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### SPEEDING UP TIDAL ANALYSES WITH PCs

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### Summary

A technique is suggested to effect the tidal analysis with PCs by using the Fast Fourier Transform applied to tidal heights obtained through a second order interpolation of the original data. If a turbo PC XT with mathematical coprocessor is available then it is possible to analyse a 369 days classical span (8856 hourly heights) and separate 170 constituents (up to the twelfth diurnal) in 13 minutes, without jeopardizing the accuracy of the results. If the same data are weighted with a cosine taper (hanning in the time domain), then a more accurate analysis of the same data can be worked out in about 3 minutes. The analyses of non tapered and tapered data will be called *normal* and *refined*, respectively.

### **1 - INTRODUCTION**

A complete set of programs to analyse and predict tides after checking the tidal observations was prepared in the 80s to be used in the main frame computer of the São Paulo University (FRANCO & HARARI, 1987). These programs are still in use in the "Diretoria de Hidrografia e Navegação" (Hydrographic Department) of the Brazilian Navy and in the "Instituto Oceanográfico" of the quoted University.

When the micro computers became available, the present author started to study the possibility of running his programs in the new excellent tool. However, the micro computers of the first generation did not have sufficient memory to work out analyses of hourly heights of one-year spans. But he did not have to wait long for improved machines where his programs could run. Thus, he bought a turbo PC XT with mathematical coprocessor and rewrote his programs in a language more suitable for this machine. Then the development of the micro computers started to run at such an amazing speed that the economical possibilities of this author could not follow any more. Thus he started to optimize these programs in order not to be

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compelled to make a "siesta" after a good launch, while his micro computer worked out lengthy computations.

The use of the method of least squares in the frequency domain is the main characteristic of these harmonic methods of analysis (FRANCO, 1988). Thus, its first step is the Fourier analysis of the either tapered or non tapered heights, for both the refined or normal analysis; this step is performed in order to separate the species. However, adapting the flexible FFT (Fast Fourier Transform) used in main frames in order to run it in micro computers proved to be very difficult. Then the Fourier analysis in the normal method was worked out with the Watt's recurrent formula (CARTWRIGHT-CATTON, 1963) applied to the frequency band of each species. But even so the harmonic analysis of a one-year span, with the normal method, necessary to separate 170 constituents, required about 1 hour and 12 minutes, in a PC XT with coprocessor, whereas the analysis of 2<sup>14</sup> hourly tapered data by the refined method, using the FFT, spent 3 minutes only, with the same machine. Then there were two different problems to be solved: a) to speed up the normal analysis by using the classical FFT, with a Brigham (1974) improvement; b) to remove the constraint of the refined method, limited to analyse time series with number of samples equal to powers of two. The solution of these problems will be shown next.

#### 2 - SUMMARY OF THE METHOD OF ANALYSIS

According to FRANCO (1988), the Fourier coefficients  $a_n$  and  $b_n$  of any tidal series worked out with the normal method can be expressed in terms of the tidal constituents by

$$a_{n}+c=\sum_{j=1}^{Q}(A_{n,j}+A_{n,-j})R_{j}\cos r_{j}$$
(2a)

$$\boldsymbol{b}_{\boldsymbol{n}} = \sum_{j=1}^{Q} (\boldsymbol{A}_{\boldsymbol{n},j} - \boldsymbol{A}_{\boldsymbol{n},-j}) \boldsymbol{R}_{j} \sin \boldsymbol{r}_{j}$$
(2b)

where Q,  $R_j$  and  $r_j$  are, respectively: the number of constituents, the amplitude and the phase of the j<sup>th</sup> constituent at the central time. c is a correction to allow the separation of the set of unknown  $R_j \cos r_j$  from that of  $R_j \sin r_j$  (FRANCO,1988). The matrices  $A_{nj}$  and  $A_{n,j}$  are given by the following relationships:

$$\boldsymbol{A}_{nj} = (-1)^{n} \frac{\sin[(\omega_{n} - \omega_{j})(N+1)\Delta t/2]}{N\sin[(\omega_{n} - \omega_{j})\Delta t/2]}$$
(2c)

$$A_{n,-j} = (-1)^n \frac{\sin[(\omega_n + \omega_j)(N+1)\Delta t/2]}{N\sin[(\omega_n + \omega_j)\Delta t/2]}$$

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and

$$c=2[\zeta(N\Delta t)-R_0]$$
(2d)

where

∆t	the sampling interval
ω	$\equiv$ the Fourier frequency
ω	= the frequency of the $j^{th}$ constituent
Ń	= the number of samples
ζ(N∆t)	= the last sample in the record
R <sub>0</sub>	≡ mean value of the sampled heights

In the refined method expressions (2a) and (2b) reduce to

$$a_{n} = \sum_{j=1}^{Q} A_{nj} R_{j} \cos r_{j}$$

$$b_{n} = \sum_{j=1}^{Q} A_{nj} R_{j} \sin r_{j}$$
(2e)

where

$$A_{ng}(-1)^{n} = \frac{\sin[(\omega_{n}-\omega_{j})(N+1)\Delta t/2]}{N\sin[(\omega_{n}-\omega_{j})\Delta t/2]} - \frac{\sin[(\omega_{n}-\omega_{j})(N+1)\Delta t/2 - \pi/N]}{2N\sin[(\omega_{n}-\omega_{j})\Delta t/2 - \pi/N]}$$

(2f)

$$\frac{\sin[(\omega_n - \omega_j)(N+1)\Delta t/2 + \pi/N]}{2N\sin[(\omega_n - \omega_j)\Delta t/2 + \pi/N]}$$

### **3 - INTERPOLATION**

Let us suppose that we need to analyse hourly heights of a classical 29 days span. Then we have in this case 696 ordinates. The question is: how to change the time series into another one, with 1024 points, in order to compute the Fourier coefficients  $a_n$  and  $b_n$  by using the classical FFT? Since 1 hour is the sampling interval in the original series, that interval in the second series should be 695/1023 = 0.6793743 hour. Thus, if we work out the interpolation of the series in order to have the heights corresponding to the 0.6793743 hour sampling interval, the ordinates to be analysed will be 1024 ( $2^{14}$ ) and it is possible to use the classical FFT algorithm to work out the Fourier analysis. However a linear interpolation is not sufficiently accurate to give the new heights.

It is well known that the ephemeris available to astronomers of the past century had right ascentions at 12 hours intervals. Then they used to interpolate these values through finite differences of several orders. CHAUVENET (1891), who wrote the first edition of his Spherical Astronomy in 1863, gave examples of interpolations by using differences up to the 5th order.

Tide is a well behaved phenomenon, with no discontinuities, thus it is possible to interpolate the curve by using  $2^{nd}$  order differences only.

Let  $\zeta_1$ ,  $\zeta_2$  and  $\zeta_3$  be three successive tidal heights and  $\zeta_4$  the interpolated tidal height nearest to  $\zeta_2$ . Then

$$\delta_{1} = [(\zeta_{3} - \zeta_{2}) + (\zeta_{2} - \zeta_{1})]/2 = (\zeta_{3} - \zeta_{1})/2$$
(3a)
$$\delta_{2} = (\zeta_{3} - \zeta_{2}) - (\zeta_{2} - \zeta_{1}) = \zeta_{3} - 2\zeta_{2} + \zeta_{1}$$

and

 $\Delta$  = the fractional interval between  $\zeta_i$  and  $\zeta_2$ 

According to CHAUVENET (1891), one of the formulae to compute  $\zeta_i$ , as reduced to the second order term, is:

$$\zeta_1 = \zeta_2 + \delta_1 \Delta + \delta_2 \Delta^2 / 2 \tag{3b}$$

where  $\Delta$  is the time interval between  $\zeta_i$  and  $\zeta_2$ . The sampling interval of the observed heights is usually short (1 hour) thus, we may consider that the low frequency background noise affecting the three heights is nearly destroyed by the operations to obtain  $\delta_1$  and  $\delta_2$ . But the measuring error affecting the observed heights must be considered.

If  $\sigma$  is the variance corresponding to the observed heights, the law of error propagation as applied to expressions (3a) gives

$$\sigma_1^2 = \sigma^2/4$$
 for  $\xi_1$ 

 $\sigma_2^2 = 6\sigma^2$  for  $\xi_2$ 

(3c)

and

The same law, as applied to expression (3b) for  $\Delta$ =0.5, will give

 $\sigma_{1}^{2} = 1.16\sigma^{2}$ 

consequently, the standard deviation is

### σ,=±1.08σ

hence the accuracy of the interpolated heights is about 8% lower than that of the observed ones. If  $\pm 1$  cm is the standard deviation corresponding to the measurement of the tidal heights, then  $\pm 1.08$  cm will be the standard deviation for the interpolated heights. In fact, the shown practical examples indicate that no decrease of the accuracy occur in the results when the observed data are replaced by the interpolated ones. The large number of observations handled in tidal analyses explains why the results are not affected when the interpolated data are used.

In order to give a visual idea of such interpolation, it was applied to the large tide of "Igarapé do Inferno" ( $02^{\circ}$  06.0' N and  $050^{\circ}$  32.0' W) in the Maracá Island (Brazil). Notwithstanding the range of about 10 meters and the strong shallow-water effects, Figure 3A shows how accurate is the interpolation.

An important remark is that the resolution of the analysis, equal to  $1/N\Delta t$ , is not altered by the interpolation, but the cut off frequency is increased, since it is equal to  $180/\Delta t^{0}/h$ .

Another remark is that this interpolation have been used in spectral analyses of randomic continuous time series with good results.





Figure 3A - • = observed heights; + = interpolated heights

### **4 - SOME RESULTS WITH THE NORMAL METHOD**

The results shown in Tables 4-I to 4-IV will convince the reader on the success of such a simple technique.

Two different classical spans were chosen: 29 and 369 days. Since the semidiurnal is the most important species, four analyses were worked out for that species. Tables 4-I and 4-II show the results of the analyses using non interpolated and interpolated heights, respectively, for a 29 days span (696 hours). The chosen tidal station is "Ponta da Madeira" (Lat.  $02^{\circ}$  06' S, Long.  $050^{\circ}$  32' W), Brazil, where the tidal range is about 7 meters; the initial time of the considered record is  $0^{h}$ , 11/1/1991. The other two analyses were worked out with 8856 data (369 days) from Cananéia (Lat.  $23^{\circ}$  O1' S,Long.  $47^{\circ}$  56' W), Brazil, started at  $0^{h}$  of 1/1/1980; the results obtained with the observed data and the interpolated ones are shown in Tables 4-III and 4-IV, respectively.

It is important to say that, according to the extension of the analysis, the statistical rejection of small constituents (FRANCO, 1972), which are flagged with an asterisc, is made by the program of analysis. The probability for the rejection is chosen by the user; in this case it was 95%. All the shallow water constituents have been considered by several researchers and are taken from an extensive table drawn by this author. These constituents showed to be stable in successive analyses of a 18.69 Julian years span (FRANCO & HARARI, 1988, 1991).

One can see by inspection of Tables 4-l and 4-II that the harmonic constants of the non flagged constituents are practically the same in both tables. In fact, the differences can be ascribed to round off errors and to the fact that the analysis with the Watt's formula was worked out with 695 heights instead of 696. The odd number of heights 5s used in order to establish pairs of heights symmetrical with respect to the central time because the Fourier analysis becomes faster and the correction c of (2c) is avoided. That correction diminishes when the length of the series increases. For a one year analysis c is nearly zero.

The spectra of residual amplitudes and the harmonic constants obtained from the analyses of a one-year span, shown in Tables 4-III and 4-IV, are so close, that no additional comments are necessary.

	CYC RESIDU	LES PER D	 AY: 2 GY: 1.09		DEGREES OF FREEDOM: 8 STANDARD DEVIATION: 0.48 cm							
		SPI	ECTRUM	OF RESID	UAL AN	APLITUDES	5					
Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.			
25.381 27.971 30.561	0.53 0.00 0.01	25.899 28.489 31.079	0.75 0.01 0.06	26.417 29.007 31.597	0.01 0.01 0.08	26.935 29.525 32.115	0.04 0.00 0.98	27.453 30.043 32.663	0.01 0.02 0.61			

## Table 4-I - 29 days analysis with the Watt's formula

	HARMONIC CONSTANTS												
No.	Symbol	Deg./h	H cm	±	G Deg.	K Deg.	GW Deg.	±					
1	*N(MU)S₂	26.4079379	1.63	1.57	133.14	123.63	212.36	****					
2	2NS <sub>2</sub>	26.8794590	3.43	1.58	255.15	247.05	335.78	36.55					
3	MNS <sub>2</sub>	27.4238337	2.02	1.57	1.77	355.30	84.04	67.94					
4	MU2	27.9682084	4.75	1.57	358.42	353.59	82.32	19.50					
5	N₂	28.4397295	38.40	1.58	195.15	191.73	280.47	2.58					
6	M,	28.9841042	213.18	1.57	198.90	197.12	285.85	0.42					
7	L <sub>2</sub>	29.5284789	10.90	1.58	164.74	164.59	253.33	7.34					
8	S <sub>2</sub>	30.0000000	53.94	1.57	230.93	232.19	320.93	1.82					
9	MSN₂	30.5443747	5.07	1.57	13.03	15.93	104.66	21.79					
10	25M2	31.0158958	4.27	1.58	76.21	80.52	169.28	26.34					
11	*25N2	31.5602705	1.75	1.57	215.54	221.49	310.22	*****					

	INFERRED CONSTANTS											
No.	No. Symbol Deg./h H cm G Deg. K Deg. C											
1	2N <sub>2</sub>	27.8953548	5.07	191.40	186.35	275.09						
2	NU <sub>2</sub>	28.5125831	7.30	195.65	192.46	281.19						
3	LAMBDA <sub>2</sub>	29.4556253	1.49	213.76	213.39	302.13						
4	T <sub>2</sub>	29.9589333	3.18	232.21	233.35	322.09						
5	R <sub>2</sub>	30.0410667	0.43	232.21	233.60	322.33						
6	K <sub>2</sub>	30.0821373	14.67	233.52	235.03	323.77						

	CYC	LES PER D JAL ENER	AY: 2 GY: 0.91	DEGREES OF FREEDOM: 8 STANDARD DEVIATION: 0.48 cm							
		SPI	ECTRUM	OF RESID	UAL AN	APLITUDE	5				
Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.		
25.357 27.944 30.531	0.43 0.01 0.01	25.874 28.461 31.049	0.67 0.01 0.04	26.391 28.979 31.566	0.02 0.00 0.02	26.909 29.496 32.084	0.01 0.01 0.94	27.426 30.014 32.601	0.00 0.01 0.56		

# Table 4-II - 29 days analysis with the FFT algorithm

	HARMONIC CONSTANTS												
No.	Symbol	Deg./h	H cm	±	G Deg.	K Deg.	GW Deg.	±					
1	*N(MU)S₂	26.4079379	1.61	1.43	138.28	128.78	217.51	****					
2	2NS <sub>2</sub>	26.8794590	3.37	1.44	256.39	248.30	337.03	33.56					
3	MNS <sub>2</sub>	27.4238337	2.04	1.43	3.03	356.56	85.30	57.09					
4	MU₂	27.9682084	4.65	1.44	359.09	354.26	83.00	18.14					
5	N <sub>2</sub>	28.4397295	38.39	1.44	195.31	191.90	280.63	2.34					
6	M <sub>2</sub>	28.9841042	213.17	1.43	198.91	197.13	285.87	0.39					
7	L <sub>2</sub>	29.5284789	10.94	1.44	165.09	164.94	253.87	6.67					
8	S <sub>2</sub>	30.0000000	54.07	1.44	230.89	232.16	320.89	1.66					
9	MSN₂	30.5443747	5.06	1.44	11.40	14.30	103.04	19.87					
10	25M2	31.0158958	4.41	1.44	76.70	81.02	169.75	23.06					
11	*25N2	31.5602705	1.83	1.43	212.47	218.42	307.15	*****					

	INFERRED CONSTANTS											
No.	No. Symbol Deg./h H cm G Deg. K Deg. GW											
1	2N <sub>2</sub>	27.8953548	5.07	191.71	186.66	275.40						
2	NU <sub>2</sub>	28.5125831	7.29	195.79	192.60	281.33						
3	LAMBDA <sub>2</sub>	29.4556253	1.49	213.75	213.39	302.12						
4	T <sub>2</sub>	29.9589333	3.19	232.17	233.31	322.05						
5	*R2	30.0410667	0.43	232.17	233.56	322.29						
6	K <sub>2</sub>	30.0821373	14.71	233.48	235.00	323.73						

## Table - 4-III - 369 days analysis with the Watt's formula

# CYCLES PER DAY: 2 DEGREES OF FREEDOM: 204 RESIDUAL ENERGY: 2.09 STANDARD DEVIATION: 0.14 cm

		SF	PECT	RUM	OF RES		L AN	APLIT	TUDE	<u></u>			<b></b>
Deg	/h Res.	Deg./h	R	es.	Deg./h	R	les.	Deg	./h	Res		Deg./h	Res.
26.3	44 0.20	26.385	0.	.09	26.426	0	.16	26.4	166	0.12	2	26.507	0.26
26.5	48 0.06	26.588	0.	.31	26.629	0	.01	26.0	570	0.18	3	26.710	0.13
26.7	51 0.13	26.792	0.	.14	26.832	0	50.	26.8	37:	0.04	ł	26.914	0.10
26.9	54 0.01	26.995	0.	.15	27.036	0	.06	27.0	076	0.18	3	27.117	0.14
27.1	58 0.24	27.198	0.	.09	27.239	0	.17	27.2	280	0.05	5	27.320	0.08
27.3	61 0.00	27.401	0.	.17	27.442	0	.12	27.4	483	0.11	l	27.523	0.09
27.5	64 0.20	27.605	0.	21	27.645	0	.11	27.6	586	0.12	2	27.727	0.17
27.7	67 0.12	27.808	0.	25	27.849		.21	27.8	389	0.01		27.930	0.06
27.9	71 0.02	28.011	0.	25	28.052		.03	28.0	<i>)</i> 93	0.05	2	28.133	0.25
20.1	77 0.02	20.213	0.	10	20.200		17	20.4	290	0.13		20.337	0.04
20.5	80 0.07	20.410	0.		20.400		.17	20.4	700	0.00		20.540	0.13
28.7	84 0.23	28.021	0.		28.865		23	20.	202	0.09	5	20.745	0.10
28.9	87 0.02	29.024	0.	02	20.003		.24	20.2	ן סטיק	0.03	2	28.540	0.02
291	90 0.02	29.020	0.	21	29.000	l ñ	14	29.	212	0.20	Ś	20.150	0.45
29.3	94 0.26	29 434	0.	09	29 475	۱ŏ	35	29.	516	0.02	,	29.556	0.21
29.5	97 025	29.637	, n	16	29.678	Ιŏ	07	20 2	710	0.2		29 759	0.13
29.8	00 0.24	29,841	0.	42	29 881	Ιŏ	20	290	222	0.02	, I	29.963	0.15
30.0	03 0.02	30.044	Ő.	03	30.085	ŏ	.04	30.1	25	0.39		30.166	0.35
30.2	07 0.17	30.247	0.	05	30.288	Ŏ	.27	30.3	329	0.29	•	30.369	0.20
30.4	10 0.15	30.451	0.	21	30.491	Ō	.14	30.5	532	0.08	3	30.573	0.14
30.6	13 0.12	30.654	0.	23	30.695	0	.03	30.2	735	0.08	3	30.776	0.12
30.8	16 0.26	30.857	0.	29	30.898	0	.15	30.9	38	0.16	5	30.979	0.14
31.0	20 0.02	31.060	0.	03	31.101	0	.13	31.1	142	0.23	3	31.182	0.34
31.2	23 0.24	31.264	0.	02	31.304	0	.22	31.3	345	0.32	2	31.386	0.12
31.4	26 0.14	31.467	0.	18	31.508	0	.28	31.5	548	0.06	5	31.589	0.09
31.6	30 0.07	31.670	0.	08	31.711	0	.15				į		
				HAR	MONIC	CON	ISTA	NTS					
No.	Symbol	Deg./	h	H cn	n e	-	GE	Deg.	К	Deg.	G	W Deg.	±
No.	Symbol	Deg./	h	H cn	n e	:	G E	Deg.	KI	Deg.	G	W Deg.	±
No.	Symbol *N(MU)S <sub>2</sub>	Deg./ 26.4079	h 379	H cm 0.34	n = 4 0.1	36	G E 33	Deg.	K 1	Deg. 6.76	G	W Deg. 112.63	±
No. 1 2	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub>	Deg./ 26.40792 26.8794	h 379 590	H cm 0.34 0.12	n = 4 0.1 2 0.1	: 36 36	G E 33 171	Deg. 1.41 1.21	K 1 15	Deg. 6.76 5.98	G	W Deg. 112.63 251.84	± ***** *****
No. 1 2 3	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub>	Deg./ 26.40792 26.87945 26.9523 27.24166	h 379 590 126	H cm 0.34 0.12 0.12	m = 4 0.1 2 0.1 6 0.1	36 36 36	G E 33 171 275	Deg. 1.41 1.21 5.30	K 1 15 26	Deg. 6.76 5.98 0.29	G	W Deg. 112.63 251.84 356.16 221 (2	± ***** ***** *****
No. 1 2 3 4 5	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNIS	Deg./ 26.40793 26.87945 26.9523 27.34169	h 379 590 126 965	H cm 0.34 0.12 0.16 0.04	n ± 4 0 2 0 6 0 4 0	36 36 36 35	G E 33 171 275 249	Deg. 1.41 1.21 5.30 0.61	K 1 15 26 23	Deg. 6.76 5.98 0.29 5.77 2.26	G	W Deg. 112.63 251.84 356.16 331.63 159.12	± ***** ***** *****
No. 1 2 3 4 5 6	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMI(2S	Deg./ 26.40793 26.87943 26.9523 27.34169 27.42383 27.5059	h 379 590 126 965 337 710	H cm 0.34 0.12 0.16 0.04 0.30	n ± 4 0 2 0 6 0 4 0 0 0 8 0	± 36 36 35 35	G E 33 171 275 249 76 295	Deg. 1.41 1.21 5.30 0.61 5.85 5.05	K 1 15 26 23 6 28	Deg. 6.76 5.98 0.29 5.77 3.26 1 70	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57	± ***** ***** ***** *****
No. 1 2 3 4 5 6 7	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N.	Deg./ 26.4079 26.8794 26.9523 27.3416 27.4238 27.5059 27.8953	h 379 590 126 965 337 710	H cm 0.34 0.12 0.14 0.04 0.30 0.28 1.70	m = 4 0.1 2 0.1 6 0.1 4 0.1 0 0.1 8 0.1 0 0 0.1	= 36 36 35 35 35 35	G E 33 171 275 249 76 295 156	Deg. .41 .21 .30 .61 .85 .05 .62	K 1 15 26 23 6 28 14	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30	± ***** ***** ***** ***** *****
No. 1 2 3 4 5 6 7 8	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub>	Deg./ 26.40794 26.87944 26.9523 27.34169 27.42385 27.50595 27.89533 27.9682	h 379 590 126 965 337 710 548 384	H cm 0.34 0.12 0.16 0.04 0.30 0.28 1.70 2.66	n     ±       4     0       2     0       6     0       4     0       0     0       8     0       00     0       6     0	36 36 36 35 35 35 35 36 36	G E 33 171 275 249 76 295 156 154	Deg. 3.41 5.30 0.61 5.85 5.05 5.62 4.49	K 1 15 26 23 6 28 14	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39	± ***** ***** ***** ***** 11.80 7.54
No. 1 2 3 4 5 6 7 8 9	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub>	Deg./ 26.40790 26.87941 26.9523 27.34169 27.4238 27.50595 27.96820 28.04100	h 379 590 126 965 337 710 548 384 520	H cm 0.34 0.12 0.16 0.04 0.30 0.28 1.70 2.66 0.28	n     ±       4     0.2       2     0.2       6     0.2       4     0.2       0     0.2       8     0.2       00     0.2       8     0.2       00     0.2       8     0.2       00     0.2       00     0.2       00     0.2       00     0.2       00     0.2       00     0.2       00     0.2       00     0.2       00     0.2	36 36 35 35 35 35 36 36 36 36	G E 33 171 275 249 76 295 156 154 251	Deg. 3.41 1.21 5.30 0.61 5.85 5.05 5.62 1.49 .92	K 1 15 26 23 6 28 14 14	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04	± ***** ***** ***** ***** 11.80 7.54 *****
No. 1 2 3 4 5 6 7 8 9 10	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub>	Deg./ 26.40790 26.87941 26.9523 27.34169 27.4238 27.50595 27.96820 28.04100 28.35759	h 379 590 126 965 337 710 548 520 548 520 922	H cm 0.34 0.12 0.14 0.04 0.30 0.28 1.70 2.66 0.28 0.20	n     ±       4     0.1       2     0.1       6     0.1       6     0.1       8     0.1       0     0.2       6     0.3       8     0.1       6     0.3       8     0.1       6     0.3       8     0.1       8     0.2       8     0.3       9     0.3	= 36 36 35 35 35 35 36 36 36 36	G E 33 171 275 249 76 295 156 154 251 7	Deg. 3.41 1.21 5.30 0.61 5.85 5.05 5.62 1.49 1.92 1.89	K 1 15 26 23 6 28 14 14 14 24 35	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96	± ***** ***** ***** 11.80 7.54 *****
No. 1 2 3 4 5 6 7 8 9 10 11	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub>	Deg./ 26.40790 26.87941 26.9523 27.34169 27.4238 27.50593 27.96820 28.04100 28.35759 28.43972	h 379 590 126 965 337 710 548 520 922 295	H cm 0.34 0.12 0.16 0.04 0.30 0.28 1.70 2.66 0.28 0.20 5.73	n         ±           4         04           2         04           6         06           8         00           8         00           8         00           9         00           10         00	= 36 36 35 35 35 35 36 36 36 36 36	G E 33 171 275 249 76 295 156 154 251 7 162	Deg. 3.41 3.21 5.30 0.61 5.05 5.05 5.62 4.49 92 7.89 2.62	K 1 15 26 23 6 28 14 14 14 24 35 15	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94	± ***** ***** ***** 11.80 7.54 ***** 3.47
No. 1 2 3 4 5 6 7 8 9 10 11 12	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MK <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub> NU <sub>2</sub>	Deg./ 26.40790 26.87941 26.9523 27.34169 27.4238 27.50599 27.96820 28.04100 28.35759 28.43977 28.51256	h 379 590 126 965 337 710 548 520 922 295 331	H cm 0.34 0.12 0.04 0.30 0.28 1.77 2.66 0.28 0.20 5.73 0.52	m         ±           4         0.3           2         0.2           6         0.3           0         0.4           0         0.3           2         0.2	36 36 35 35 35 35 36 36 36 36 36 36 36	G E 33 171 275 249 76 295 156 154 251 7 162 152	Deg. 3.41 3.21 5.30 0.61 5.05 5.05 5.62 4.49 2.92 7.89 2.62 2.16	K 1 15 26 23 6 28 14 14 24 35 15 15	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70	± ***** ***** ***** 11.80 7.54 ***** 3.47 41.63
No. 1 2 3 4 5 6 7 8 9 10 11 12 13	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NNK <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OV	Deg./ 26.40790 26.87941 26.9523 27.34169 27.4238 27.50599 27.96820 28.04100 28.35759 28.43977 28.51258 28.60400	h 379 590 126 965 337 710 548 520 922 295 331 041	H cm 0.34 0.12 0.04 0.30 0.28 1.77 2.66 0.28 0.20 5.73 0.52 0.22	m         ±           4         0.1           2         0.1           6         0.3           6         0.3           0         0.1           6         0.3           0         0.3           2         0.2           3         0.3           2         0.3           7         0.1	36 36 35 35 35 35 36 36 36 36 36 36 36 36 36 36 36	G E 33 171 275 249 76 295 156 154 251 7 162 152 175	Deg. 3.41 3.21 5.30 0.61 5.85 5.05 5.62 1.49 1.92 7.89 2.62 1.16 5.94	K 1 15 26 23 6 28 14 14 24 35 15 14 16	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75	± ***** ***** ***** 11.80 7.54 ***** 3.47 41.63 *****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>5</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub>	Deg./ 26.40790 26.87941 26.9523 27.34169 27.4238 27.50599 27.89533 27.96820 28.04100 28.35755 28.43977 28.51256 28.43977 28.51256 28.60400 28.90190	h 379 590 126 965 337 710 548 084 620 922 295 331 041 670	H cm 0.34 0.12 0.04 0.04 0.28 0.28 0.28 0.28 0.22 0.57 0.52 0.52	m         ±           4         0.1           2         0.2           6         0.3           4         0.1           8         0.1           0         0.1           8         0.2           0         0.3           0         0.3           2         0.3           2         0.3           9         0.3	36 36 35 35 35 35 36 36 36 36 36 36 36 36 36 35	G E 33 171 275 249 76 295 156 154 251 7 162 152 175 65	Deg. 3.41 3.21 3.30 0.61 5.65 5.65 5.62 4.49 	K 1 15 26 23 6 28 14 14 24 35 15 15 14 16 5	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97	± ***** ***** 11.80 7.54 ***** 3.47 41.63 ***** 44.24
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 15 15 15 10 11 12 13 14 15 10 11 12 15 10 10 10 10 10 10 10 10 10 10	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>5</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub>	Deg./ 26.40790 26.87941 26.9523 27.34169 27.4238 27.50599 27.89533 27.96820 28.04100 28.35755 28.43977 28.51256 28.43977 28.51256 28.60400 28.90190 28.94300	h 379 590 126 965 337 710 548 884 620 922 295 331 041 670 375	H cm 0.34 0.12 0.04 0.04 0.28 0.28 0.28 0.28 0.28 0.28 0.28 0.25 0.55 0.55 0.55 0.55 0.55	m         ±           4         0.1           2         0.2           6         0.3           4         0.1           5         0.3           0         0.3           0         0.3           0         0.3           2         0.3           2         0.3           3         0.2           7         0.3           9         0.3	2 36 36 335 335 335 335 336 336 336 336 3	G E 33 171 275 249 76 295 156 154 251 7 162 152 175 65 87	Deg. 3.41 3.21 3.30 3.61 3.85 5.05 5.62 4.49 .92 7.89 2.62 1.16 5.94 3.26 7.98	K 1 15 26 23 6 28 14 14 24 35 15 15 15 14 16 5 7	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81	± ***** ***** ***** 11.80 7.54 ***** 3.47 41.63 ***** 44.24 36.52
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 17 17 17 17 17 17 17 17 17	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> M <sub>2</sub> M <sub>2</sub> T	Deg./ 26.40792 26.87942 26.9523 27.34162 27.42382 27.50592 27.89533 27.96820 28.04100 28.35755 28.60400 28.351256 28.60400 28.90190 28.94300 28.94300 28.94300	h 379 590 126 965 3337 710 548 620 922 295 331 041 670 0375 042	H cm 0.34 0.12 0.04 0.04 0.28 0.28 0.28 0.28 0.28 0.28 0.25 0.55 0.55 0.55 0.55 0.55 0.55 0.55	m         ±           4         0.1           2         0.2           6         0.3           4         0.2           6         0.3           6         0.3           0         0.2           3         0.2           7         0.3           9         0.3           9         0.2	2 36 36 36 335 335 335 335 336 36 36 36 36 36 35 35 35 35 35 35 35 35 35 35	G E 33 171 275 249 76 295 156 295 156 251 7 162 152 175 65 87 93	Deg. 3.41 .21 3.30 .61 5.05 5.62 .49 .92 .89 .62 .16 .94 .26 .94 .26 .98 .16	K 1 15 26 23 6 28 14 14 24 35 15 14 16 5 77 8	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11	± ***** ***** ***** 11.80 7.54 ***** 3.47 41.63 ***** 44.24 36.52 0.54
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 1°	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> M <sub>2</sub> MST <sub>2</sub> MYC	Deg./ 26.4079: 26.8794; 26.9523; 27.3416; 27.4238; 27.5059; 27.8953; 27.9682; 28.0410; 28.0410; 28.3575; 28.4397; 28.4397; 28.5125; 28.6040; 28.9019; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9430; 28.9441; 29.9251; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.9551; 29.955	h 379 590 126 965 337 710 548 620 922 295 331 670 375 042 709	H cm 0.34 0.12 0.04 0.30 0.28 1.77 0.266 0.28 0.22 0.22 0.55 0.25 0.25 0.25 0.55 0.25 0.2	n         4           4         0.3           2         0.3           6         0.3           6         0.3           6         0.3           6         0.3           6         0.3           7         0.3           9         0.3           9         0.3           1         0.3	2 36 36 36 335 335 335 335 336 36 36 36 36 36 36 36 35 35 35 35 35 35 35 35 35 35	G L 333 171 275 2499 76 295 156 251 154 251 152 152 175 65 87 93 187	Deg. 3.41 .21 .30 0.61 5.85 5.605 5.62 1.49 2.62 2.89 2.62 2.16 5.94 5.94 5.94 5.94 5.94 5.94 5.94 5.94 5.94 5.94 5.94 5.94 5.94 5.94 5.96 5.94 5.96 5.94 5.96 5.96 5.94 5.96 5.96 5.94 5.96 5.96 5.96 5.96 5.94 5.96 5	K 1 15 26 23 6 28 28 14 14 24 35 15 14 16 5 7 7 8 8 17	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75	± ***** ***** ***** 11.80 7.54 ***** 3.47 41.63 ***** 44.24 36.52 0.54 57.45
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 10 11 12 13 14 15 16 17 10 10 10 10 10 10 10 10 10 10	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> NU <sub>2</sub> *2(NU)M <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MKS <sub>2</sub> *1 AMED A	Deg./ 26.40790 26.87940 26.9523 27.34160 27.42380 27.9533 27.96820 28.04100 28.35755 28.43977 28.51255 28.60400 28.90190 28.94300 28.94300 28.94300 28.94300 28.94300 28.94410 29.02512 29.06622	h 379 590 126 965 337 710 548 620 922 295 331 041 670 375 042 709 415	H cm 0.34 0.12 0.16 0.04 0.30 0.30 0.30 0.30 0.28 1.77 0.26 0.28 0.22 0.55 0.55 0.55 0.55 0.55 0.55 0.55	n         4           4         0.3           6         0.3           6         0.3           6         0.3           7         0.3           9         0.3           9         0.3           9         0.3           8         0.3	± 36 36 335 335 335 335 336 36 36 36 36 36 36 36 35 35 35 35 35 35 35 35 35 35	G E 333 171 275 249 76 295 156 154 251 154 251 152 175 655 87 93 187 93	Deg. 3.41 .21 3.30 0.61 5.85 5.05 5.62 4.49 9.92 2.62 1.16 5.94 1.26 7.98 3.16 7.68 5.94 1.21 5.94 1.21 5.94 1.21 5.94 1.21 5.94 5.26 5.94 5.26 5.94 5.26 5.94 5.26 5.94 5.26 5.94 5.26 5	K 1 15 26 23 6 28 14 14 24 35 15 15 14 16 5 7 7 8 8 17 18	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 3.58	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.65	± ***** ***** ***** ***** ***** ***** ****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MKS <sub>2</sub> *LAMBDA <sub>2</sub>	Deg./ 26.40790 26.87941 26.9523 27.34160 27.42383 27.5059 27.89533 27.96820 28.04100 28.35759 28.43977 28.51255 28.43977 28.51255 28.43979 28.94300 28.94300 28.94300 28.94300 28.94300 28.94300 28.94310 29.02512 29.06620 29.055512 29.06620 29.055512 29.06620 29.055512 29.06620 29.055512 29.05620 29.055512 29.055512 29.05552 20.555512 29.05552 29.555512 29.5555512 29.5555512 29.5555512 29.5555512 29.55555555555555555555555555555555555	h 379 590 126 965 337 710 548 620 922 295 331 041 670 375 042 709 415 253	H cm 0.34 0.12 0.04 0.30 0.30 0.30 0.30 0.28 1.70 2.66 0.28 0.22 0.55 0.55 0.55 0.55 0.55 0.55 0.55	n         4           4         0.3           6         0.3           8         0.3           0	± 36 36 335 335 335 335 336 36 36 36 36 36 36 335 335	G E 333 171 275 2499 76 2959 1566 154 2511 1552 1552 1555 57 933 1877 1922 56	Deg. 3.41 .21 3.30 0.61 5.85 5.05 5.62 4.49 9.62 1.16 5.94 4.26 7.98 3.16 7.68 5.50 5	K 1 15 26 23 6 28 14 14 24 35 15 15 15 14 16 5 7 7 8 8 17 18	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 3.58 9.00	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20	± ***** ***** ***** ***** ***** ***** ****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> M <sub>2</sub> MST <sub>2</sub> MKS <sub>2</sub> *LAMBDA <sub>2</sub> L <sub>2</sub> 2SK	Deg./ 26.40790 26.87941 26.9523 27.34160 27.42383 27.96820 28.04100 28.35759 28.43977 28.51256 28.43977 28.51256 28.43979 28.94300 28.94300 28.94300 28.98410 29.02512 29.06620 29.45566 29.52842 29.9179	h 379 590 126 965 337 710 548 620 922 295 331 041 670 375 042 709 415 253 789 627	H cm 0.34 0.12 0.16 0.04 0.28 1.70 2.66 0.28 0.20 5.77 0.55 0.55 0.55 0.55 0.55 0.55 0.5	m         d           4         0.3           6         0.3           6         0.3           0	2 36 36 36 335 335 335 335 336 36 36 36 335 335	G E 333 171 279 2499 76 299 76 299 156 251 152 251 152 251 152 255 57 93 187 192 56 68	Deg. 3.41 3.30 0.61 8.85 5.05 5.62 4.49 2.62 2.16 7.68 2.25 5.50 8.62 3.16 7.68 2.25 5.50 8.62 3.62 3.64 5.55 5.50	K 1 15 26 23 6 28 14 14 24 35 15 15 15 15 15 15 15 15 15 15 15 17 18 4 4 6 6	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 3.58 9.00 1.34 9.00	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75	± ***** ***** ***** ***** ***** ***** ****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> *NMU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MKS <sub>2</sub> *LAMBDA <sub>2</sub> L <sub>2</sub> 2SK <sub>2</sub> T.	Deg./ 26.40790 26.87941 26.9523 27.34160 27.4238 27.50597 27.89533 27.96820 28.04100 28.35759 28.43977 28.51256 28.43977 28.51256 28.43979 28.90190 28.98410 28.98430 28.984410 29.02517 29.06624 29.45566 29.52844 29.91786	h 379 590 126 965 337 710 548 620 922 295 331 041 670 375 042 709 415 253 789 9415	H cm 0.34 0.12 0.04 0.28 1.70 2.66 0.28 0.20 5.73 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.5	m         d           4         0.3           2         0.3           6         0.3           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3           1         0.3           9         0.3           9         0.3           9         0.3           5         0.3	2 36 336 335 335 335 335 335 335	G E 333 171 279 2499 76 295 156 295 156 154 251 152 152 175 65 57 93 187 192 56 68 8186	2008. 3.41 3.30 0.61 5.30 0.61 5.85 5.62 3.16 5.94 7.98 3.16 7.98 3.16 7.98 3.16 7.98 3.16 7.98 3.16 7.98 3.16 7.98 3.16 7.98 3.16 7.98 3.16 7.98 7.99	K 1 15 26 23 6 28 14 14 24 35 15 15 15 15 15 15 15 15 15 15 15 15 15	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 3.58 9.00 1.34 0.88 3.40	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75 219.27	± ***** ***** ***** ***** ***** ***** ****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> *NMK2 *SNK <sub>2</sub> NU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MKS <sub>2</sub> *LAMBDA <sub>2</sub> L <sub>2</sub> 2SK <sub>2</sub> T <sub>2</sub> S	Deg./ 26.40790 26.87941 26.9523 27.34160 27.4238 27.50597 27.89533 27.96820 28.04100 28.35759 28.43977 28.51256 28.60400 28.94300 28.94300 28.94300 28.94300 28.94300 28.945662 29.05562 29.52844 29.91786 29.95892 30.00000	h 379 590 126 965 337 710 548 620 922 295 331 041 670 375 042 709 415 253 789 627 333	H cm 0.34 0.12 0.16 0.04 0.30 0.28 1.70 2.66 0.28 0.20 0.55 0.55 0.55 0.55 0.55 0.55 0.55	m         d           4         0.1           2         0.2           6         0.3           6         0.3           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3           3         0.3           9         0.3           8         0.3           9         0.3           5         0.3           4         0.3		G E 333 171 279 2499 76 299 156 154 251 7 7 162 152 175 65 57 93 187 192 56 68 186 182 99	Deg. 4.41 5.30 0.61 5.85 5.62 5.62 7.89 7.89 7.89 7.89 7.89 7.89 7.89 7.89 7.89 7.89 7.89 7.89 7.98 7.99 7.98 7.98 7.98 7.98 7.98 7.99 7.98 7.98 7.98 7.99 7.99 7.98 7.98 7.98 7.99 7.99 7.98 7.98 7.98 7.98 7.99 7.99 7.98 7.99 7.98 7.99 7.98 7.99 7.99 7.99 7.98 7.98 7.99 7.99 7.99 7.98 7.98 7.99 7.99 7.99 7.99 7.98 7.98 7.99 7	K 1 15 26 23 6 28 28 28 28 28 28 28 14 14 24 35 15 15 14 16 5 7 7 8 8 17 18 4 4 6 188 18 28 28 28 28 28 28 28 28 28 28 28 28 28	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.888 3.58 9.00 1.34 0.88 3.40 9.29	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75 219.27 185.15	± ***** ***** ***** ***** ***** ***** ****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MK5 <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MKS <sub>2</sub> *LAMBDA <sub>2</sub> L <sub>2</sub> 2SK <sub>2</sub> T <sub>2</sub> S <sub>2</sub> S <sub>2</sub>	Deg./ 26.40790 26.87941 26.9523 27.34160 27.4238 27.50590 27.89533 27.96820 28.04100 28.35759 28.43977 28.51256 28.60400 28.94300 28.94300 28.94300 28.94300 28.94300 28.945662 29.02517 29.06624 29.91786 29.91786 29.91786 29.91786 29.91786 29.91786 29.95889 30.00000 30.0410	h 379 5590 126 965 337 710 548 528 884 520 922 295 331 670 941 525 375 941 525 709 4115 2253 789 527 789 527 7333 667	H cm 0.34 0.12 0.16 0.04 0.30 0.28 1.70 2.66 0.28 0.20 5.73 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.5	m         d           4         0.1           2         0.2           6         0.3           0         0.3           8         0.3           0         0.3           8         0.3           0         0.3           3         0.3           2         0.3           3         0.3           9         0.3           1         0.3           8         0.4           9         0.3           4         0.4           0         0.4		G E 333 171 275 2499 76 295 156 295 152 152 152 152 152 152 152 152 152 15	Deg. 3.41 3.30 0.61 5.85 5.05 5.65 5.65 5.94 5.25 5.50 6.62 5.99 6.15 5.00 6.62 5.99 6.15 5.00 6.62 5.99 6.15 5.00 5.00 5.00	K 1 15 26 23 6 28 28 28 28 28 28 28 35 15 14 16 55 77 8 17 18 18 44 6 18 12 28 13	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 8.88 3.58 9.00 1.34 0.88 3.40 9.29 8.25	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75 219.27 185.15 234.12	± ***** ***** ***** ***** ***** ***** ****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MKS <sub>2</sub> *LAMBDA <sub>2</sub> L <sub>2</sub> 2SK <sub>2</sub> T <sub>2</sub> S <sub>2</sub> R <sub>2</sub> K <sub>3</sub>	Deg./ 26.40790 26.87941 26.9523 27.34169 27.4238 27.50597 27.89533 27.96820 28.04100 28.35759 28.43977 28.51259 28.60400 28.90199 28.94300 28.984310 29.02517 29.06624 29.45565 29.52847 29.91788 29.91788 29.91788 29.95890 30.004100 30.08211	h 379 590 126 965 337 710 548 520 922 295 331 541 670 941 570 941 575 042 779 415 253 789 627 333 000 667 373	H cm 0.34 0.12 0.16 0.04 0.30 0.28 1.70 2.66 0.22 0.52 0.52 0.55 0.55 0.55 0.55 0.55	m         ±           4         0.1           2         0.2           6         0.3           0         0.3           6         0.3           6         0.3           0         0.3           0         0.3           1         0.3           1         0.3           1         0.3           1         0.3		G E 333 171 275 2499 76 295 156 154 251 7 7 162 152 175 65 87 7 93 3 87 192 56 68 188 188 182 9 9 9 9 144	Deg. 3.41 3.30 0.61 5.85 5.05 5.62 1.16 5.94 1.26 5.94 1.26 5.94 1.26 5.94 1.25 5.50 4.62 5.99 1.15 1.00 5.55	K 1 15 26 23 6 28 28 14 14 14 24 35 15 14 16 55 77 8 17 18 17 18 4 4 6 18 12 2 8 3 8	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.888 8.358 9.00 1.34 0.88 3.40 9.29 8.25 0.93	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75 219.27 185.15 234.12 176.79	± ***** ***** ***** ***** ***** ***** ****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MKS <sub>2</sub> *LAMBDA <sub>2</sub> L <sub>2</sub> 2SK <sub>2</sub> T <sub>2</sub> S <sub>2</sub> R <sub>2</sub> K <sub>2</sub> *MSN,	Deg./ 26.40790 26.87941 26.9523 27.34169 27.4238 27.50597 27.89533 27.96820 28.04100 28.35759 28.43977 28.51256 28.60400 28.90199 28.94300 28.984310 29.06624 29.02517 29.06624 29.02517 29.06624 29.91788 29.91788 29.91788 29.91788 29.91788 29.91788 29.91788 29.91788 29.95890 30.004100 30.08217 30.54437	h 379 590 126 965 337 710 548 528 8331 922 295 333 1 941 570 941 570 941 570 941 573 789 627 333 000 667 373 7747	H cm 0.34 0.12 0.04 0.30 0.28 1.70 2.66 0.22 0.55 0.55 0.55 0.55 0.55 0.55 0.55	m         ±           4         0.2           2         0.2           6         0.3           8         0.0           0         0.3           6         0.3           6         0.3           0         0.3           0         0.3           1         0.4           0         0.4           1         0.4           0         0.3		G E 333 171 275 249 76 295 156 154 251 7 7 162 152 175 65 87 93 387 192 566 88 1866 81 868 1829 95 9 144 866 231	Deg. 3.41 3.21 3.30 0.61 5.85 5.605 5.602 5.94 1.16 5.94 1.26 5.94 1.26 5.94 1.26 5.94 1.26 5.99 1.15 5.55 5.27	K 1 15 26 23 6 28 14 14 24 35 15 14 16 5 7 7 8 17 18 4 4 6 18 12 2 8 8 13 8 22	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 8.358 9.00 1.34 0.88 3.40 9.29 8.25 0.93 7.03	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75 219.27 185.15 234.12 176.79 322.90	± ***** ***** ***** 11.80 7.54 ***** 3.47 41.63 ***** 44.24 36.52 0.54 57.45 38.90 ***** 6.68 76.25 28.64 0.86 71.07 3.29 ****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 22 22 24 25 26 27 27 22 22 22 22 22 22 22 22	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub> NU <sub>4</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MKS <sub>2</sub> *LAMBDA <sub>2</sub> L <sub>2</sub> 2SK <sub>2</sub> T <sub>2</sub> S <sub>2</sub> R <sub>2</sub> K <sub>2</sub> *MSN <sub>2</sub> KJ <sub>2</sub>	Deg./ 26.40792 26.87942 26.9523 27.34162 27.42382 27.5059 27.89533 27.96820 28.04100 28.35755 28.60400 28.90190 28.94300 28.94300 28.94300 28.94300 28.94300 28.94300 28.94300 29.952842 29.91780 29.95892 30.00000 30.04100 30.08212 30.54433 30.6265	h 379 590 126 965 337 710 548 620 922 295 331 620 922 295 331 641 670 375 042 709 415 5253 789 627 333 000 667 373 7747	H cm 0.34 0.12 0.04 0.30 0.28 1.70 2.66 0.22 0.55 0.22 0.55 0.55 0.55 0.55 0.55	m         ±           4         0.2           2         0.2           6         0.3           6         0.3           6         0.3           6         0.3           6         0.3           6         0.3           7         0.3           9         0.3           9         0.3           9         0.3           9         0.4           9         0.5           9         0.4           10         0.4           10         0.4	± 36 36 335 335 335 335 335 335	G E 333 171 275 249 76 295 156 154 251 152 152 152 152 152 152 152 152 152	Deg. 3.41 3.21 3.30 0.61 5.85 5.60 5.62 5.94 1.16 5.94 1.26 5.94 1.26 5.94 1.26 5.50 6.62 5.99 0.39 1.15 5.55 1.27 3.26	K 1 15 26 23 6 28 14 14 24 35 15 14 16 55 77 8 17 18 17 18 4 4 6 18 13 8 8 22 27	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 8.358 9.00 1.34 0.88 3.40 9.29 8.25 0.93 7.03 9.27	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75 219.27 185.15 234.12 176.79 322.90 15.14	± ***** ***** ***** 11.80 7.54 **** 3.47 41.63 **** 44.24 36.52 0.54 57.45 38.90 ***** 6.68 76.25 28.64 0.86 71.07 3.29 *****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 27 28 27 28 20 21 22 23 24 25 26 27 28 20 21 22 23 24 25 26 27 28 20 20 21 22 23 24 25 26 27 27 28 20 20 20 20 20 20 20 20 20 20	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MKS <sub>2</sub> *LAMBDA <sub>2</sub> L <sub>2</sub> 2SK <sub>2</sub> T <sub>2</sub> S <sub>2</sub> R <sub>2</sub> K <sub>2</sub> *MSN <sub>2</sub> KJ <sub>2</sub> 2SM <sub>2</sub>	Deg./ 26.40792 26.87942 26.9523 27.34166 27.42383 27.5059 27.89533 27.96820 28.04100 28.35755 28.43977 28.51256 28.60400 28.90190 28.94300 28.98410 29.02517 29.06626 29.52842 29.917866 29.9176	h 379 590 126 965 337 710 548 620 922 295 331 041 670 375 042 709 415 2253 374 709 415 2253 373 3000 667 373 3747 119 958	H cm 0.34 0.12 0.16 0.28 0.28 1.77 0.266 0.28 0.20 5.73 0.55 0.22 0.55	m         d           4         0.3           2         0.2           6         0.3           8         0.3           0         0.3           6         0.3           7         0.4           9         0.3           9         0.3           2         0.2           5         0.3           5         0.3           4         0.4           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3           0         0.3	± 36 36 36 36 36 36 36 36 36 36	G E 333171 275 249 76 295 156 154 251 152 152 152 152 152 152 152 152 152	Deg. 3.41 3.21 3.30 0.61 5.85 5.60 5.62 5.94 1.16 5.94 1.26 5.94 1.26 5.99 9.39 1.15 5.50 1.27 3.26 2.18	K 1 15 266 28 28 14 14 24 35 5 15 14 16 5 7 7 8 17 18 17 18 4 4 6 18 12 8 8 13 8 22 27 26	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 3.58 9.00 1.34 0.88 3.40 9.29 8.25 0.93 7.03 9.27 9.36	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75 219.27 185.15 234.12 176.79 322.90 15.14 5.23	± ***** ***** ***** 11.80 7.54 **** 3.47 41.63 ***** 44.24 36.52 0.54 57.45 38.90 ***** 6.68 76.25 28.64 0.86 71.07 3.29 ***** 48.31 16.72
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 20 21 22 23 24 25 26 27 28 29 20 20 20 20 20 20 20 20 20 20	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> ZSK <sub>2</sub> T <sub>2</sub> S <sub>2</sub> R <sub>2</sub> K <sub>2</sub> K <sub>2</sub> K <sub>2</sub> XSSN <sub>2</sub> ZSSN <sub>2</sub> 2MS2N <sub>2</sub>	Deg./ 26.40792 26.87942 26.9523 27.34166 27.42383 27.50597 27.89533 27.96820 28.04100 28.35755 28.60400 28.9199 28.94300 28.94300 28.94300 28.94300 28.94300 28.94300 28.94300 29.92847 29.92847 29.91788	h 379 590 126 965 337 710 548 620 922 295 331 627 333 1 670 375 042 709 415 225 331 041 670 375 225 333 747 711 958 494	H cm 0.34 0.12 0.16 0.00 0.28 1.77 2.66 0.28 0.22 0.22 0.22 0.55 0.22 0.22 0.55 36.79 0.44 0.68 0.29 3.66 0.29 3.62 0.22 0.22 0.25 0.22 0.22 0.22 0.25 0.22 0.22	m         d           4         0.3           2         0.3           6         0.3           8         0.3           0         0.3           0         0.3           2         0.3           0         0.3           3         0.3           9         0.3           2         0.4           9         0.5           0.4         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4           0         0.4	2 36 36 335 335 336 36 36 36 36 36 36 36 36 3	G E 333171 275 249 76 295 156 154 251 152 152 152 152 152 152 152 152 152	Deg.           3.41	K 1 15 266 28 28 28 28 28 28 28 14 14 24 35 5 7 7 8 15 15 14 16 5 7 7 7 8 8 17 18 8 4 4 6 18 12 8 8 13 8 8 22 27 7 26 23 27 26 28 28 28 28 28 28 28 28 28 28 28 28 28	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 3.58 6.10 8.94 4.24 8.88 3.58 9.00 1.34 0.88 3.40 9.29 5.27 7.03 9.27 9.36 7.09	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75 219.27 185.15 234.12 176.79 322.90 15.14 5.23 332.96	± ***** ***** ***** ***** ***** ***** ****
No. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 32 4 25 26 27 28 29 30	Symbol *N(MU)S <sub>2</sub> *2NS <sub>2</sub> *M(MU)S <sub>2</sub> *OQ <sub>2</sub> *MNS <sub>2</sub> *NMK2S <sub>2</sub> 2N <sub>2</sub> MU <sub>2</sub> *2(NU)M <sub>2</sub> *2(NU)M <sub>2</sub> *SNK <sub>2</sub> N <sub>2</sub> NU <sub>2</sub> *2KN2S <sub>2</sub> OP <sub>2</sub> MTS <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MST <sub>2</sub> MSS <sub>2</sub> *LAMBDA <sub>2</sub> L <sub>2</sub> 2SK <sub>2</sub> T <sub>2</sub> S <sub>2</sub> R <sub>2</sub> K <sub>3</sub> K <sub>3</sub> 2SSM <sub>2</sub> 2SSM <sub>2</sub> 2SSM <sub>2</sub> 2SSM <sub>2</sub>	Deg./ 26.4079: 26.8794; 26.9523; 27.3416; 27.4238; 27.5059; 27.8953; 27.9682( 28.04100; 28.35755; 28.4397; 28.51256; 28.60400; 28.9019( 28.9430; 28.9430; 28.9430; 29.0251; 29.0662; 29.4556; 29.5284; 29.9178; 20.9178; 20	h 379 590 126 965 337 710 548 8284 620 922 295 331 647 833 10570 3375 042 709 415 253 789 667 333 3000 667 373 333 747 119 958 494	H cm 0.34 0.12 0.16 0.04 0.28 1.7( 2.66 0.28 0.20 5.7( 0.27 0.55 0.27 0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.5	m         ±           4         0.3           6         0.3           6         0.3           6         0.3           7         0.4           0         0.3           1         0.3           9         0.3           9         0.3           1         0.4           0         0.4           1         0.3           0         0.4           0.5 <t< td=""><td>2 36 36 36 36 36 36 36 36 36 36</td><td>G E 333 171 275 2499 76 295 156 154 251 152 152 152 152 152 152 152 155 65 87 93 187 192 56 68 88 186 231 129 95 144 86 231 128 3277 239 95 144</td><td>Deg.           3.41           3.30           0.61           5.30           6.62           4.49           9.62           1.16           5.30           7.68           1.25           5.50           5.51           5.52           1.16           7.68           1.25           5.50           5.30           5.51           1.27           3.26           2.18           9.69</td><td>K 1 15 26 23 6 28 14 28 14 24 35 15 15 15 15 15 15 15 15 15 15 15 15 15</td><td>Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 3.58 9.00 1.34 0.88 3.40 9.29 8.25 0.93 9.27 9.36 7.09 1.71</td><td>G</td><td>W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75 219.27 185.15 234.12 176.79 322.90 15.14 5.23 332.96 137.58</td><td>± ***** ***** ***** ***** ***** ***** ****</td></t<>	2 36 36 36 36 36 36 36 36 36 36	G E 333 171 275 2499 76 295 156 154 251 152 152 152 152 152 152 152 155 65 87 93 187 192 56 68 88 186 231 129 95 144 86 231 128 3277 239 95 144	Deg.           3.41           3.30           0.61           5.30           6.62           4.49           9.62           1.16           5.30           7.68           1.25           5.50           5.51           5.52           1.16           7.68           1.25           5.50           5.30           5.51           1.27           3.26           2.18           9.69	K 1 15 26 23 6 28 14 28 14 24 35 15 15 15 15 15 15 15 15 15 15 15 15 15	Deg. 6.76 5.98 0.29 5.77 3.26 1.70 4.44 2.53 0.18 7.09 2.08 1.83 5.88 6.10 8.94 4.24 8.88 3.58 9.00 1.34 0.88 3.40 9.29 8.25 0.93 9.27 9.36 7.09 1.71	G	W Deg. 112.63 251.84 356.16 331.63 159.12 17.57 240.30 238.39 336.04 92.96 247.94 237.70 261.75 151.97 174.81 180.11 274.75 279.45 144.86 157.20 276.75 219.27 185.15 234.12 176.79 322.90 15.14 5.23 332.96 137.58	± ***** ***** ***** ***** ***** ***** ****

	CYCLES PER DAY: 2 DEGREES OF FREEDOM: 204 RESIDUAL ENERGY: 2.09 STANDARD DEVIATION: 0.14 cm															
			S	PEC	TRUM	OF	RESIL	DUA	L AN	APLI	TUD	ES				
De	g./h	Res.	Deg./h	1	Res.	De	g./h	Re	<b>25</b> .	De	g./h	Re	s	Deg./h	Res.	
26. 26	.343 .546	0.20 0.06	26.383 26.587		0.07	26.424 26.627		0. 0.(	15 04	26 26	.465 .668	0.1 0.2	3 0	26.505 26.709	0.26	
26.	749	0.11	26.790		0.14 2		26.831		08	26	.871	0.0	5	26.912	0.09	
26.	953 156	0.00	26.993		0.15	27	.034	0.0	J/ 16	27	.075 .278	0.1	8 6	27.115	0.14	
27.	359	0.00	27.400	ļ	0.16	27	.440	0.	10	27	.481	0.1	1	27.522	0.08	
27.	562 766	0.19	27.603		).19	27	.644	0.0	08 22	27. 97	.684 888	0.1	4	27.725	0.18	
27.	969	0.01	28.010	Ċ	0.12	28	.050	0.0	33	28	.091	0.0	6	28.132	0.26	
28.	172 375	0.16	28.213		0.26	28	.253	0.3	32	28.	294	0.1	2	28.335	0.03	
28.	579	0.07	28.619	Ċ	0.03	28	.660	0.2	23	28.	.701	0.0	0	28.338	0.13	
28.	782	0.21	28.823		80.0	28	.863	0.2	25	28.	904	0.0	1	28.945	0.01	
20.	189	0.01	29.026		0.20	29.	.06/	0.1	14	29. 29.	.107	0.2	ь 8	28.148	0.45	
29.	392	0.25	29.432	0	).08	29.	.473	0.3	31	29.	514	0.3	3	29.554	0.26	
29.	595 798	0.26	29.636 29.839		0.17	29.	.676	0.0	)7 20	29.	717   020	0.3	4	29.758	0.11	
30.	002	0.01	30.042	Ö	0.01	30.	.083	0.0	5	30.	124	0.4	2	30.164	0.35	
30.	205	0.15	30.245	0	0.05	30.	286	0.2	5	30.	327	0.3	0	30.367	0.19	
30.	408   611	0.16	30.449		23	30.	.489	0.1	15 12	30. 30.	530 733	0.0	8	30.571	0.13	
30.	815	0.25	30.855	0	.30	30.	896	0.1	4	30.	937	0.1	4	30.977	0.15	
31.	018   221	0.01	31.059		03	31.	302	0.1	0	31.	140 343	020	6	31.180	0.33	
31.4	424	0.14	31.465	ŏ	.17	31.	506	0.2	0.28 31		1.546 0		7	31.587	0.08	
31.0	528	0.09	31.668	0	.09	31.	709	0.1	5							
	T		<del></del>		HAR	MO	NIC C	ONS	STAN	<b>NTS</b>			<b></b>			
No.	5	Symbol	Deg./	<u>h</u>	На	n	±		G D	eg.	K	Deg.	G	W Deg.	±	
1	*N(	MU)S2	26.4079	379	0.34	4	0.36		34.	08	1	7.44		113.30	*****	
3	*M	MU)S,	26.95231	126	0.13	5	0.36		172.2		26	2.69		252.91 358.56	*****	
4	*00	22	27.34169	65	0.04	4	0.35		260.27		7 246.43			342.30	*****	
5		N52 1K25	27.4238	337	0.31	$\frac{1}{2}$	0.35		76.93		28	3.34		159.21 17.19	*****	
7	$2N_2$	2	27.89535	548	1.70	5	0.36		156.	23	14	1.05		239.92	11.78	
8 9	MU *20	JUM.	27.96820	)84 520	2.65		0.36	ļ	154.	69 10		2.72		238.59	7.58	
10	-SN	K <sub>2</sub>	28.35759	22	0.20	, j	0.36		- 	72	358	3.93		94.79	*****	
11			28.43972	295 297	5.72	2	0.36		162.	60	152	2.05		247.92	3.47	
12	*2K	, N2S,	28.51258	41 41	0.53	?	0.36		152.	02 00	14	/4 1.95		237.61 260.81	40.73	Ì
14	OP,		28.90196	570	0.58	3	0.35		65.	82	56	5.66		152.53	45.07	
15 16	MT M	5 <sub>2</sub>	28.94303	375 No	0.57		0.35		88. 02	00 17	78	3.96		174.83	36.83	
17	MS]	Γ2	29.02517	709	0.41	i	0.35		93. 187.	41	178			274.48	57.10	
18	MK	S.	29.06624	15	0.68	3	0.35		191.	79	183	3.13		278.99	39.19	
19 20		MBDA <sub>2</sub>	29.45562	153   789	0.29	;	0.36		57. 68	40 63	49	90 35		145.76 157.21	***** 6 70	
21	2SK	2	29.91786	27	0.44	í	0.35		186.	77	180	).66		276.52	84.18	
22	T <sub>2</sub>		29.95893	33	0.74		0.35		130.	22	124	.23		220.10	28.61	
23 24	$R_{2}$		30.0000	67	23.54		0.35		95. 143.	10 34	137	7.59 7.59		233.46	0.86 72.60	
25	K <sub>2</sub>		30.08213	73	7.72		0.35	1	86.	48	80	.86		176.72	3.29	
26 27	I MS	N <sub>2</sub>	30.54437	47	0.30	2	0.35		231.	97	227	7.73	:	323.60	*****	
28		30.6265119 0.6 31.0158958 1.1		0.00	0.35			283.95		279.97		15.83 5.19		40.27	1	
	25M	2	31.01589	31.0158958 1.19 J. 31.0887494 0.47			0.36 272.		272.	15	269	9.33		5.19	16.79	
29	25M 2MS	$^{2}{_{2}}N_{2}$	31.01589 31.08874	58 94	1.19 0.47		0.36		272. 240.	15 72	269 238	0.33 0.12		5.19 333.99	16.79 42.68	

# Table - 4 - IV - 369 days analysis with the FFT algorithm

### **5 - GENERALIZATION OF THE REFINED METHOD**

The refined method was devised to analyse spans of  $2^{14}$  hours (FRANCO, 1988). However we replaced the classical FFT, in the original program by a flexible one so that it became possible to analyse series of any length (usually spans larger than one-year) with main frames; but the program written by this author to be run in PCs kept the classical FFT subroutine to work out analyses of series with samples from  $2^8$  to  $2^{14}$  hours.

In order to establish the sets of constituents to be separated with the mentioned spans, attention was paid to the spectra of residual amplitudes. Then, analyses of spans from 1024 hours onwards did show residual peaks much larger than those expected, near the frequencies of constituents  $P_{1\nu}v_2$  and  $K_2$ .

After introducing the interpolation scheme in the program, analyses from a 1368 hours span onwards, were worked out. Then it became clear from the spectra of residual amplitudes that it was necessary to include new constituents in the analyses of the diurnal and semidiurnal species in order to get rid of the contamination of the secondary constituents of the groups  $(K_1, P_1)$ ,  $(N_2, v_2)$  and  $(S_2, K_2, T_2)$  on the main constituents  $K_1$ ,  $N_2$  and  $S_2$ , respectively. Thus, it was concluded that the Rayleigh's rule was not applicable when tapered heights were analysed. In fact, the spectra of residual amplitudes of an analysis of a 1368 hourly heights (57 days) did show peaks on the Fourier frequencies near the ones of  $P_1$ ,  $v_2$  and  $K_2$ . It was really surprising the possibility of separating the pairs  $(P_1, K_1)$ ,  $(v_2, N_2)$  and  $(S_2,$  $K_2)$  through the analysis of such a short span. However, it can be mentioned that the method devised by DOODSON (1941) for the British Admiralty, to analyse a 15 days span allowed the separation of  $N_2$  and  $M_2$ , with a phase shift of about 180<sup>o</sup> during the period analysed.

Table 5-I shows some results obtained from analyses of data taken at Cananéia. One sees there that the harmonic constants for the above mentioned pairs of constituents, obtained through the analysis of a 57 days span, are very reliable. Let us investigate the reason for that.

The Hanning window considerably reduces the lateral contamination of the other constituents in a group as well as the background noise near these frequencies. Hence each group is contaminated only by the background noise and the measuring error under the spectral window. This is why it is not necessary a  $360^{\circ}$  phase shift of two neighbouring constituents, during the period covered by the series, in order to separate them. In fact, if no noise and measuring error existed, the separation would be possible even for small values of the phase shift. Table 5-II shows the redundant matrix for the semidiurnal constituents for a 1368 hourly tapered heights span. One can see that the largest value in each column is preceeded and followed by only four important contributions of neighbouring constituents. Values beyond these four Fourier harmonics are meaningless.

The reduction of these lateral contaminations by the hanning window allows the analysis of spans of any extension, not tied to the lunations.

Obviously, the accuracy of the analysis increases with the span length; the analysis of a 2<sup>14</sup> hourly heights span is extremely accurate.

For series up to 8500 hourly heights the refined analyses are faster than the normal ones. Expression (2e) and (2f) shows that matrix  $A_{nj}$  holds for both sets of unknowns,  $R_j \cos r_j$  and  $R_j \sin r_j$ . Thus there will be also a single normal matrix and the respective inversion for each species. For spans equal to or larger than 8500 hourly heights, the system covering each species can be split into several small ones, with the result that the analyses of these spans are much faster than those which work out lesser ones.

	1	8.69 years	(1972-198	<del>)</del>	682.67 days						
	cm	cm cm ±		deg. ±	cm	cm ±	deg.	deg. ±			
$ \begin{array}{c} P_1 \\ K_1 \\ N_2 \\ \nu_2 \\ S_2 \\ K_2 \end{array} $	2.36 6.36 5.78 0.77 23.68 7.47	0.03 0.00 0.04 0.04 0.06 0.06	149.31 145.65 163.30 168.93 95.96 86.73	0.68 0.25 0.35 0.27 0.15 0.43	2.70 6.31 5.87 0.57 23.69 8.07	0.17 0.17 0.14 0.14 0.22 0.22	152.39 144.43 161.53 162.45 95.25 66.63	3.63 1.69 1.36 14.28 0.54 1.88			
		369	days			57 c	lays				
P, K <sub>1</sub> N <sub>2</sub> ×, S <sub>2</sub> K <sub>2</sub>	2.67 6.37 8.68 0.69 23.71 7.51	0.36 0.45 0.25 0.24 0.31 0.23	153.63 145.56 163 55 156.95 96.19 87.19	7 77 4.39 2.40 19.39 1.41 3.60	2.59 5.99 6.75 *0.67 23.30 7.83	1.62 1.59 1.68 1.68 1.51 1.51 1.52	171.52 141.74 158.34 215.78 94.21 91.74	38.15 17.09 14.01 ***** 3.52 14.39			

Table 5-I - Comparing some results of refined analysis

Table 5-II - Value of 1000xAnj for the semi diurnal species

٩ -	Rows correspond to Fourier frequencies rows from 25.789 to 32.105 o/h $\Delta f = 0.263$													
$N\mu S_2$	2NS <sub>z</sub>	MNS.	μ2	N <sub>2</sub>	γ <sub>2</sub>	M <sub>2</sub>	L <sub>2</sub>	Sz	K <sub>2</sub>	MSN₂	25M2	2SN <sub>2</sub>		
-27	-1	0	0	()	e	0	0	υ	0	0	0	0		
-271	-4	0	0	Ó	Ō	0	J	0	0	0	0	0		
933	-16	-3	-1	0	0	0	0	0	0	0	0	0		
-739	-413	-6	-2	0		J	0	0	0	- 0	0	0		
90	991	-21	-3	0	-1	0	0	Û	0	0	0	0		
17	-588	-365	-7	0	-2	0	0	0	0	0	0	0		
24	978	-25	-1	-4	0	0	0	9	0	0	0	0		
3	5	-639	-518	-2	-3	-1	0	C	0	0	0	0		
2	2	42	958	-7	-26	-2	-1	0	0	0	0	0		
1	1	9	-688	-476	-273	-4	-1	0	-1	0	0	0		
ì	1	3	64	999	935	-15	-2	0	-1	0	0	0		
0	0	21	13	-534	-736	-6	9	-2	Ű	0	0	0		
0	0	1	5	8	88	952	-21	0	-3	0	0	0		
6	0	1	2	2	15	-538	-368	1	-8	0	1	0		
0	0	0	1	1	3	- 23	979	5	-26	-1	1	0		
0	0	0	1	0	31	0	-636	-521	-297	-2	1	0		
Û	υ	0	0	0	2	2	41	1000	949	-6	3	0		
0	0	Û	0	0		1	9	-479	-710	-470	8	1		
0	0	0	0	0	1	Í.	3	-1	74	999	37	2		
C	0	0	5	0	0	9	0	-1	14	-530	-626	5		
0	0	0	01	0	0	0	1	Э	5	7	982	20		
0	0	0	0.	0		0	1	0	2	2	-377	-575		
0	0	0	0	0	0	0	0	0	1	1	-20	994		
0	0	0	0	0	0	0	0	0	1	0	-6	-426		
0	0	0	0	0	0	0	0	0	0	0	-2	-14		

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### **6 - FINAL COMMENTS**

Excellent observations of tidal heights at "São Pedro and São Paulo" Rocks (Lat.  $00^{\circ}$  56' S, Long.  $29^{\circ}$  17' W), Brazil, with a pressure gauge, during a little more than 29 days, with a sampling interval of 0.25 hour were also analysed, for observed and interpolated data, with the normal method. The results were identical since the interpolation through smaller sampling interval is more effective.

In the interpolation technique here presented, the successive differences of the various orders are convenient approximations to the derivatives, thus it is understandable that good results are reached through the analysis of interpolated data.

As it was said before the Doodson's method of tidal analysis for a 15 days span, then used by the Hydrographic Department of the British Admiralty (1941) allowed the separation of  $M_2$  and  $N_2$ , with a phase shift of about 180°. Thus it is not surprising that constituents with a phase shift less than 180° are separated through the refined method of analysis.

This author has prepared a set of eight programs, to be used in PCs, to check tidal observations, predict and analyse tides and tidal currents and to solve related problems. These programs are a complement to his book on tides (FRANCO, 1988), thus an English version of that software is also available (FRANCO, 1989).

Spectral analyses of interpolated random time series is also well succeeded (J. R. MARTINS, private communication). Hence, it is possible that the same improvement can be introduced in the harmonic analysis of tidal currents. However, the background noise in the current observation is usually so high that the data filtering of such spurious interference must preceed the interpolation procedure.

The above mentioned programs run in any IBM PC and the processing speed depends on the kind of used machine. If, for example, an IBM 486 is available, the normal analysis of a 8856 hours span is worked out in about 10 seconds.

It is in fact, very remarkable that so good results can be reached by using such a simple technique.

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