demonstrated how error statements can be derived from available test data. At the same time, this work presents a documentation about the capabilities of current laser polar positioning systems for hydrography.

References

[1] KAYTON/MYRON (1969): Avionics Navigation Systems, John Wiley & Sons, New York.

[2] D.R. CHILDS *et al.* (1978): Error Measures for Normal Random Variables, IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-14, No. 1, January.

[3] BURT, KAPLAN *et al.* (1965): Mathematical Considerations Pertaining the Accuracy of Position Location and Navigation System, Stanford Research Institute.

[4] H.L. HARTER (1960): Circular Error Probabilities, American Statistical Association Journal, December.

[5] I. HARRE (1987): Modellieren von Navigationsfehlern — Fehlertypen und Entwicklung eines Fehlerkreismodells, Ortung & Navigation, Heft 3.