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CALCULATION OF THE TIDE OFFSHORE FOR SOUNDING REDUCTION

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Foreword

Prior to 1978, sounding reduction at the French Hydrographic Office (EPSHOM) was obtained from co-tidal charts which provided the range factor k (X,Y) and the time lag σ(X,Y) to be used in the formula:

$$h_L\,(t,X,Y) = k\,(X,Y)\,[h_R\,(t - \sigma(X,Y))]$$

where h_R is the height of the tide observed at a coastal station and
h_L the height of the tide calculated at the position of sounding.

This method, which pre-supposes a similitude between tides at any point of the sounding area, had proved to be ill-adapted to the Bay of Seine where large ranges are associated with important distortions of the tidal wave propagating over shallow water.

The harmonic method made possible an appreciable improvement of the results. This method, which was first applied to the Bay of Seine, is now applied to the whole continental shelf off the French coast.

1. HARMONIC METHOD — GENERAL

Computation of height of tide by the harmonic method is application of the following formula:

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$$h(t,X,Y) = Z_0(X,Y) + \sum_{i=1}^{N} A_i(X,Y) \cos [V_i(t) - G_i(X,Y)]$$

where the harmonic constants A_i (X,Y) and G_i (X,Y) represent amplitude and phase lag of the constituent i at the point (X,Y).

V_i (t) is the value of the constituent i astronomical argument at time t and Z_0(X,Y) is the mean level above chart datum.

Harmonic analysis makes it possible to calculate amplitudes and phase lags of constituents at any point, provided a sufficiently long period of observation is available.

Owing to scarce offshore observations, this method was initiated with the help of harmonic constants issued from a hydraulic reduced model of the English Channel from the Institut de Mécanique de Grenoble (IMG) (Ref. 1).

This model provided the main harmonic constants of tide at many locations in the English Channel. The density of these points is variable, depending on the constituent and the area, but it is generally sufficient to allow, by interpolation, calculation of harmonic constants at any location.

The use of automatic seabed tide gauges provides the means to an increase in the density of harmonic constants and an improvement of the accuracy of the results.

It allowed extension of the method to be used from the South of the North Sea to the continental shelf limit and from the latitude of the Scilly Islands to the coast of Spain.

Figures 1 and 2 show the locations for which harmonic constants are available.

The IMG hydraulic model provided, at some locations, only the most important constituent constants which are: Q_1, O_1, P_1, K_1, μ_2, N_2, M_2, S_2, K_2, M_4 and M_4.

Generally, this is not sufficient for the accurate calculation of the tide, which requires more constituents at the exact location of the sounding. Therefore the calculation of main constants at given points and the calculation of additional important constants are the problems to be solved.

Paragraph 2 deals with the first point and paragraph 3 with the second one.

2. SPATIAL INTERPOLATION OF CONSTANTS

Main constituents of the tide are available at N locations, the coordinates of which are (x_i, y_i).

We have to calculate these constituents at location (x,y).

Each constituent k of the tide at location (x,y) may be presented as a vector V_k (x,y), the modulus and direction of which are the amplitude and phase of this constituent.

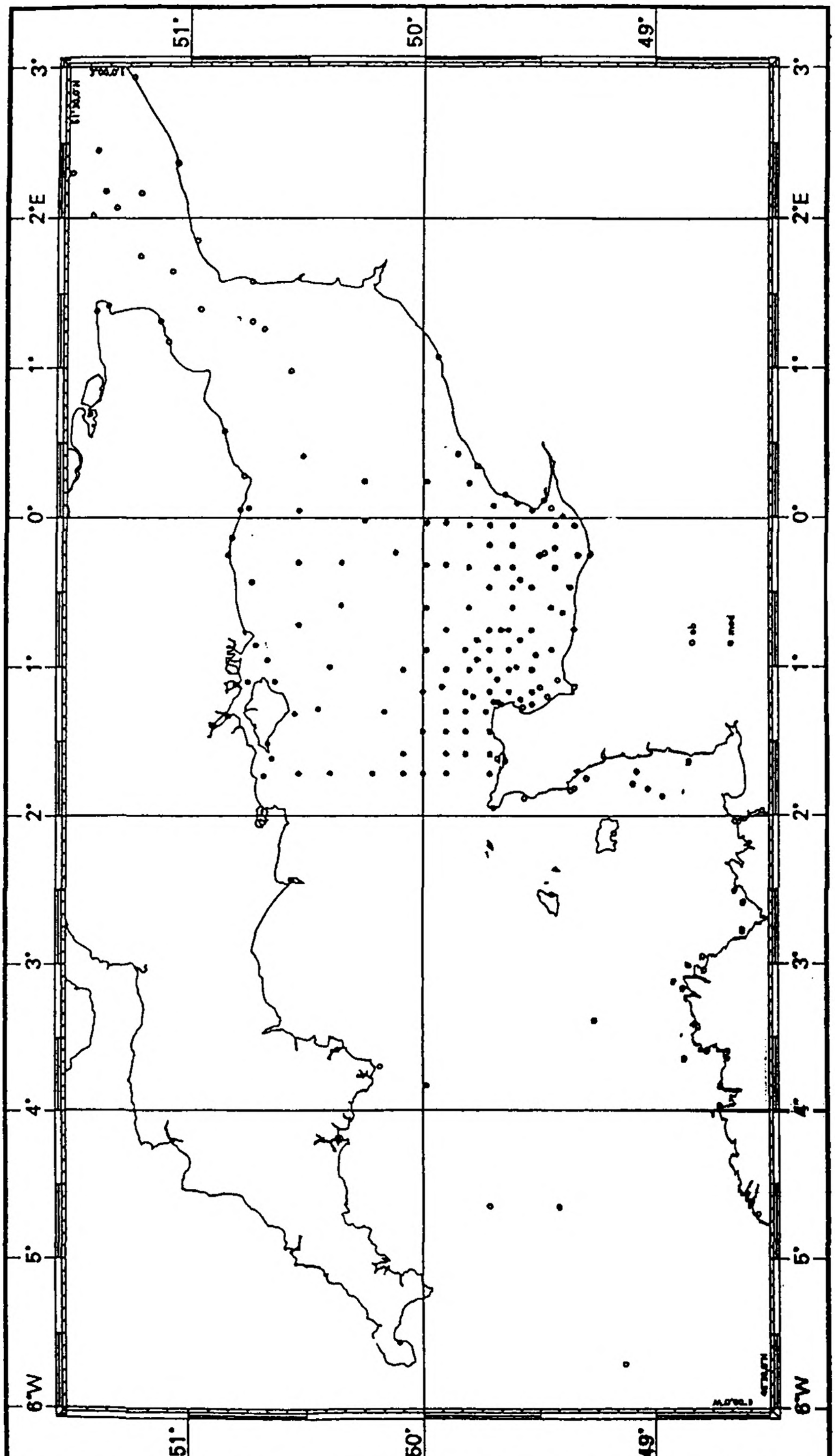


FIG. 1—. Harmonic constants available in the English Channel.

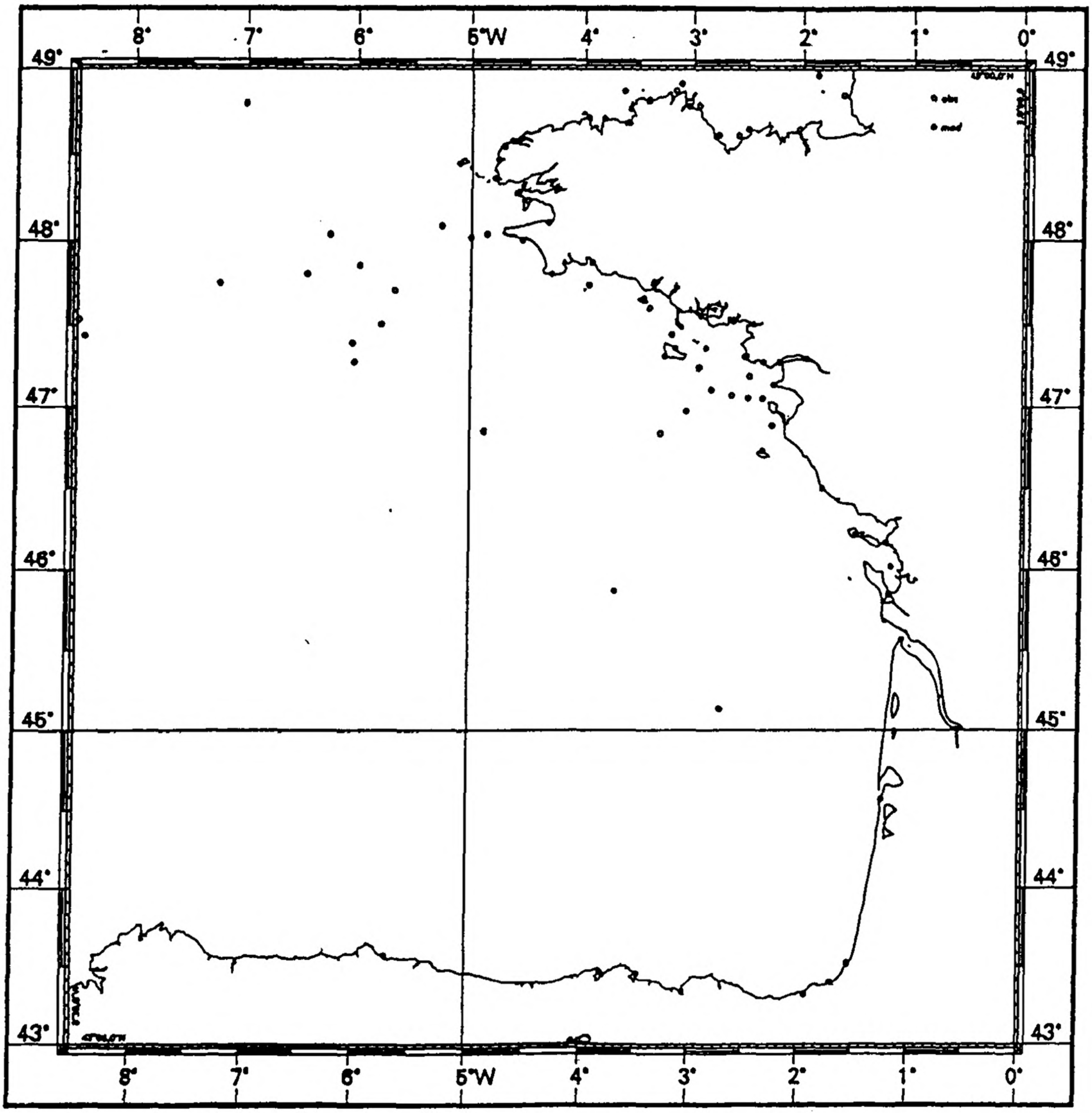


FIG. 2.— Harmonic constants available on the Atlantic Coast.

The following function of two variables describes the shape of a thin sheet constrained to pass through given points (x_i, y_i)

$$V_k(x,y) = \sum_{i=1}^{N} \lambda_i R_i^2 \operatorname{Log} R_i + ax_i + by_i + c,$$

$$\text{where } R_i = [(x - x_i)^2 + (y - y_i)^2]^{1/2}$$

with additional conditions:

$$\sum_{i=1}^{N} \lambda_i = 0$$

$$\sum_{i=1}^{N} \lambda_i x_i = 0$$

$$\sum_{i=1}^{N} \lambda_i y_i = 0$$

This set of simultaneous equations has to be solved for each constituent V_k (x_i, y_i) whose values are known at N points (x_i, y_i).

The N + 3 unknown values $\{\lambda_i\}$, a,b,c may be obtained by N + 3 equations:

$$V_k(x_i,y_j) = \sum_{i=1}^{N} \lambda_i R_{ij}^2 \operatorname{Log} R_{ij} + ax_j + by_j + c,$$

$$\text{for } \{j = 1,N\} \text{ with } R_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2} \text{ and,}$$

$$R_{ij}^2 \operatorname{Log} R_{ij} = 0 \text{ when } j = i$$

and

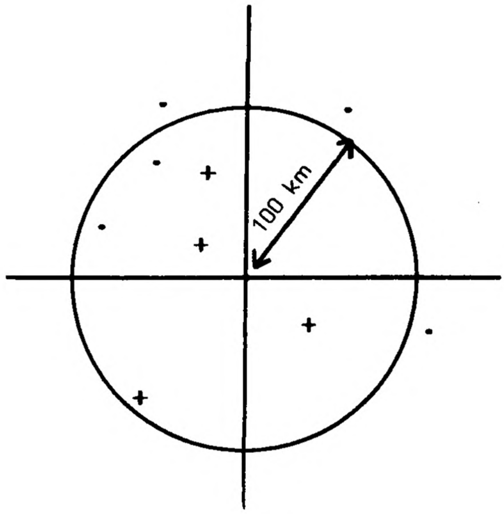
$$\sum_{i=1}^{N} \lambda_i = 0$$

$$\sum_{i=1}^{N} \lambda_i x_i = 0$$

$$\sum_{i=1}^{N} \lambda_i y_i = 0$$

Practical use of this method involves a suitable arrangement of locations where harmonic constants are available and the location where the tide is to be calculated. In particular, it is necessary to avoid extrapolations. For this purpose, the measurement points are sorted in increasing distance order from the point (x,y), and only the two nearest ones in each quadrant are retained. Points which are more than 100 km distant are discarded. If two adjacent quadrants do not contain any available measurement point, the precision is judged insufficient.

Exemple:
- . measurement points
- + measurement points retained for interpolation



Moreover, interpolation across coasts must be avoided.

Justification of the method is given by Duchon (Ref. 2).

— It provides the smoothest surface constrained to pass through given points
— The solution is independent of coordinates (origin, orientation, scales).

Accuracy of results is a function of the density of the reference points and the spatial variation of the harmonic constants.

Figure 3 portrays a co-tidal chart of the Dover Strait obtained for M_2 in application of this method of interpolation.

3. CALCULATION OF ADDITIONAL CONSTITUENTS

As stated previously, interpolated constants are not numerous enough to allow a precise calculation of the tide. Additional constituents are obtained from considerations of Le Provost (Ref. 1):

a) Figures 4 to 7 show that the constituents MS_4, MN_4, $2MS_6$ and $2MN_6$ can be deduced from M_4 and M_6 with sufficient precision.

The following formulae can be deduced:

$$A_{MS_4} = A_{M_4} / 1,5$$

$$A_{MN_4} = A_{MN_4} / 2,9$$

$$G_{MS_4} = G_{M_4} + 53^{\circ}$$

$$G_{MN_4} = G_{M_4} - 25^{\circ}$$

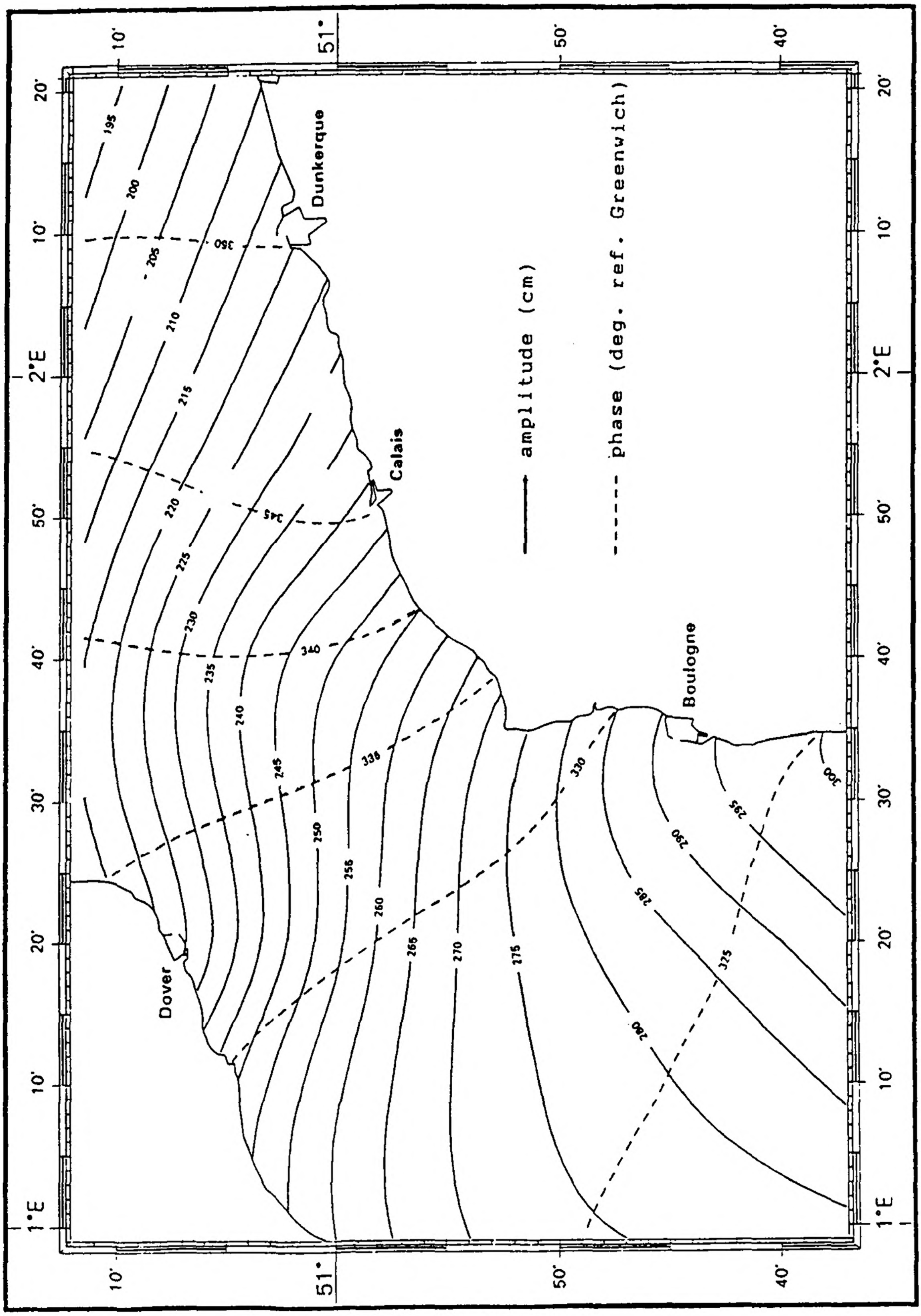


FIG. 3.— Co-tidal lines of M_2 constituent.

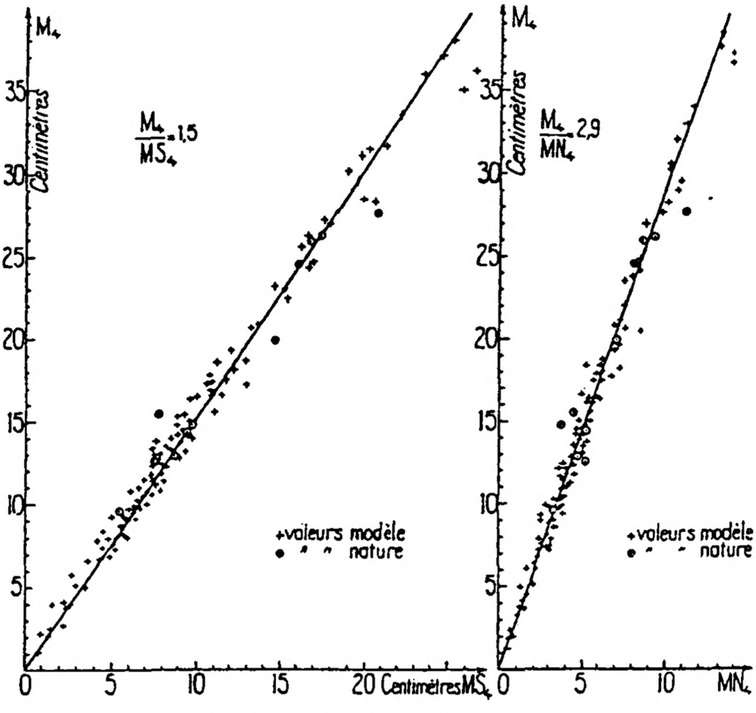


FIG. 4.— Amplitudes of MS_4 and MN_4 constituents.

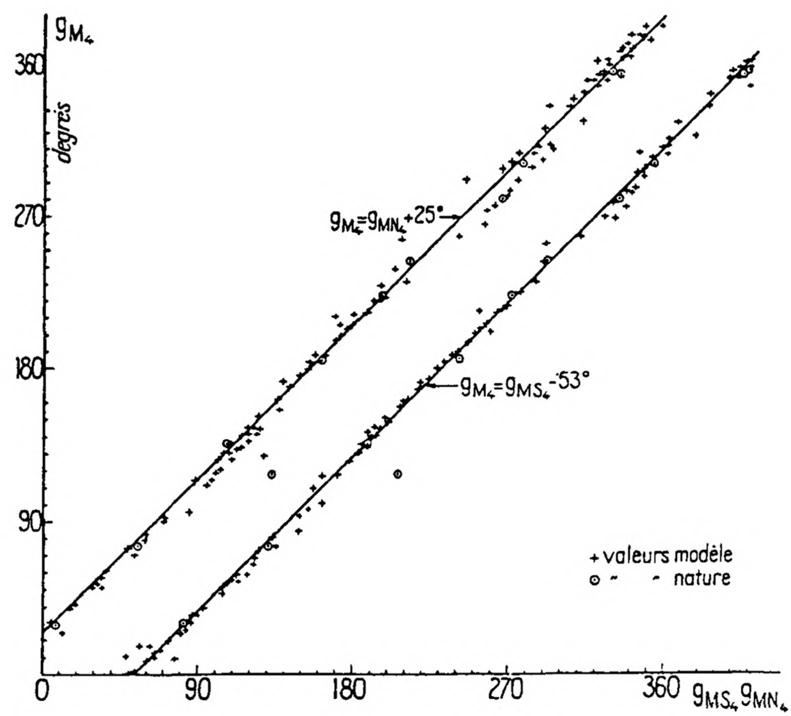


FIG. 5.— Phases of constituents MS_4 and MN_4.

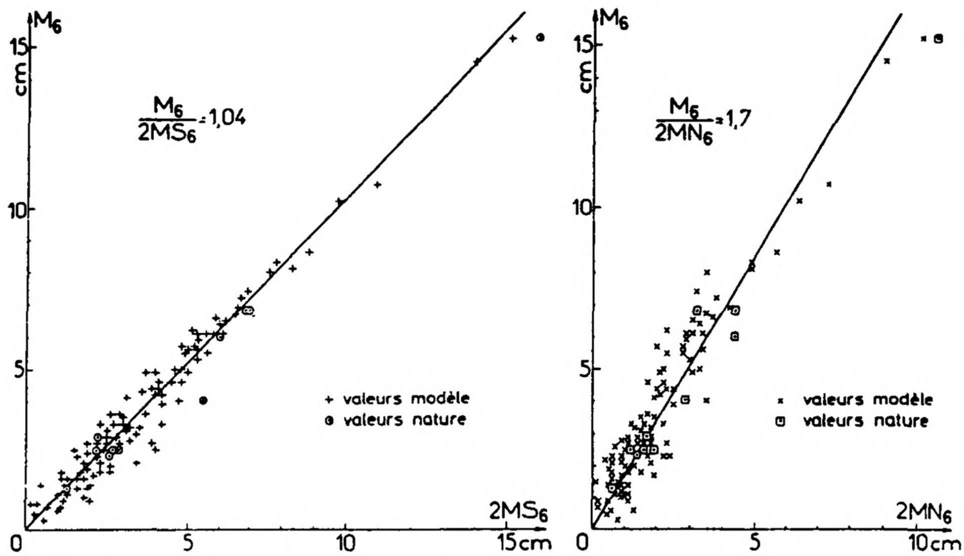


FIG. 6.— Amplitudes of constituents $2MS_6$ and $2MN_6$.

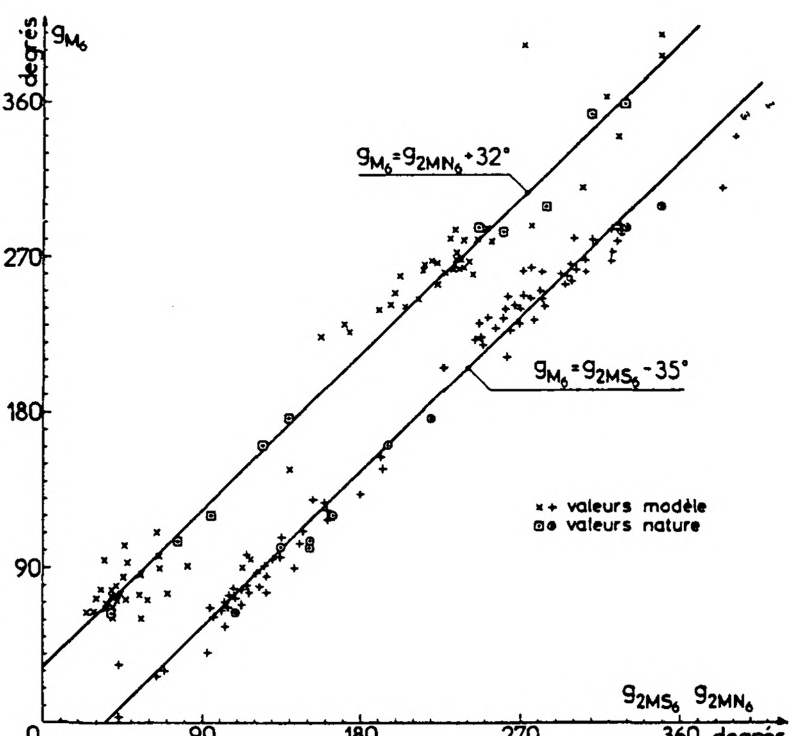


FIG. 7.— Phases of constituents $2MS_6$ and $2MN_6$.

The following formulae can be deduced from figures 6 and 7.

$$A_{2MS_6} = A_{M_6} / 1,04 \simeq A_{M_6}$$

$$A_{2MN_6} = A_{M_6} / 1,7$$

$$G_{2MS_6} = G_{M_6} + 35°$$

$$G_{2MS_6} = G_{M_6} - 32°$$

Other constituents are deduced from the following hypotheses:

$$\frac{A_{MK_4}}{A_{MS_4}} = \frac{A_{K_2}}{A_{S_2}} \Longrightarrow A_{MK_4} = A_{K_2} - A_{M_4} / 1,5\ A_{S_2}$$

$$G_{MK_4} - G_{MS_4} = G_{K_2} - G_{S_2} \longrightarrow G_{MK_4} = G_{M_4} + G_{K_2} - G_{S_2} + 53°$$

$$\frac{A_{2MK_6}}{A_{2MS_6}} = \frac{A_{K_2}}{A_{S_2}} \longrightarrow A_{2MK_6} = A_{K_2} \times A_{M_6} / A_{S_2}$$

$$G_{2MK_6} - G_{2MS_6} = G_{K_2} - G_{S_2} \longrightarrow G_{2MK_6} = G_{M_6} + G_{K_2} - G_{S_2} + 35°$$

Constituents $2N_2$, ν_2, L_2, λ_2 and T_2 are calculated from application of the following formulae, based on the hypothesis that phase lag and ratio of amplitude to corresponding coefficient of generating potential vary linearly with frequency.

$$A_{2N_2} = 0,265\ A_{N_2} - 0,0253\ A_{M_2} \qquad G_{2N_2} = 2\ G_{N_2} - G_{M_2}$$

$$A_{\nu_2} = 0,165\ A_{N_2} - 4,71 \times 10^{-3} A_{M_2} \qquad G_{\nu_2} = 0,87\ G_{N_2} + 0,13\ G_{M_2}$$

$$A_{L_2} = 3,27 \times 10^{-2} A_{S_2} + 1,3 \times 10^{-2} A_{M_2} \qquad G_{L_2} = 0,54\ G_{S_2} + 0,46\ G_{M_2}$$

$$A_{\lambda_2} = 7,46 \times 10^{-3} A_{S_2} + 3,91 \times 10^{-3} A_{M_2} \qquad G_{\lambda_2} = 0,47\ G_{S_2} = 0,53\ G_{M_2}$$

$$A_{T_2} = 0,059\ A_{S_2} \qquad G_{T_2} = G_{K_2} - 0,1122\ (G_{K_2} - G_{M_2})$$

Annual constituent Sa is also introduced, but its value at Brest is uniformly adopted:

$$A_{Sa} = 5,8 \text{ cm}$$

$$G_{Sa} = 229,5°$$

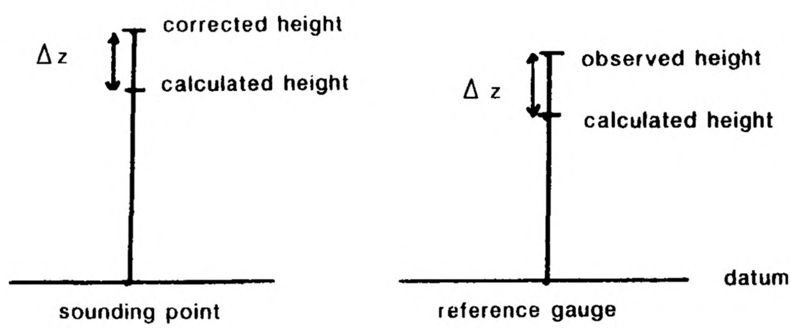
4. CALCULATION OF CORRECTIVE TERMS

The height of the tide is therefore calculated by means of 23 constituents.

In order to improve precision, we have to take into account meteorological effects and prediction faults. As a basic hypothesis, it is stated that the differences between predictions and observations are equal in a nearby harbour and at the sounding location. For that purpose, a tide gauge is set up as close as possible to the sounding area.

The water height at the tide gauge location is calculated by means of 23 constituents, even if, as is usual, a more precise calculation might be possible.

The difference between observed and predicted tides at the tide gauge location is used to correct the predicted tide at the sounding location.



Utilization of automatic seabed tide gauges has made it possible to improve the efficiency of this procedure as it enables tide observations close to the area of interest.

These tide gauges are left in place for a period of one month in order to make it possible to determine the datum related to the reference harbour datum.

This duration is also sufficient to calculate reliable harmonic constants, so that the density of the measurement points of the model can be increased.

5. MEAN LEVEL, TIDE AREA

In the harmonic formula, Z_0 is the mean sea level, a constant the value of which depends on the choice of datum. The latter is situated close to the lowest astronomical tides (lowest low water).

However, in many cases, chart datums were chosen at a time when tide observations were not sufficient for a precise determination to be made. In France, the traditional rule was to choose chart datum so that mariners should rely on more water depth than that shown on the chart. For the sake of security, it resulted that chart datum, in France, was generally situated slightly below lowest low-tide level.

Due to difficulty in changing traditional datums of important harbours and in order to preserve as far as possible, the continuity of chart datum, tide areas have been defined.

A tide area is defined by its geographical boundaries and a reference harbour where chart datum has been determined.

Figure 8 represents the boundaries of tide areas along the north coast of France (Ref. 3). By means of a 'concordance' relation, it is possible to determine the datum at any point in the area from simultaneous observations at that point

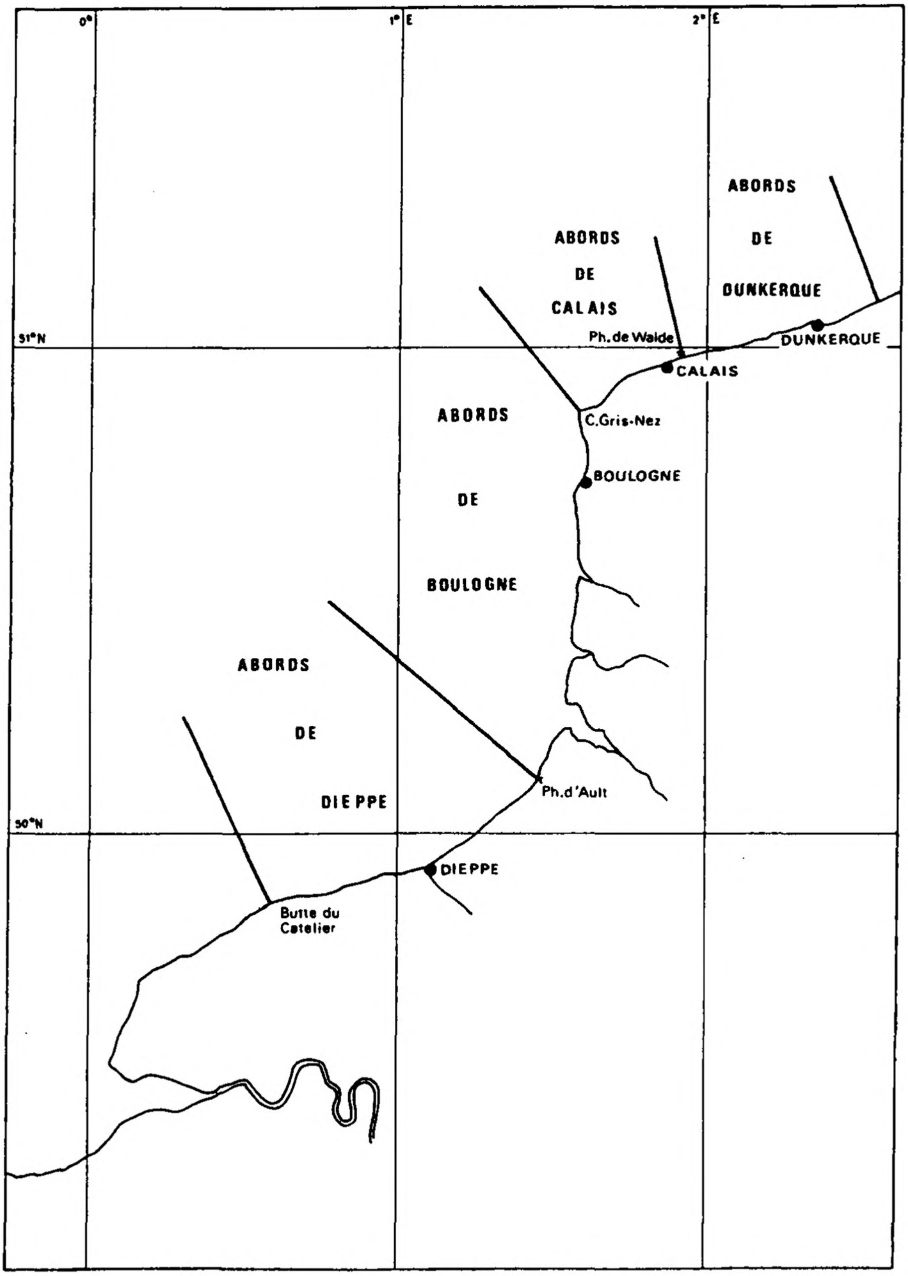
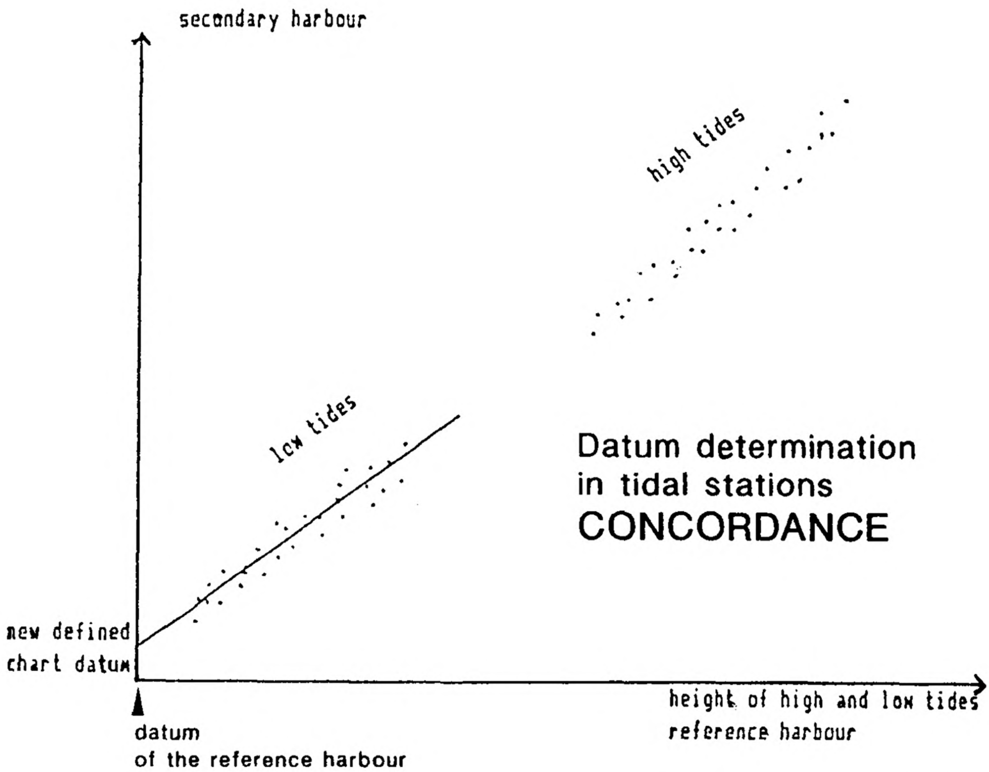


FIG. 8.— Tide areas, North Coast of France.

and at the reference harbour. The points whose coordinates are the heights of corresponding high and low tide at the reference harbour (X-axis) and at the secondary harbour (Y-axis) are shown on the same graph.



The intersection of the regression line fitting the low tide points with the X-ordinate of the reference harbour datum defines the secondary harbour datum.

When no simultaneous observation is available at sounding location (it is the common situation for offshore sounding) the following method is used:

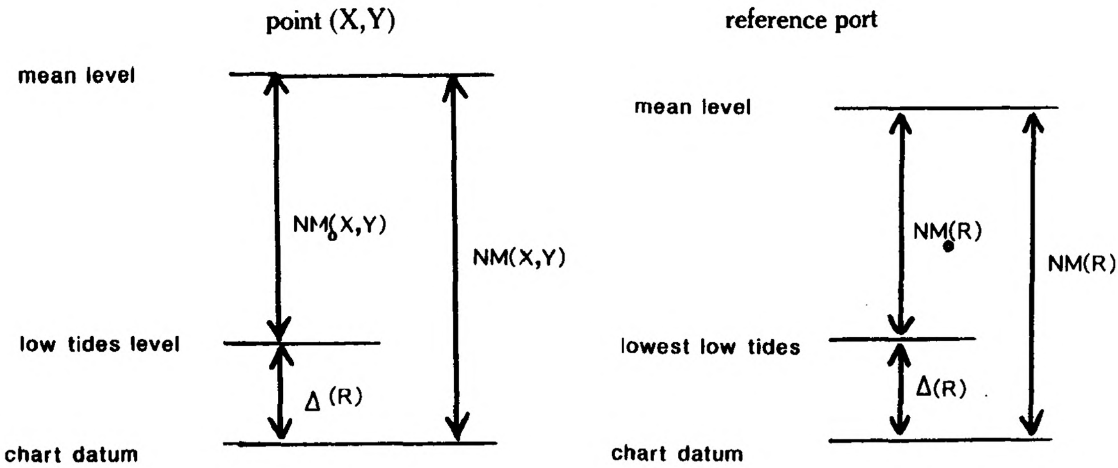
The lowest low-tide that had ever been predicted in France occurred on 14th March 1918. The mean sea level referred to the lowest low tide is obtained by resolving

$$h(t,X,Y) = 0 = Z_{00}(X,Y) + \sum_{i=1}^{N} A_i\,(X,Y) \cos\,[V_i(t) - G_i(X,Y)]$$

for the time t of the lowest low-tide close to this date.

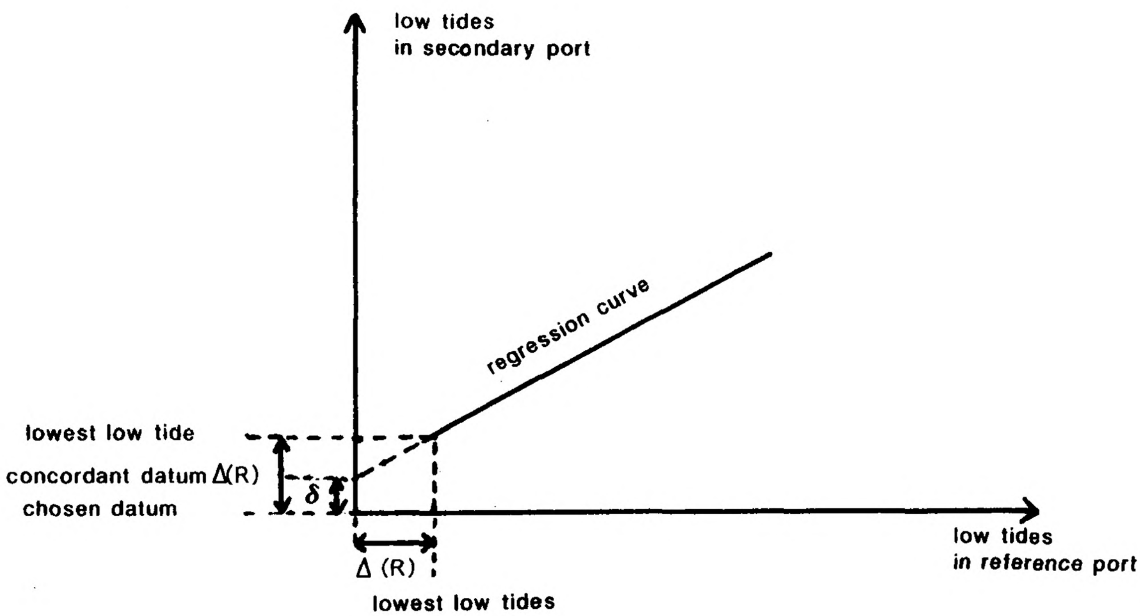
The same calculation is performed for the reference harbour. We obtain $Z_{00}(R)$ which is the mean level referred to the lowest low-tide at the reference harbour. Let $Z_0(R)$ be the mean level refered to the datum, the difference $\Delta(R) = Z_0(R) - Z_{00}(R)$ gives the means to obtain mean sea level $Z_0(X,Y)$.

$$Z_0(X,Y) = Z_{00}(X,Y) + \Delta(R)$$



This method is not exactly equivalent to a concordance, but the induced error is very small as shown below.

The gradient p of the regression curve nearly equals 1 in a same tide area. On account of the generally small value of Δ(R), the error $\delta = \Delta(R)\ (1-p)$ can be considered as negligeable.



6. APPLICATIONS

a) Deferred time

Offshore survey hydrographers (mission hydrographique) are able to make tide computation with the help of the French Navy Hydrographic and Oceanographic Service's Main Establishment (EPSHOM) computer, thanks to a telephone connection.

The following operations are required:

— Tide observations in the reference port.
These data should be faultless for any observation error is wholly transmitted to the sounding correction.

— Offshore tide observation close to the sounding area permits model precision verification by means of comparison between observed and computed tides.

Large divergences are investigated by the team in charge of tide studies and may lead to an adjustment of the model.

Furthermore, these observations are analysed to improve density and quality of the harmonic constants of the model.

Successive model states are saved so that previous computations might be performed again.

Heights of tide are calculated for every location in the survey.

The required equipment comprises:

HP 9814 (Hewlett Packard) computer
HP 2225 printer
HP 9122 diskette reader
Telephone connection, equipped with MODEM.

b) Real time

EPSHOM has just acquired a shallow-water multibeam echo-sounder, installed on a new hydrographic survey vessel.

Performing the survey involves real-time processing of results.

The offshore tide prediction program was transcripted to an IBM-PC compatible micro-computer, but this version does not involve observed tide correction, owing to the absence — up to now — of transmitting tide gauges.

7. FUTURE IMPROVEMENTS

Offshore tide computation as described here involves some faults, especially connected, for some areas, to the insufficient number of harmonic constants taken into account and to the coarse surge correction. Studies of these problems are now in hand at EPSHOM.

Tide table computation for Le Havre requires 143 constituents which were calculated from 10 years of continuous observations. With only 23 constituents, the model for sounding reduction obviously cannot attain the same precision. However, each of the additional constituents is neglibeable. Their importance derives from their number.

This means that an appreciable improvement of the results would require a greater number of additional constituents to be taken into account. This is not feasible at the present time.

The new method will be based on the fact that a small number of complex coefficients are sufficient to compute the spectral representation of the tide in a port from that of another port, if not too distant. This is the basis of the species concordance method which provides an efficient method of analysis of short duration tide and current observations.

It can be presented as a description of heights of water ratios at the sounding location and at the reference port, in terms of input — output of a 'black box'. It is therefore natural to look for relations between input and output use function of frequency. These relations are not linear.

Meteorological perturbations, introduced as long period species are easily taken into account by this method.

At present, meteorological correction is simply transported from reference port to sounding location. A mapping of long period species transfert function should help to improve this correction. It could be obtained from hydrodynamic storm-surge models available for North Sea and English Channel.

A study is in hand on at EPSHOM to adapt the species concordance method to the sounding reduction model with the help of objective analysis techniques in order to take advantage of all available information: observations, tide, current and surge models.

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