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ALGORITHM FOR THE CALCULATION OF GEODETIC DISTANCES FOR MARITIME JURISDICTIONAL BOUNDARIES

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Abstract

This paper presents an algorithm which can be applied for the calculations of distances of hundred kilometres from the co-ordinates of their end points.

Analysis of the simplifications are also presented and could be adopted for applications related to the Law of the Sea.

INTRODUCTION

Various solutions are possible to solve the problem of calculating the distances from latitude and longitude coordinates (so-called "second fundamental geodetic problem"). The complexity of the solutions increases with the longitude between end points.

Among the different programmes available, not all allow a quick change of elliptic parametres when their application to different reference systems is necessary. A few years ago, the accuracy in the calculation of distances larger than 150 km was hardly needed for Law of the Sea purposes. In most cases, distances involved did not exceed 40 km. The application of the United Nations Convention on the Law of the Sea has resulted in a need for the calculation of distances of 200 international nautical miles (1,852 m), i.e. 370,4 km.

In the exploitation of resources and during the control of operations related to this subject, it is frequently necessary to check if the distance of a position (obtained from a satellite positioning system) to the baseline or the line from which

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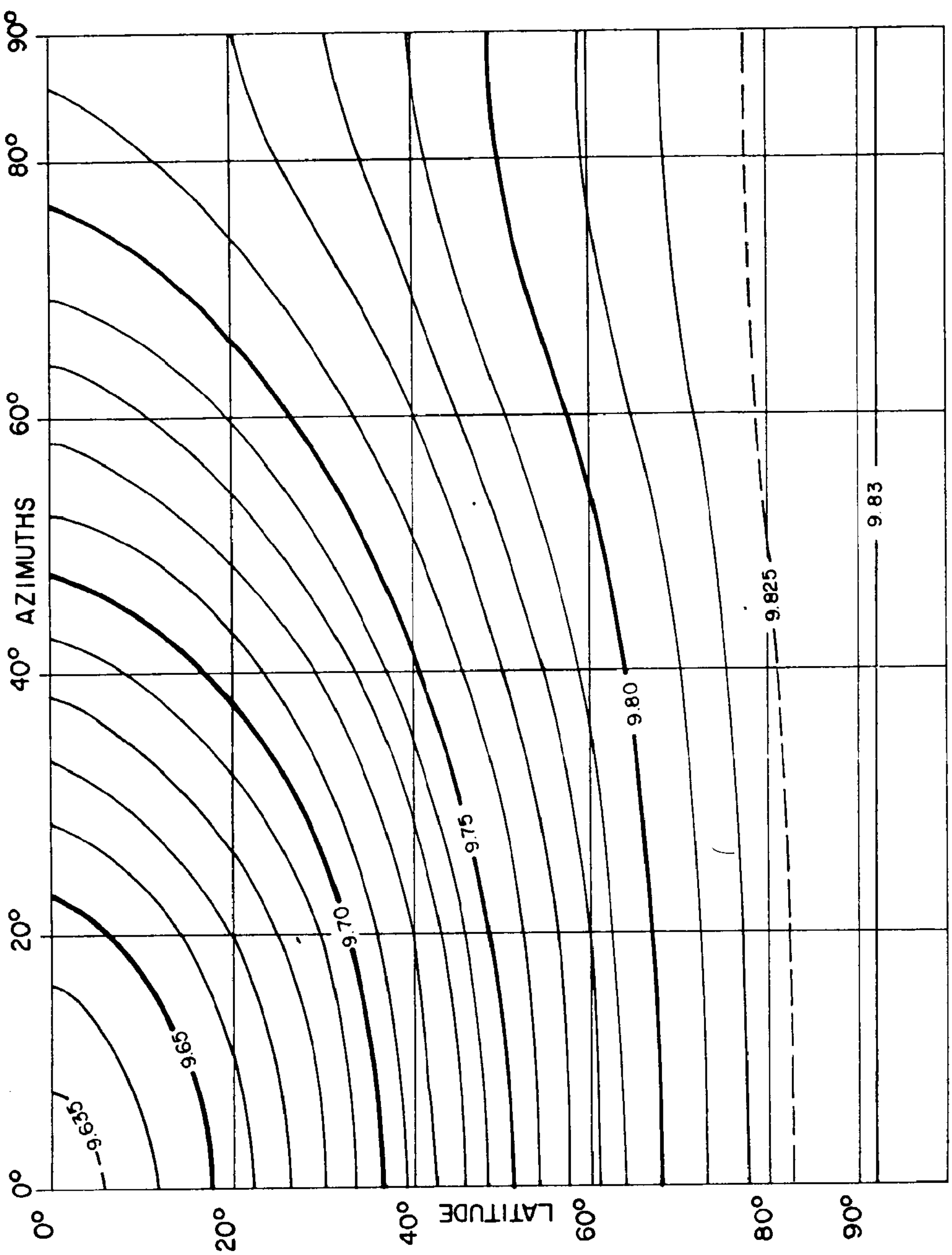

DIAGRAM FOR OBTAINING K (As a function of Latitude and Azimuth)

After obtaining K, the geodetic arc S must be calculated:

$$S = c + \frac{c^3}{K \times 10^{14}}$$

c and S being expressed in metres.

The following examples could clarify the application of the expressions given above:

Ex.	Latitude	Longitude	c	K	S
1	-45 00 00.000 -42 28 02.765	-65 00 00.000 -61 38 49.700	389939.18	9.763	389999.91
2	-45 00 00.000 -47 29 04.821	-65 00 00.000 -61 14 49.203	399934.56	9.763	400000.00
3	-45 00 00.000 -47 21 48.485	-65 00 00.000 -68 33 25.715	379943.88	9.763	380000.02
4	-45 00 00.000 -42 39 56.504	-65 00 00.000 -68 06 17.272	359952.17	9.763	359999.94
5	-45 00 00.000 -44 34 31.558	-65 00 00.000 -59 58 48.644	399934.71	9.795	400000.02
6	-34 3500.0000 -30 2122.2148	-58 2200.0000 -53 3607.5169	647920.06	9.742	648199.24
7	-34 3500.0000 -34 2245.4562	-58 2200.0000 -51 1848.5387	647922.15	9.785	648200.12
8	-51 1755.0000 -45 2810.3983	-58 2758.0000 -58 2758.0000	647920.49	9.747	648199.50
9	-51 1755.0000 -47 0041.2043	-58 2758.0000 -52 2616.4277	647920.95	9.774	648199.25
10	-51 1755.0000 -50 5553.9524	-58 2758.0000 -49 1318.8437	647922.99	9.803	648200.46

Ellipsoid used "Int. Madrid 1924".
(a = 6378388 m and 1/f = 297)

(*) The values of Latitude and Longitude are given in degrees, minutes and seconds, and c and S are given in metres.

The adopted approximation $C^3/24\ R^2$ is reduced to the second term of the series. The difference with the complete expression reaches the following values:

Distance (Km)	Error (m)
400	0.04
600	0.27
800	0.90

These are acceptable within the range of maritime jurisdiction (Exclusive Economic Zone, Continental Shelf and Edge).

APPROXIMATE FORMULAS

Making some simplifications, the expressions given above are transformed into simpler ones. Taking:

$$\frac{1}{\sqrt{l-f\,(2-f\,)\,\sin^2\varphi_1}} \sim \frac{1}{\sqrt{l-f\,(2-l\,)\,\sin^2\varphi_2}} \sim 1 + f\,\sin^2\left(\frac{\varphi_1+\varphi_2}{2}\right)$$

$$(l-f\,)^2 \sim (l-2f\,) \qquad K \sim 9.77$$

$$c = a\left[1+f\,\sin^2\left(\frac{\varphi_1+\varphi_2}{2}\right)\right]\sqrt{\left[\cos\varphi_2\cos(\omega_2-\omega_1) - \cos\varphi_1\right]^2 + \left[\cos\varphi_2\sin(\omega_2-\omega_1)\right]^2 + \left[(1-2f\,)(\sin\varphi_2 - \sin\varphi_1)\right]^2}$$

$$S = c + \frac{c^3}{9.77\times10^{14}}$$

From the examples given above, the following results are obtained:

Example	c	S
1	389940.95	390001.64
2	389936.13	400001.61
3	379945.38	380001.52
4	359953.80	360001.53
5	399934.23	399999.54
6	647920.92	648200.12
7	647922.20	648200.17
8	647927.39	648206.45
9	647924.04	648202.33
10	647921.21	648198.68

If WGS 84 is used instead of that ellipsoid (a=6378137 m and 1/f=298.25722), the distances are:

Example	c	S
1	389925.97	389986.65
2	399920.02	399985.49
3	379930.10	379986.23
4	359939.95	359987.68
5	399915.76	399981.06
6	647899.64	648178.82
7	647893.77	648171.71
8	647904.87	648183.90
9	647897.54	648175.80
10	647890.18	648167.61

SPHERICAL APPROXIMATION

The ellipsoid calculation gives important differences with respect to the spherical calculation. Navigators generally assume that, if distances are calculated from the formula:

$$\cos s = \sin \varphi_1, \sin \varphi_2 + \cos \varphi_1 \cdot \cos \varphi_2 \cdot \cos \Delta \omega$$

and the value of S in minutes is transformed into miles, then the accuracy of the calculation will be sufficient. The list below gives various examples:

Example	cos S	S	Sm(S *1852)
1	0.9981304278	210.2459717	389375.54
2	0.9980350963	215.5413790	399182.63
3	0.9982265793	204.7665307	379227.62
4	0.9984069733	194.0698558	359417.37
5	0.9980406311	215.2374922	398619.83
6	0.9948246922	349.9004112	648015.56
7	0.9948517232	348.9846485	646319.57
8	0.9948293329	349.7433616	647724.71
9	0.9948443944	349.2331739	646779.84
10	0.9948615579	348.6508715	645701.42

This demonstrates that differences of various kilometers can result from the above mentioned assumption.

APPLICATION TO DEFINING THE LIMITS OF MARITIME JURISDICTIONS

One of the more concrete applications of the calculation of long geodetic distances is the establishment and the verification of the limits of the United Nations Convention on the Law of the Sea.

In addition to the problem discussed in the Introduction about checking if the distance of a given position to the points defining baselines is larger or shorter than 200 international nautical miles, the limits for exploitation of resources must be frequently drawn.

On the other hand, when two coastal States with adjacent or opposite coasts have interests in superposed areas, these States have to solve their delimitation problems by means of bilateral treaties. These treaties are generally based on equidistant lines.

To face this problem, the calculation of long distances from given positions becomes a routine of frequent application.

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