International Hydrographic Review, Monaco, LXX(1), March 1993

ON THE STABILITY OF LONG SERIES TIDAL ANALYSES

by A. S. FRANCO and J. HARARI [1]

Abstract

Fourteen analyses with a nodal cycle resolution of filtered daily values of the 'mean' sea level at Cananéia (Brazil) were worked out, with a one-year shift. The results were examined for the stability of the harmonic constants of constituents Sa, Ssa, Mm, MSf, Mf and Mtm. Simultaneous barometric pressure data were available to correct the filtered tidal heights and the analyses were worked out with and without these corrections.The general conclusion is that the harmonic constants of Sa and Ssa were not stable and that their phase lags increased in the successive analyses.

The harmonic constants of the remaining long period constituents were absolutely unstable and completely unreliable. Other fourteen harmonic analyses of $10*2^{14}$ hourly tidal heights were worked out, for determining the harmonic constants of constituents with higher frequencies, in the same port. The results of these analyses, with a one-year shift,were very stable and gave an excellent insight of the micro-structure of the phenomenon.

1 - INTRODUCTION

This research used tidal and atmospheric pressure data from Cananéia, Brazil, located at 25^0 01.0′ S 47^0 55.5′ W, from 1957 to 1989; it was developed into three steps.

The first step started with fourteen harmonic analyses of the filtered daily values of the 'mean' sea level (GODIN, 1972). This analysis was performed using the refined method of tidal analysis (FRANCO, 1978), where the mentioned values were weighted with a cosine taper before working out each analysis with the

[1] Instituto Oceanográfico of the São Paulo University, Brazil.

one-year shift. These processings were performed in order to study the stability of constituents Sa, Ssa, Mm, MSf, Mf and Mtm.

The second step was to study the behaviour of the results when the filtered pressure corrections were straightforward applied to the filtered values of the 'mean' sea level and when these corrections were introduced according to the CRAWFORD (1982) technique.

The third and last step was to work out 14 analyses of $10*2^{14}$ hourly heights, in order to study the stability of the harmonic constants of the constituents with one cycle per day onwards.

The results shown in this paper will be examined in detail.

2 - LOW FREQUENCY CONSTITUENTS

Table 2-I shows the harmonic constants H (cm) and G (degrees) of constituents Sa and Ssa, obtained from analyses worked out without pressure corrections. Rejected constituents are flagged with asterisks, which means that they did not pass the statistical test of significance.

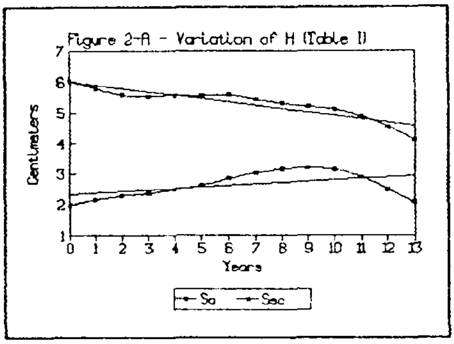
Table 2-I

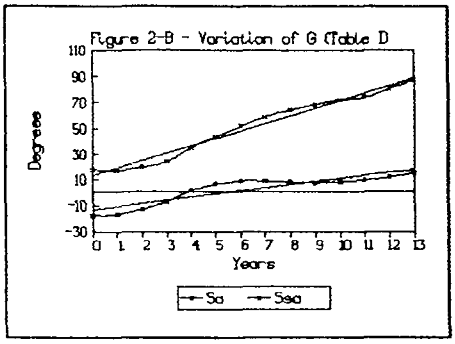
Harmonic constants of Sa and Ssa from analyses without pressure corrections

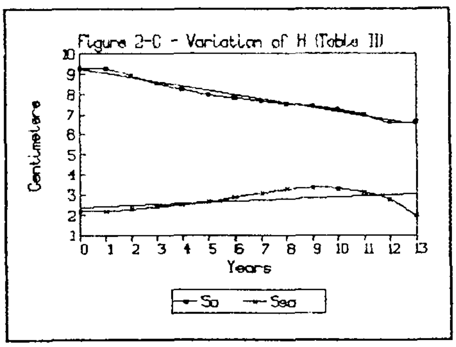
Years	Sa				Ssa			
	H (cm)	±	G (deg.)	±	H (cm)	±	G (deg.)	±
57-76	6.02	2.43	342.33	23.82	1.98	0.79	17.87	23.39
58-77	5.79	2.41	342.78	24.56	2.15	0.85	16.92	23.37
59-78	5.57	2.33	347.38	24.71	2.25	0.91	19.71	22.73
60-79	5.53	2.19	353.01	23.27	2.35	0.94	23.86	23.56
61-80	5.56	1.93	1.18	20.26	2.48	0.94	38.85	22.32
62-81	5.55	1.58	6.15	16.54	2.61	0.88	47.70	19.64
63-82	5.57	1.31	8.66	13.58	2.84	0.79	52.92	16.12
64-83	5.43	1.09	8.51	11.62	3.02	0.69	58.88	13.21
65-84	5.31	1.01	7.80	10.95	3.16	0.62	64.44	11.35
66-85	5.20	0.99	7.33	11.01	3.20	0.64	68.65	11.61
67-86	5.10	1.03	8.00	11.62	3.14	0.74	71.89	13.72
68-87	4.88	1.13	9.25	13.32	2.89	0.88	73.69	17.67
69-88	4.55	1.29	11.82	16.54	2.49	1.00	80.67	23.72
70-89	4.14	1.47	15.58	20.81	2.06	1.06	87.39	31.01

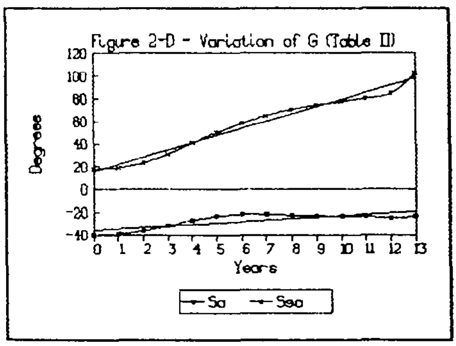
One can see from that table that both amplitude and phase lags, especially the latter, exhibit considerable variations.

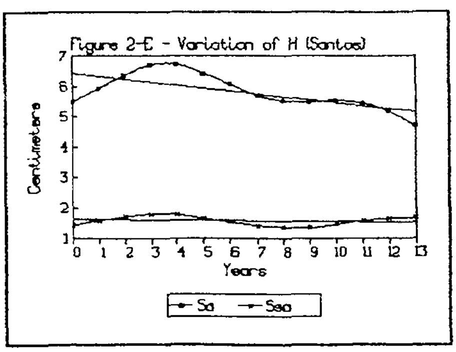
It is usually assumed that constituents Sa and Ssa are generated by meteorological forces. So, in order to detect the effect of these forces,fourteen analyses after correcting the tidal heights for the atmospheric pressure were performed, by considering the sea as an inverted barometer. The results of these analyses are shown in Table 2-II. As one can see, these results also show large variations in amplitudes and phase lags, exactly in the same directions as before.

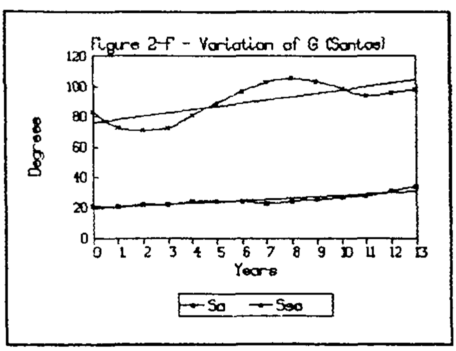












Again fourteen analyses were worked out, now with pressure corrections according to CRAWFORD (1982). In this technique, only the FOURIER lines for which the coherence is high (≥0.86) were corrected for the pressure effect. The corresponding results are nearly the same as shown in Table 2-II.

From Figures 2-A to 2-D one can see that the variations of H and G for Ssa are almost unaffected by the pressure corrections, notwithstanding the method used to make them. However, different characteristics are found when checking the harmonic constants of Sa. In fact,the G values for Sa are decreased by about 20 to 40 degrees when the pressure corrections are used, although keeping the same increasing pattern (Fig. 2-B and 2-D). Figures 2-A and 2-C suggest that H varies periodically about an inclined straight line with one cycle in thirteen years, approximately. The slope of the straight line indicates that the average value of H decreases.

How to justify the large deviation of G for Sa when pressure corrections are used? In addition, how to explain the considerable drifts of G for both Sa and Ssa? As far as these drifts are concerned, one can think of wrong frequencies used in the analysis but there is no theoretical basis to change those frequencies by empirical ones. Is it possible that the meteorological effect does not have an exact cycle of one-year?

Table 2-II

Harmonic constants of Sa and Ssa from analyses of data corrected for atmospheric pressure

Years	Sa				Ssa			
	H (cm)	±	G (deg.)	±	H (cm)	±	G (deg.)	±
57-76	9.40	2.40	321.10	15.36	2.20	0.87	19.57	23.00
58-77	9.27	2.46	321.10	15.36	2.22	0.87	19.05	23.05
59-78	8.90	2.43	323.66	15.82	2.29	0.95	23.75	24.60
60-79	8.53	2.31	328.34	15.69	2.41	1.00	31.29	24.51
61-80	8.24	2.06	332.91	14.49	2.52	1.02	40.84	23.87
62-81	7.97	1.70	336.45	12.31	2.64	0.95	50.22	20.93
63-82	7.82	1.39	338.57	10.25	2.89	0.85	58.47	17.03
64-83	7.66	1.15	338.31	8.61	3.09	0.73	65.35	13.72
65-84	7.53	1.05	337.51	8.01	3.26	0.64	70.68	11.41
66-85	7.42	1.04	336.81	8.04	3.34	0.64	74.44	11.01
67-86	7.27	1.08	336.61	8.53	3.31	0.71	77.29	12.42
68-87	6.99	1.17	336.31	9.67	3.11	0.84	80.76	15.64
69-88	6.57	1.34	335.01	11.74	2.75	0.98	84.77	20.90
70-89	5.65	1.72	336.71	17.61	1.99	1.04	102.36	31.39

Fourteen analyses for Santos (23° 55′ S,46° 18′ W) were also worked out and the results are shown in Figures 2-E and 2-F. No pressure corrections were made.The behaviour of constituent Sa is nearly the same as for Cananéia. However, constituent Ssa has a smaller amplitude and the drift of G is only 26° in the 14 years. During the IUGG meeting (August, 1991) Dr. Eckart SCHUMANN (personal communication) presented the interannual wind variability on the South and East Coasts of South Africa, where he showed that the average wind direction both at Port Elizabeth and Cape Town changed about 20° in 32 years. Probably such variation has something to do with the drift of Ssa, which is less in Santos where its amplitude is also less than that at Cananéia.

The next point of interest is concerned with the behaviour of the harmonic constants of the other long period constituents. Table 2-III gives the results of an analysis (without pressure corrections) for the span from 1961 to 1980. The conclusions based on these results hold for other spans. Even when applying either the complete pressure correction or CRAWFORD's technique, the results were not improved.

In Table 2-III the asterisks indicate the rejected constituents and the subscripts A to E correspond to the satellites in the decreasing order of magnitudes. This table shows that several satellites are larger than the respective main constituents. Such anomally occurs in several of the 14 analyses, even for the MSf constituent, which seems to be well behaved in the given example. Thus, in conclusion, all the results corresponding to frequencies above 0.5421921 °/h are not relieable. A last remark on the long period constituents discussion is that they were also included in the analyses of 163840 hourly tidal data but the results were worse than those obtained from the analyses of the daily values of the 'mean' sea level.

Table 2-III

Results of an analysis of data taken at Cananéia from 1961 to 1980

No.	Symbol	Frequency cm	H	C.I. ±	G Deg.	C.I. ±
1	Sa	0.0410686	5.56	1.93	1.18	20.26
2	Ssa	0.0821372	2.48	0.94	34.85	22.32
3	*Mm(A)	0.5421921	****	0.94	137.12	****
4	*Mm	0.5443985	****	1.24	16.08	****
5	Mm(B)	0.5466049	1.20	0.94	157.03	51.66
6	Mm(C)	0.5536345	2.64	0.94	88.86	20.84
7	Mm(D)	0.5558409	2.72	1.24	283.53	27.16
8	Mm(E)	0.5580473	2.20	0.94	124.27	25.24
9	MSf(A)	1.0136894	1.22	0.70	174.22	35.26
10	MSf	1.0158958	2.86	0.93	78.27	18.97
11	*MSf(B)	1.0181022	0.28	****	48.70	****
12	Mf(A)	1.0887970	1.35	0.61	42.24	26.70
13	Mf	1.0980330	1.26	0.94	318.04	48.47
14	Mf(B)	1.1002394	1.45	1.25	23.13	59.23
15	Mf(C)	1.1024458	1.09	0.94	257.81	59.47
16	Mtm(A)	1.6331955	1.03	0.45	351.79	25.63
17	*Mtm	1.6424315	1.41	****	248.61	****
18	*Mtm(B)	1.6446379	0.35	****	15.65	****
19	*Mtm(C)	1.6468443	0.68	****	315.37	****
20	Mtm(D)	1.6560803	0.98	0.45	42.39	27.06

3 - STABILITY OF CONSTITUENTS OF HIGHER FREQUENCY

Notwithstanding the enormous quantity of constituents (1014) which can be separated by the 163840 hourly data analysis, some of them were selected, to study their behaviour (Fig. 3-A to 3-R). Such figures were drawn from tables similar to 2-I to 2-III,which were not reproduced here in order to lighten the text.

Much attention must be paid to the very different scales of the mentioned figures,otherwise some wrong conclusions may be formulated about the long term variations of the harmonic 'constants'. At first glance, it is amazing the extraordinary accuracy of the results,which allows to detect variations of about tenths of millimeters for the amplitudes and tenths of degrees for the phases.

By considering only the linear regressions of the variations shown in Figures 3-A to 3-R, Table 3-I was drawn with the mean amplitudes and phase lags (H_0 and G_0), the average yearly variations (AYV) and the standard deviations (SD),in amplitudes and phase lags,for the nine constituents taken into consideration.

Figures 3-A to 3-R show that both the amplitudes and phase lags oscillate about values which are not strictly constants. In fact, the straight lines clearly indicate drifts in the results of subsequent analyses. Only the amplitude of M^3, shown in Figure 3-M, suggests some kind of periodic variation. An interesting remark is that all the phase lags, except those of K_1 and K_2, decrease, but with different rates; on the other hand, the amplitudes of the diurnal constituents decrease (Fig. 3-A and 3-C) while the amplitudes of all the other constituents increase.

Table 3-I - Accuracy of the results

Const.	H_0 (cm)	AYV (cm)	S.D. (cm) ±	G0 (deg.)	AYV (deg.)	S.D. (deg.) ±
O_1	11.3	-0.00046	0.07	84.39	-0.09118	0.23
K_1	6.34	-0.00105	0.03	144.86	0.02505	0.45
N_2	5.76	0.00446	0.05	163.68	-0.19667	0.70
M_2	36.22	0.06057	0.08	94.44	-0.09070	0.36
S_2	23.27	0.06132	0.03	95.89	-0.03158	0.44
K_2	7.44	0.02012	0.07	86.59	0.00448	0.36
M_2	7.41	0.08460	0.08	238.65	-0.24758	0.32
M_4	6.77	0.01611	0.06	210.90	-0.19039	0.56
MS_4	3.40	0.01123	0.03	318.93	-0.16587	0.75

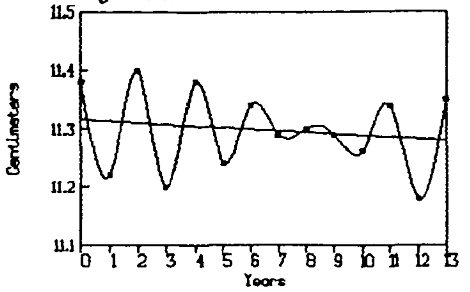


Figure 3-A - Variation of H for O1

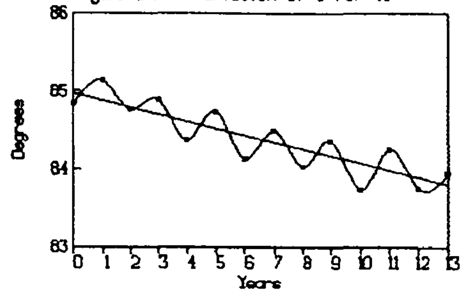


Figure 3-B - Variation of G for O1

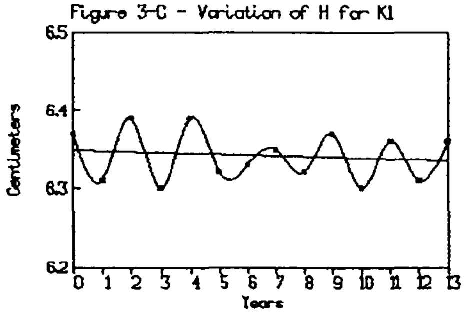


Figure 3-C - Variation of H for K1

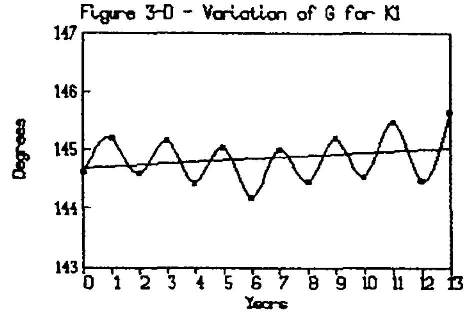


Figure 3-D - Variation of G for K1

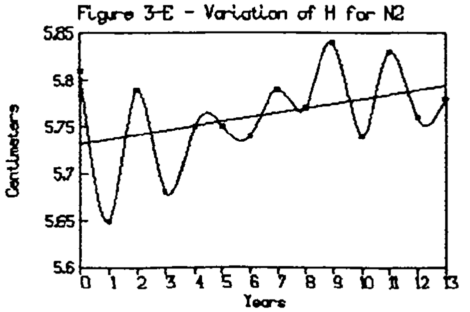


Figure 3-E - Variation of H for N2

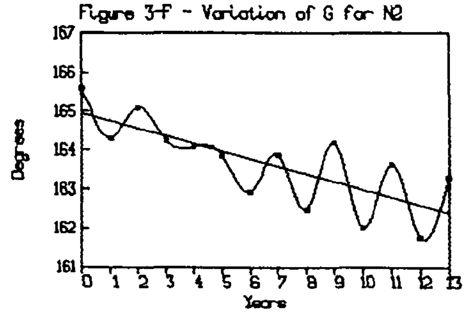


Figure 3-F - Variation of G for N2

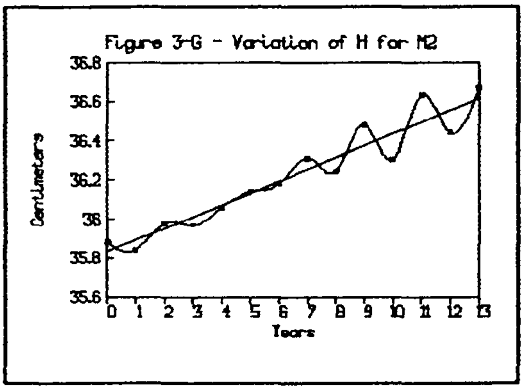


Figure 3-G - Variation of H for M2

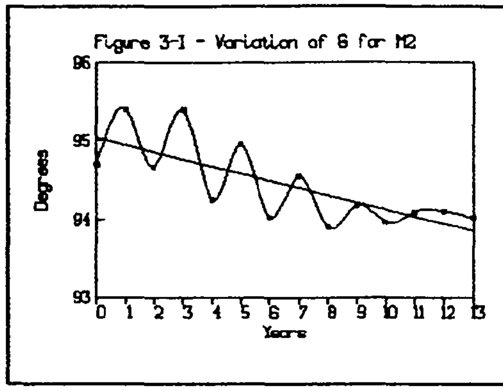


Figure 3-I - Variation of G for M2

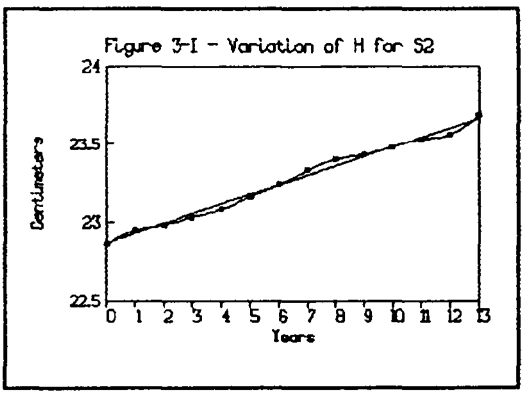


Figure 3-I - Variation of H for S2

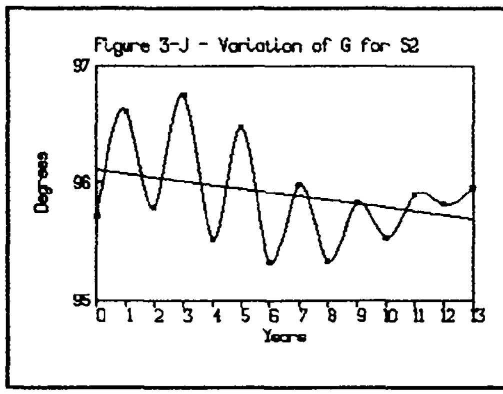


Figure 3-J - Variation of G for S2

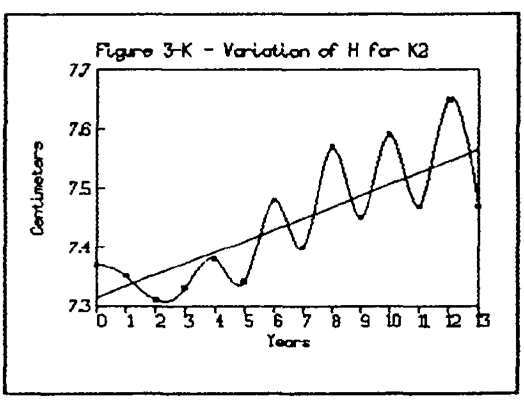


Figure 3-K - Variation of H for K2

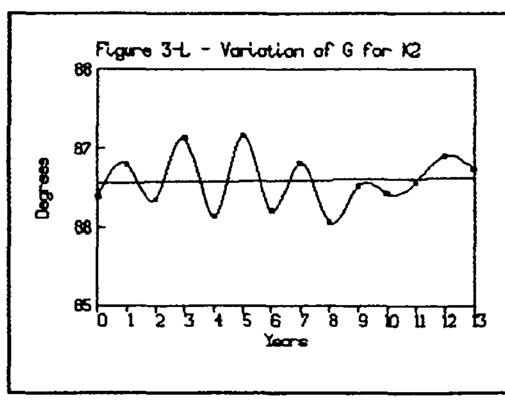


Figure 3-L - Variation of G for K2

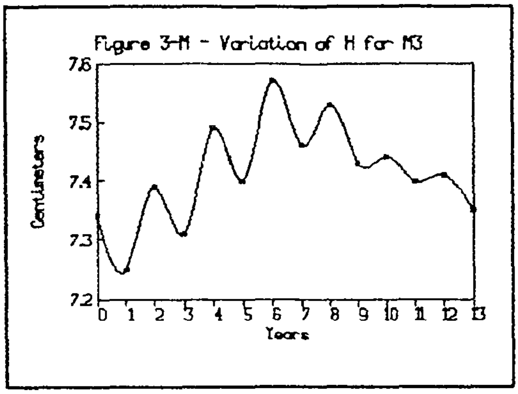


Figure 3-M - Variation of H for M3

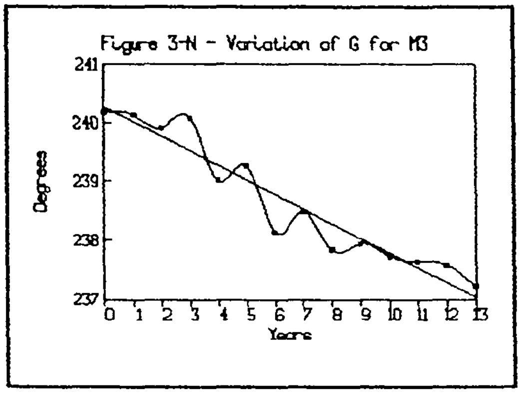


Figure 3-N - Variation of G for M3

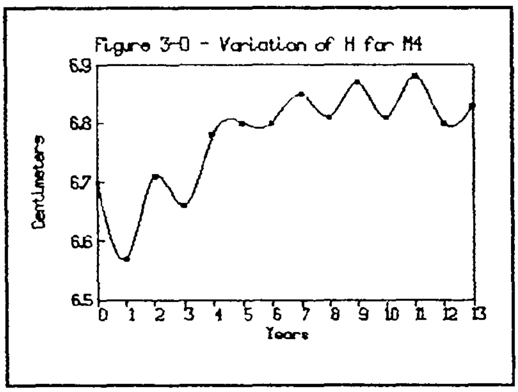


Figure 3-O - Variation of H for M4

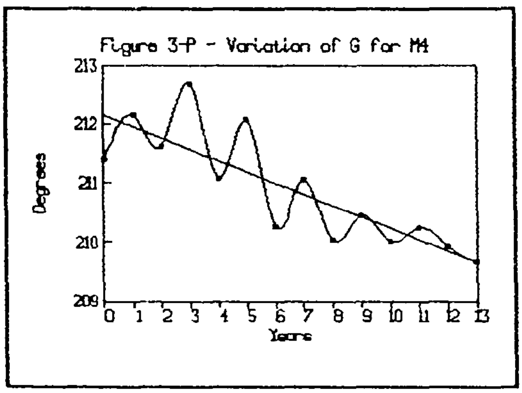


Figure 3-P - Variation of G for M4

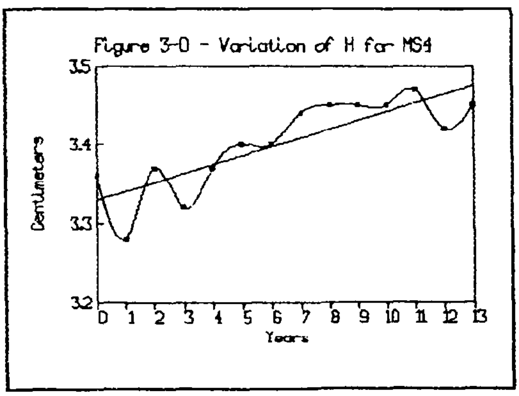


Figure 3-O - Variation of H for MS4

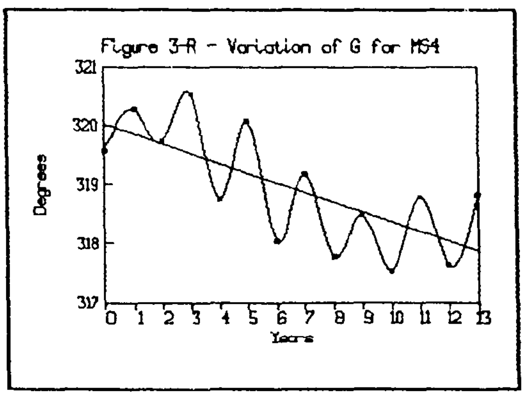


Figure 3-R - Variation of G for MS4

It is well known that each constituent has its own amphidromic system, which can eventually move with the change of the physical characteristics of the water masses and the geophysical medium. If so the very clear variations shown in Figures 3-A to 3-R may be explained by some displacement of the amphidromic systems, as suggested by Professor A.R. de MESQUITA (personal communication).

However, some details of the variations shown in Figures 3-A to 3-R still lack explanation. In fact, there are some very regular oscillations about the fitted straight lines, with an approximate period of two years. Anyway, there is still no plausible reason for the almost linear increasing of the S_2 amplitude (Fig. 3-I), which oscillates very little about the straight line given by the linear regression. In other cases, these oscillations are not so regular, as for the amplitude of M_2 (Fig. 3-G).

An interesting remark is that the shallow-water constituents M_4 and MS_4 show variations in amplitudes and phase lags with patterns similar to the astronomical constituents.

Generally speaking,it has not been possible to draw final reliable conclusions about the observed oscillations in the tidal'constants'.

But similar tendencies have also been observed in other ports, such as the ones in the Bay of Fundy, by GODIN (1991). He detected a 0.15 cm/year increase of the M^2 amplitude, with signs of accelerating, while the S_2 is diminishing at the rate of 0.04 to 0.05 cm/year. GODIN (1991) considers that the tide in the Bay of Fundy, which is strongly affected by frictional effects, is evolving rapidly, probably due to the continuous increase in sea level and a redistribution of sediments in the area.

4 - ACCURACY OF THE RESULTS

Since the results shown in the previous section are given in tenths of a millimeter for the amplitudes and hundredths of degree for the phase lags,it is necessary to comment the accuracy of these results.

In FRANCO-HARARI (1988) the rejection of small constituents was effected as in the analysis of a one-year span. However, this research suggests to reformulate the rejection criterium. In fact, Table 3-I shows such a high accuracy of the results that it may be concluded that the former confidence intervals were exaggerated.

FRANCO & ROCK (1972) established that,according to the theory,the standard deviation of each unknown resulting from the solution of a normal system for the tidal constituents is given by the expression:

$$\sigma_j = \sigma\sqrt{\alpha_{jj}}$$

where:

σ ≡ sampling standard deviation;

σ_j ≡ standard deviation of the j^{th} unknown;

and

α_{jj} ≡ diagonal term of the inverted matrix corresponding to the j^{th} unknown.

In FRANCO & ROCK (1972) α_{jj} was very close to 1 and such value had been kept in the long series analysis too. But, in this case, the diagonal of the inverted matrices of each cluster is, in fact, always very close to 0.0340, the square root of which is nearly 0.18. Consequently, the former confidence intervals of the amplitudes had been wrongly multiplied by 1/0.18 = 5.5. Hence the modification is in order to include the square root of α_{jj} , in the amplitude confidence intervals, for the long period analysis.

The confidence intervals for the phase lags were recomputed, but keeping here the procedure of FRANCO & ROCK (1972).The results are given in Table 4-I which shows how accurate are the harmonic constants derived from the harmonic analyses of a $10*2^{14}$ hours span. Consequently, the detected variations of those 'constants', as given in Table 3-I and Figures 3-A to 3-R, are in fact reliable.

5 - CONCLUSIONS

The first aspect to point out in this research is about the efficiency of the tidal analysis based on harmonic techniques, which low cost gave us the possibility of having a so deep insight of the micro-structure of the phenomenon (FOREMAN & NEUFELD, 1991). In fact, 42 analyses of the filtered 'mean' sea level were worked out to extract the harmonic 'constants' of six long period constituents and their satellites; additionally, 14 analyses of $10*2^{14}$ hourly tidal heights computed the harmonic 'constants' of 1014 constituents. The total computer time consumming was about 50 minutes, in a B-7900 Unisys computer.

From the scientific point of view, the first reached conclusion is that the determination of constituents Sa and Ssa is far from being perfect. As far as constituent Sa is concerned, the spectrum of residual amplitudes shows a constituent with close frequency and from ignored source, which modulates Sa.

With respect to Ssa, it hasn't been possible to formulate any plausible hypothesis about the strong drifts found both in amplitude and phase lag.

The second important conclusion is that the remaining long period constituents were completely unreliable in the series of analyses of 18.69 years data from Cananéia harbour, even using the inverted barometer correction or the Crawford technique to correct the pressure effect.

Table 4-I

Confidence intervals for amplitudes CIH (cm) and phase lags CIG (deg.)

Years	O_1		K_1		N_2	
	CIH ±	CIG ±	CIH ±	CIG ±	CIH ±	CIG ±
57-76	0.03	0.18	0.03	0.25	0.03	0.33
58-77	0.04	0.23	0.04	0.32	0.06	0.62
59-78	0.04	0.18	0.05	0.42	0.04	0.38
60-79	0.04	0.23	0.03	0.30	0.06	0.57
61-80	0.04	0.20	0.03	0.27	0.05	0.48
62-81	0.04	0.21	0.03	0.29	0.05	0.49
63-82	0.04	0.20	0.03	0.29	0.05	0.51
64-83	0.04	0.19	0.03	0.23	0.04	0.42
65-84	0.04	0.21	0.03	0.30	0.06	0.55
66-85	0.03	0.17	0.03	0.23	0.04	0.36
67-86	0.05	0.23	0.04	0.34	0.06	0.59
68-87	0.04	0.19	0.03	0.23	0.04	0.38
69-88	0.05	0.26	0.04	0.34	0.06	0.62
70-89	0.04	0.19	0.03	0.25	0.03	0.35

Years	M_2		S_2		K_2	
	CIH ±	CIG ±	CIH ±	CIG ±	CIH ±4	CIG ±
57-76	0.05	0.08	0.04	0.10	0.04	0.30
58-77	0.10	0.15	0.08	0.21	0.08	0.64
59-78	0.06	0.09	0.05	0.11	0.05	0.36
60-79	0.09	0.14	0.08	0.20	0.08	0.63
61-80	0.07	0.11	0.05	0.13	0.05	0.40
62-81	0.08	0.13	0.08	0.19	0.08	0.61
63-82	0.08	0.13	0.06	0.14	0.06	0.45
64-83	0.07	0.10	0.07	0.16	0.07	0.51
65-84	0.09	0.14	0.07	0.16	0.07	0.50
66-85	0.06	0.09	0.06	0.15	0.06	0.48
67-86	0.10	0.15	0.07	0.18	0.07	0.55
68-87	0.06	0.09	0.06	0.15	0.06	0.46
69-88	0.11	0.18	0.09	0.22	0.09	0.67
70-89	0.06	0.10	0.06	0.15	0.06	0.48

Years	M_3		M_4		MS_4	
	CIH ±	CIG ±	CIH ±	CIG ±	CIH ±	CIG ±
57-76	0.05	0.39	0.02	0.20	0.03	0.50
58-77	0.05	0.43	0.03	0.19	0.03	0.58
59-78	0.05	0.38	0.02	0.20	0.03	0.50
60-79	0.05	0.40	0.03	0.28	0.03	0.57
61-80	0.05	0.03	0.03	0.26	0.03	0.56
62-81	0.05	0.40	0.03	0.28	0.03	0.59
63-82	0.05	0.40	0.04	0.31	0.04	0.62
64-83	0.05	0.37	0.03	0.29	0.03	0.58
65-84	0.05	0.40	0.04	0.34	0.04	0.61
66-85	0.04	0.34	0.03	0.28	0.04	0.61
67-86	0.05	0.42	0.04	0.36	0.04	0.64
68-87	0.05	0.35	0.03	0.24	0.03	0.58
69-88	0.06	0.48	0.05	0.39	0.04	0.64
70-89	0.06	0.45	0.03	0.28	0.03	0.58

The third conclusion is that the amplitudes H and phase lags G of the main harmonic tidal constituents are not strictly constants, but the computed drifts have yet to be explained.

ACKNOWLEDGEMENTS

We are very much indebted to Mr. C.A. de SAMPAIO FRANÇA, who was kind enough to work out the spectral analyses which were necessary to arrive at some important conclusions, and to Dr. SCHUMANN who kindly sent us his not yet published paper presented in the IUGG meeting (Vienna, August, 1991). We are also grateful to Professor A.R. de MESQUITA with whom we had a very fruitful discussion about the results of our research.

References

CRAWFORD, W.R. (1982): Analysis of fortnightly and monthly tides. *International Hydrographic Review*, Vol. LIX(1), Monaco, January.

FOREMAN, M.G.G. & NEUFELD, E.T. (1991): Harmonic tidal analysis of long time series. *International Hydrographic Review*, Vol. LXVIII(1), Monaco, January.

FRANCO, A.S. & ROCK, N.J. (1972): Comparative accuracy of Fourier Tidal Analysis employing different spans with reference to a Doodson analysis. The 2nd International Ocean Development Conference, October, Tokyo, Japan.

FRANCO, A.S. (1978): A refined method of tidal analysis. The Belle Baruch Library in Marine Science No. 7. *Estuarine Transport Process*. Edited by Björn Kjerfve.

FRANCO, A.S. & HARARI, J. (1988): Tidal analysis of long series. *International Hydrographic Review*. Monaco, LXV(1), January.

GODIN, G. (1972): The analysis of tides. Liverpool, University Press.

GODIN, G. (1991): A study of the tide reaching Saint John, New Brunswick, in the Bay of Fundy. Canadian Contractor Report of Hydrography and Ocean Sciences, No. 40.

SCHUMANN, E.H. (1991): Interannual wind variability on the South and East Coasts of South Africa (personnal communication).