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SPEEDING UP TIDAL ANALYSES WITH PCs

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Summary

A technique is suggested to effect the tidal analysis with PCs by using the Fast Fourier Transform applied to tidal heights obtained through a second order interpolation of the original data. If a turbo PC XT with mathematical coprocessor is available then it is possible to analyse a 369 days classical span (8856 hourly heights) and separate 170 constituents (up to the twelfth diurnal) in 13 minutes, without jeopardizing the accuracy of the results. If the same data are weighted with a cosine taper (hanning in the time domain), then a more accurate analysis of the same data can be worked out in about 3 minutes. The analyses of non tapered and tapered data will be called *normal* and *refined*, respectively.

1 - INTRODUCTION

A complete set of programs to analyse and predict tides after checking the tidal observations was prepared in the 80s to be used in the main frame computer of the São Paulo University (FRANCO & HARARI, 1987). These programs are still in use in the "Diretoria de Hidrografia e Navegação" (Hydrographic Department) of the Brazilian Navy and in the "Instituto Oceanográfico" of the quoted University.

When the micro computers became available, the present author started to study the possibility of running his programs in the new excellent tool. However, the micro computers of the first generation did not have sufficient memory to work out analyses of hourly heights of one-year spans. But he did not have to wait long for improved machines where his programs could run. Thus, he bought a turbo PC XT with mathematical coprocessor and rewrote his programs in a language more suitable for this machine.Then the development of the micro computers started to run at such an amazing speed that the economical possibilities of this author could not follow any more. Thus he started to optimize these programs in order not to be

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compelled to make a "siesta" after a good launch, while his micro computer worked out lengthy computations.

The use of the method of least squares in the frequency domain is the main characteristic of these harmonic methods of analysis (FRANCO, 1988). Thus, its first step is the Fourier analysis of the either tapered or non tapered heights, for both the refined or normal analysis; this step is performed in order to separate the species. However, adapting the flexible FFT (Fast Fourier Transform) used in main frames in order to run it in micro computers proved to be very difficult.Then the Fourier analysis in the normal method was worked out with the Watt's recurrent formula (CARTWRIGHT-CATTON, 1963) applied to the frequency band of each species.But even so the harmonic analysis of a one-year span, with the normal method, necessary to separate 170 constituents, required about 1 hour and 12 minutes, in a PC XT with coprocessor, whereas the analysis of 2^{14} hourly tapered data by the refined method, using the FFT, spent 3 minutes only, with the same machine. Then there were two different problems to be solved: a) to speed up the normal analysis by using the classical FFT, with a Brigham (1974) improvement; b) to remove the constraint of the refined method, limited to analyse time series with number of samples equal to powers of two. The solution of these problems will be shown next.

2 - SUMMARY OF THE METHOD OF ANALYSIS

According to FRANCO (1988), the Fourier coefficients a_n and b_n of any tidal series worked out with the normal method can be expressed in terms of the tidal constituents by

$$a_n + c = \sum_{j=1}^{Q} (A_{nj} + A_{n,-j}) R_j \cos r_j \tag{2a}$$

$$b_n = \sum_{j=1}^{Q} (A_{nj} - A_{n,-j}) R_j \sin r_j \tag{2b}$$

where Q, R_j and r_j are, respectively: the number of constituents, the amplitude and the phase of the j^{th} constituent at the central time. c is a correction to allow the separation of the set of unknown $R_j \cos r_j$ from that of $R_j \sin r_j$ (FRANCO,1988). The matrices A_{nj} and $A_{n,-j}$ are given by the following relationships:

$$A_{nj} = (-1)^n \frac{\sin[(\omega_n - \omega_j)(N+1)\Delta t/2]}{N \sin[(\omega_n - \omega_j)\Delta t/2]}$$

(2c)

$$A_{n,-j} = (-1)^n \frac{\sin[(\omega_n + \omega_j)(N+1)\Delta t/2]}{N \sin[(\omega_n + \omega_j)\Delta t/2]}$$

and

$$c=2[\zeta(N\Delta t)-R_0] \tag{2d}$$

where

Δt	≡ the sampling interval
ω_n	≡ the Fourier frequency
ω_j	≡ the frequency of the j^{th} constituent
N	≡ the number of samples
$\zeta(N\Delta t)$	≡ the last sample in the record
R_0	≡ mean value of the sampled heights

In the refined method expressions (2a) and (2b) reduce to

$$\begin{aligned} a_n &= \sum_{j=1}^{Q} A_{nj} R_j \cos r_j \\ b_n &= \sum_{j=1}^{Q} A_{nj} R_j \sin r_j \end{aligned} \tag{2e}$$

where

$$A_{nj}(-1)^n = \frac{\sin[(\omega_n-\omega_j)(N+1)\Delta t/2]}{N\sin[(\omega_n-\omega_j)\Delta t/2]} - \frac{\sin[(\omega_n-\omega_j)(N+1)\Delta t/2-\pi/N]}{2N\sin[(\omega_n-\omega_j)\Delta t/2-\pi/N]} - \frac{\sin[(\omega_n-\omega_j)(N+1)\Delta t/2+\pi/N]}{2N\sin[(\omega_n-\omega_j)\Delta t/2+\pi/N]} \tag{2f}$$

3 - INTERPOLATION

Let us suppose that we need to analyse hourly heights of a classical 29 days span. Then we have in this case 696 ordinates. The question is: how to change the time series into another one, with 1024 points, in order to compute the Fourier coefficients a_n and b_n by using the classical FFT? Since 1 hour is the sampling interval in the original series, that interval in the second series should be 695/1023 = 0.6793743 hour. Thus, if we work out the interpolation of the series in order to have the heights corresponding to the 0.6793743 hour sampling interval, the ordinates to be analysed will be 1024 (2^{14}) and it is possible to use the classical FFT algorithm to work out the Fourier analysis. However a linear interpolation is not sufficiently accurate to give the new heights.

It is well known that the ephemeris available to astronomers of the past century had right ascentions at 12 hours intervals. Then they used to interpolate these values through finite differences of several orders. CHAUVENET (1891), who wrote the first edition of his Spherical Astronomy in 1863, gave examples of interpolations by using differences up to the 5th order.

Tide is a well behaved phenomenon, with no discontinuities, thus it is possible to interpolate the curve by using 2[nd] order differences only.

Let ζ_1, ζ_2 and ζ_3 be three successive tidal heights and ζ_i the interpolated tidal height nearest to ζ_2. Then

$$\delta_1=[(\zeta_3-\zeta_2)+(\zeta_2-\zeta_1)]/2=(\zeta_3-\zeta_1)/2$$

$$\delta_2=(\zeta_3-\zeta_2)-(\zeta_2-\zeta_1)=\zeta_3-2\zeta_2+\zeta_1 \qquad \text{(3a)}$$

and

Δ = the fractional interval between ζ_i and ζ_2

According to CHAUVENET (1891), one of the formulae to compute ζ_i, as reduced to the second order term, is:

$$\zeta_i=\zeta_2+\delta_1\Delta+\delta_2\Delta^2/2 \qquad \text{(3b)}$$

where Δ is the time interval between ζ_i and ζ_2. The sampling interval of the observed heights is usually short (1 hour) thus, we may consider that the low frequency background noise affecting the three heights is nearly destroyed by the operations to obtain δ_1 and δ_2. But the measuring error affecting the observed heights must be considered.

If σ is the variance corresponding to the observed heights, the law of error propagation as applied to expressions (3a) gives

$$\sigma_1^2 = \sigma^2/4 \textit{ for } \xi_1$$

and (3c)

$$\sigma_2^2 = 6\sigma^2 \textit{ for } \xi_2$$

The same law, as applied to expression (3b) for Δ=0.5, will give

$$\sigma_i^2=1.16\sigma^2$$

consequently, the standard deviation is

$$\sigma_I = \pm 1.08\sigma$$

hence the accuracy of the interpolated heights is about 8% lower than that of the observed ones. If ±1 cm is the standard deviation corresponding to the measurement of the tidal heights, then ±1.08 cm will be the standard deviation for the interpolated heights. In fact, the shown practical examples indicate that no decrease of the accuracy occur in the results when the observed data are replaced by the interpolated ones. The large number of observations handled in tidal analyses explains why the results are not affected when the interpolated data are used.

In order to give a visual idea of such interpolation, it was applied to the large tide of "Igarapé do Inferno" (02° 06.0′ N and 050° 32.0′ W) in the Maracá Island (Brazil). Notwithstanding the range of about 10 meters and the strong shallow-water effects, Figure 3A shows how accurate is the interpolation.

An important remark is that the resolution of the analysis, equal to $1/N\Delta t$, is not altered by the interpolation, but the cut off frequency is increased, since it is equal to $180/\Delta t$ °/h.

Another remark is that this interpolation have been used in spectral analyses of randomic continuous time series with good results.

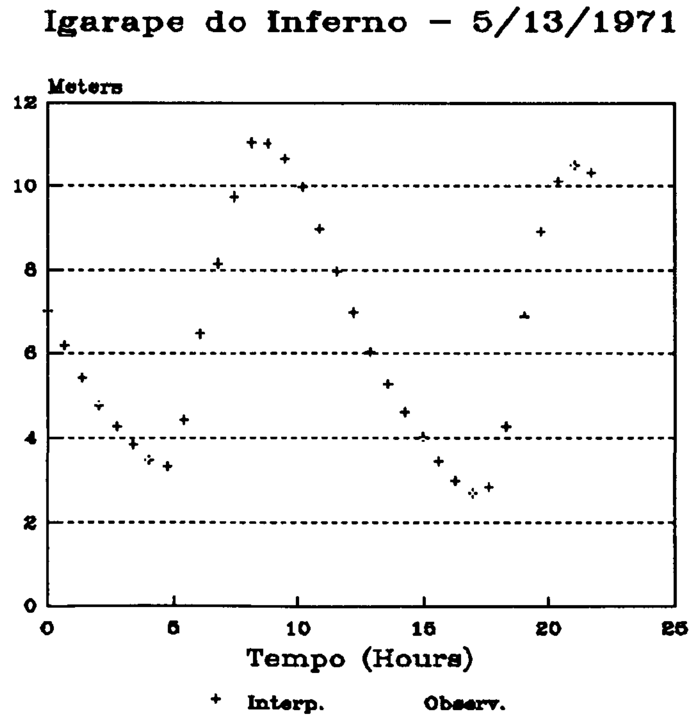

Figure 3A - • ≡ observed heights; + ≡ interpolated heights

4 - SOME RESULTS WITH THE NORMAL METHOD

The results shown in Tables 4-I to 4-IV will convince the reader on the success of such a simple technique.

Two different classical spans were chosen: 29 and 369 days. Since the semidiurnal is the most important species, four analyses were worked out for that species. Tables 4-I and 4-II show the results of the analyses using non interpolated and interpolated heights, respectively, for a 29 days span (696 hours). The chosen tidal station is "Ponta da Madeira" (Lat. 02^0 06′ S, Long. 050^0 32′ W), Brazil, where the tidal range is about 7 meters; the initial time of the considered record is 0^h, 11/1/1991. The other two analyses were worked out with 8856 data (369 days) from Cananéia (Lat. 23^0 O1′ S,Long. 47^0 56′ W), Brazil, started at 0^h of 1/1/1980; the results obtained with the observed data and the interpolated ones are shown in Tables 4-III and 4-IV, respectively.

It is important to say that, according to the extension of the analysis, the statistical rejection of small constituents (FRANCO, 1972), which are flagged with an asterisc, is made by the program of analysis. The probability for the rejection is chosen by the user; in this case it was 95%. All the shallow water constituents have been considered by several researchers and are taken from an extensive table drawn by this author. These constituents showed to be stable in successive analyses of a 18.69 Julian years span (FRANCO & HARARI, 1988, 1991).

One can see by inspection of Tables 4-I and 4-II that the harmonic constants of the non flagged constituents are practically the same in both tables. In fact, the differences can be ascribed to round off errors and to the fact that the analysis with the Watt's formula was worked out with 695 heights instead of 696. The odd number of heights 5s used in order to establish pairs of heights symmetrical with respect to the central time because the Fourier analysis becomes faster and the correction c of (2c) is avoided. That correction diminishes when the length of the series increases. For a one year analysis c is nearly zero.

The spectra of residual amplitudes and the harmonic constants obtained from the analyses of a one-year span, shown in Tables 4-III and 4-IV, are so close, that no additional comments are necessary.

Table 4-I - 29 days analysis with the Watt's formula

CYCLES PER DAY: 2 RESIDUAL ENERGY: 1.09	DEGREES OF FREEDOM: 8 STANDARD DEVIATION: 0.48 cm

SPECTRUM OF RESIDUAL AMPLITUDES									
Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.
25.381	0.53	25.899	0.75	26.417	0.01	26.935	0.04	27.453	0.01
27.971	0.00	28.489	0.01	29.007	0.01	29.525	0.00	30.043	0.02
30.561	0.01	31.079	0.06	31.597	0.08	32.115	0.98	32.663	0.61

HARMONIC CONSTANTS								
No.	Symbol	Deg./h	H cm	±	G Deg.	K Deg.	GW Deg.	±
1	*$N(MU)S_2$	26.4079379	1.63	1.57	133.14	123.63	212.36	*****
2	$2NS_2$	26.8794590	3.43	1.58	255.15	247.05	335.78	36.55
3	MNS_2	27.4238337	2.02	1.57	1.77	355.30	84.04	67.94
4	MU_2	27.9682084	4.75	1.57	358.42	353.59	82.32	19.50
5	N_2	28.4397295	38.40	1.58	195.15	191.73	280.47	2.58
6	M_2	28.9841042	213.18	1.57	198.90	197.12	285.85	0.42
7	L_2	29.5284789	10.90	1.58	164.74	164.59	253.33	7.34
8	S_2	30.0000000	53.94	1.57	230.93	232.19	320.93	1.82
9	MSN_2	30.5443747	5.07	1.57	13.03	15.93	104.66	21.79
10	$2SM_2$	31.0158958	4.27	1.58	76.21	80.52	169.28	26.34
11	*$2SN_2$	31.5602705	1.75	1.57	215.54	221.49	310.22	*****

INFERRED CONSTANTS						
No.	Symbol	Deg./h	H cm	G Deg.	K Deg.	GW Deg.
1	$2N_2$	27.8953548	5.07	191.40	186.35	275.09
2	NU_2	28.5125831	7.30	195.65	192.46	281.19
3	$LAMBDA_2$	29.4556253	1.49	213.76	213.39	302.13
4	T_2	29.9589333	3.18	232.21	233.35	322.09
5	R_2	30.0410667	0.43	232.21	233.60	322.33
6	K_2	30.0821373	14.67	233.52	235.03	323.77

Table 4-II - 29 days analysis with the FFT algorithm

CYCLES PER DAY: 2 RESIDUAL ENERGY: 0.91	DEGREES OF FREEDOM: 8 STANDARD DEVIATION: 0.48 cm

SPECTRUM OF RESIDUAL AMPLITUDES									
Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.
25.357	0.43	25.874	0.67	26.391	0.02	26.909	0.01	27.426	0.00
27.944	0.01	28.461	0.01	28.979	0.00	29.496	0.01	30.014	0.01
30.531	0.01	31.049	0.04	31.566	0.02	32.084	0.94	32.601	0.56

HARMONIC CONSTANTS								
No.	Symbol	Deg./h	H cm	±	G Deg.	K Deg.	GW Deg.	±
1	*N(MU)S_2	26.4079379	1.61	1.43	138.28	128.78	217.51	*****
2	2NS_2	26.8794590	3.37	1.44	256.39	248.30	337.03	33.56
3	MNS_2	27.4238337	2.04	1.43	3.03	356.56	85.30	57.09
4	MU_2	27.9682084	4.65	1.44	359.09	354.26	83.00	18.14
5	N_2	28.4397295	38.39	1.44	195.31	191.90	280.63	2.34
6	M_2	28.9841042	213.17	1.43	198.91	197.13	285.87	0.39
7	L_2	29.5284789	10.94	1.44	165.09	164.94	253.87	6.67
8	S_2	30.0000000	54.07	1.44	230.89	232.16	320.89	1.66
9	MSN_2	30.5443747	5.06	1.44	11.40	14.30	103.04	19.87
10	2SM_2	31.0158958	4.41	1.44	76.70	81.02	169.75	23.06
11	*2SN_2	31.5602705	1.83	1.43	212.47	218.42	307.15	*****

INFERRED CONSTANTS						
No.	Symbol	Deg./h	H cm	G Deg.	K Deg.	GW Deg.
1	2N_2	27.8953548	5.07	191.71	186.66	275.40
2	NU_2	28.5125831	7.29	195.79	192.60	281.33
3	LAMBDA_2	29.4556253	1.49	213.75	213.39	302.12
4	T_2	29.9589333	3.19	232.17	233.31	322.05
5	*R_2	30.0410667	0.43	232.17	233.56	322.29
6	K_2	30.0821373	14.71	233.48	235.00	323.73

Table - 4-III - 369 days analysis with the Watt's formula

CYCLES PER DAY: 2
DEGREES OF FREEDOM: 204
RESIDUAL ENERGY: 2.09
STANDARD DEVIATION: 0.14 cm

SPECTRUM OF RESIDUAL AMPLITUDES

Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.
26.344	0.20	26.385	0.09	26.426	0.16	26.466	0.12	26.507	0.26
26.548	0.06	26.588	0.31	26.629	0.01	26.670	0.18	26.710	0.13
26.751	0.13	26.792	0.14	26.832	0.08	26.873	0.04	26.914	0.10
26.954	0.01	26.995	0.15	27.036	0.06	27.076	0.18	27.117	0.14
27.158	0.24	27.198	0.09	27.239	0.17	27.280	0.05	27.320	0.08
27.361	0.00	27.401	0.17	27.442	0.12	27.483	0.11	27.523	0.09
27.564	0.20	27.605	0.21	27.645	0.11	27.686	0.12	27.727	0.17
27.767	0.12	27.808	0.25	27.849	0.21	27.889	0.01	27.930	0.06
27.971	0.02	28.011	0.11	28.052	0.03	28.093	0.05	28.133	0.25
28.174	0.17	28.215	0.25	28.255	0.33	28.296	0.13	28.337	0.04
28.377	0.08	28.418	0.10	28.458	0.17	28.499	0.08	28.540	0.13
28.580	0.07	28.621	0.04	28.662	0.23	28.702	0.09	28.743	0.18
28.784	0.23	28.824	0.09	28.865	0.24	28.906	0.03	28.946	0.02
28.987	0.02	29.028	0.02	29.068	0.03	29.109	0.25	28.150	0.45
29.190	0.08	29.231	0.21	29.272	0.14	29.312	0.09	29.353	0.21
29.394	0.26	29.434	0.09	29.475	0.35	29.516	0.29	29.556	0.25
29.597	0.25	29.637	0.16	29.678	0.07	29.719	0.35	29.759	0.13
29.800	0.24	29.841	0.42	29.881	0.20	29.922	0.02	29.963	0.02
30.003	0.02	30.044	0.03	30.085	0.04	30.125	0.39	30.166	0.35
30.207	0.17	30.247	0.05	30.288	0.27	30.329	0.29	30.369	0.20
30.410	0.15	30.451	0.21	30.491	0.14	30.532	0.08	30.573	0.14
30.613	0.12	30.654	0.23	30.695	0.03	30.735	0.08	30.776	0.12
30.816	0.26	30.857	0.29	30.898	0.15	30.938	0.16	30.979	0.14
31.020	0.02	31.060	0.03	31.101	0.13	31.142	0.23	31.182	0.34
31.223	0.24	31.264	0.02	31.304	0.22	31.345	0.32	31.386	0.12
31.426	0.14	31.467	0.18	31.508	0.28	31.548	0.06	31.589	0.09
31.630	0.07	31.670	0.08	31.711	0.15				

HARMONIC CONSTANTS

No.	Symbol	Deg./h	H cm	±	G Deg.	K Deg.	GW Deg.	±
1	*N(MU)S$_2$	26.4079379	0.34	0.36	33.41	16.76	112.63	*****
2	*2NS$_2$	26.8794590	0.12	0.36	171.21	155.98	251.84	*****
3	*M(MU)S$_2$	26.9523126	0.16	0.36	275.30	260.29	356.16	*****
4	*OQ$_2$	27.3416965	0.04	0.35	249.61	235.77	331.63	*****
5	*MNS$_2$	27.4238337	0.30	0.35	76.85	63.26	159.12	*****
6	*NMK2S$_2$	27.5059710	0.28	0.35	295.05	281.70	17.57	*****
7	2N$_2$	27.8953548	1.70	0.36	156.62	144.44	240.30	11.80
8	MU$_2$	27.9682084	2.66	0.36	154.49	142.53	238.39	7.54
9	*2(NU)M$_2$	28.0410620	0.28	0.36	251.92	240.18	336.04	*****
10	*SNK$_2$	28.3575922	0.20	0.36	7.89	357.09	92.96	*****
11	N$_2$	28.4397295	5.73	0.36	162.62	152.08	247.94	3.47
12	NU$_2$	28.5125831	0.52	0.36	152.16	141.83	237.70	41.63
13	*2KN2S$_2$	28.6040041	0.27	0.36	175.94	165.88	261.75	*****
14	OP$_2$	28.9019670	0.59	0.35	65.26	56.10	151.97	44.24
15	MTS$_2$	28.9430375	0.53	0.35	87.98	78.94	174.81	36.52
16	M$_2$	28.9841042	36.79	0.35	93.16	84.24	180.11	0.54
17	MST$_2$	29.0251709	0.41	0.35	187.68	178.88	274.75	57.45
18	MKS$_2$	29.0662415	0.68	0.35	192.25	183.58	279.45	38.90
19	*LAMBDA$_2$	29.4556253	0.29	0.36	56.50	49.00	144.86	*****
20	L$_2$	29.5284789	3.62	0.36	68.62	61.34	157.20	6.68
21	2SK$_2$	29.9178627	0.45	0.35	186.99	180.88	276.75	76.25
22	T$_2$	29.9589333	0.74	0.35	129.39	123.40	219.27	28.64
23	S$_2$	30.0000000	23.54	0.35	95.15	89.29	185.15	0.86
24	R$_2$	30.0410667	0.30	0.35	144.00	138.25	234.12	71.07
25	K$_2$	30.0821373	7.71	0.35	86.55	80.93	176.79	3.29
26	*MSN$_2$	30.5443747	0.30	0.35	231.27	227.03	322.90	*****
27	KJ$_2$	30.6265119	0.60	0.35	283.26	279.27	15.14	48.31
28	2SM$_2$	31.0158958	1.20	0.36	272.18	269.36	5.23	16.72
29	2MS2N$_2$	31.0887494	0.46	0.36	239.69	237.09	332.96	43.09
30	*2SN$_2$	31.5602705	0.23	0.36	42.90	41.71	137.58	*****
31	*SKN$_2$	31.6424078	0.23	0.36	298.05	297.11	32.97	*****

Table - 4 -IV - 369 days analysis with the FFT algorithm

CYCLES PER DAY: 2
DEGREES OF FREEDOM: 204
RESIDUAL ENERGY: 2.09
STANDARD DEVIATION: 0.14 cm

SPECTRUM OF RESIDUAL AMPLITUDES

Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.	Deg./h	Res.
26.343	0.20	26.383	0.07	26.424	0.15	26.465	0.13	26.505	0.26
26.546	0.06	26.587	0.30	26.627	0.04	26.668	0.20	26.709	0.14
26.749	0.11	26.790	0.14	26.831	0.08	26.871	0.05	26.912	0.09
26.953	0.00	26.993	0.15	27.034	0.07	27.075	0.18	27.115	0.14
27.156	0.23	27.197	0.10	27.237	0.16	27.278	0.06	27.318	0.07
27.359	0.00	27.400	0.16	27.440	0.10	27.481	0.11	27.522	0.08
27.562	0.19	27.603	0.19	27.644	0.08	27.684	0.14	27.725	0.18
27.766	0.11	27.806	0.25	27.847	0.22	27.888	0.02	27.928	0.06
27.969	0.01	28.010	0.12	28.050	0.03	28.091	0.06	28.132	0.26
28.172	0.16	28.213	0.26	28.253	0.32	28.294	0.12	28.335	0.03
28.375	0.07	28.416	0.10	28.457	0.16	28.497	0.09	28.538	0.13
28.579	0.06	28.619	0.03	28.660	0.23	28.701	0.10	28.741	0.17
28.782	0.21	28.823	0.08	28.863	0.25	28.904	0.01	28.945	0.01
28.985	0.01	29.026	0.01	29.067	0.00	29.107	0.26	28.148	0.45
29.189	0.08	29.229	0.20	29.270	0.14	29.310	0.08	29.351	0.19
29.392	0.25	29.432	0.08	29.473	0.31	29.514	0.33	29.554	0.26
29.595	0.26	29.636	0.17	29.676	0.07	29.717	0.34	29.758	0.11
29.798	0.24	29.839	0.43	29.880	0.20	29.920	0.02	29.961	0.01
30.002	0.01	30.042	0.01	30.083	0.01	30.124	0.42	30.164	0.35
30.205	0.15	30.245	0.05	30.286	0.25	30.327	0.30	30.367	0.19
30.408	0.16	30.449	0.19	30.489	0.15	30.530	0.09	30.571	0.13
30.611	0.13	30.652	0.23	30.693	0.02	30.733	0.08	30.774	0.12
30.815	0.25	30.855	0.30	30.896	0.14	30.937	0.14	30.977	0.15
31.018	0.01	31.059	0.03	31.099	0.10	31.140	0.26	31.180	0.33
31.221	0.24	31.262	0.02	31.302	0.23	31.343	0.32	31.384	0.12
31.424	0.14	31.465	0.17	31.506	0.28	31.546	0.07	31.587	0.08
31.628	0.09	31.668	0.09	31.709	0.15				

HARMONIC CONSTANTS

No.	Symbol	Deg./h	H cm	±	G Deg.	K Deg.	GW Deg.	±
1	*N(MU)S_2	26.4079379	0.34	0.36	34.08	17.44	113.30	*****
2	*$2NS_2$	26.8794590	0.13	0.36	172.27	157.04	252.91	*****
3	*M(MU)S_2	26.9523126	0.15	0.36	277.70	262.69	358.56	*****
4	*OQ_2	27.3416965	0.04	0.35	260.27	246.43	342.30	*****
5	*MNS_2	27.4238337	0.31	0.35	76.93	63.34	159.21	*****
6	*$NMK2S_2$	27.5059710	0.29	0.35	294.67	281.33	17.19	*****
7	$2N_2$	27.8953548	1.70	0.36	156.23	144.05	239.92	11.78
8	MU_2	27.9682084	2.65	0.36	154.69	142.72	238.59	7.58
9	*2(NU)M_2	28.0410620	0.28	0.36	251.19	239.45	335.31	*****
10	*SNK_2	28.3575922	0.20	0.36	9.72	358.93	94.79	*****
11	N_2	28.4397295	5.72	0.36	162.60	152.05	247.92	3.47
12	NU_2	28.5125831	0.53	0.36	152.07	141.74	237.61	40.73
13	*$2KN2S_2$	28.6040041	0.27	0.36	175.00	164.95	260.81	*****
14	OP_2	28.9019670	0.58	0.35	65.82	56.66	152.53	45.07
15	MTS_2	28.9430375	0.57	0.35	88.00	78.96	174.83	36.83
16	M_2	28.9841042	36.76	0.35	93.17	84.25	180.12	0.54
17	MST_2	29.0251709	0.41	0.35	187.41	178.61	274.48	57.10
18	MKS_2	29.0662415	0.68	0.35	191.79	183.13	278.99	39.19
19	*$LAMBDA_2$	29.4556253	0.29	0.36	57.40	49.90	145.76	*****
20	L_2	29.5284789	3.60	0.36	68.63	61.35	157.21	6.70
21	$2SK_2$	29.9178627	0.44	0.35	186.77	180.66	276.52	84.18
22	T_2	29.9589333	0.74	0.35	130.22	124.23	220.10	28.61
23	S_2	30.0000000	23.54	0.35	95.16	89.29	185.16	0.86
24	R_2	30.0410667	0.30	0.35	143.34	137.59	233.46	72.60
25	K_2	30.0821373	7.72	0.35	86.48	80.86	176.72	3.29
26	*MSN_2	30.5443747	0.30	0.35	231.97	227.73	323.60	*****
27	KJ_2	30.6265119	0.60	0.35	283.95	279.97	15.83	48.27
28	$2SM_2$	31.0158958	1.19	0.36	272.15	269.33	5.19	16.79
29	$2MS2N_2$	31.0887494	0.47	0.36	240.72	238.12	333.99	42.68
30	*$2SN_2$	31.5602705	0.23	0.36	40.52	39.34	135.20	*****
31	*SKN_2	31.6424078	0.22	0.36	296.37	295.43	31.30	*****

5 - GENERALIZATION OF THE REFINED METHOD

The refined method was devised to analyse spans of 2^{14} hours (FRANCO, 1988). However we replaced the classical FFT, in the original program by a flexible one so that it became possible to analyse series of any length (usually spans larger than one-year) with main frames; but the program written by this author to be run in PCs kept the classical FFT subroutine to work out analyses of series with samples from 2^8 to 2^{14} hours.

In order to establish the sets of constituents to be separated with the mentioned spans, attention was paid to the spectra of residual amplitudes. Then, analyses of spans from 1024 hours onwards did show residual peaks much larger than those expected, near the frequencies of constituents P_1,ν_2 and K_2.

After introducing the interpolation scheme in the program, analyses from a 1368 hours span onwards, were worked out. Then it became clear from the spectra of residual amplitudes that it was necessary to include new constituents in the analyses of the diurnal and semidiurnal species in order to get rid of the contamination of the secondary constituents of the groups (K_1, P_1), (N_2,ν_2) and (S_2,K_2,T_2) on the main constituents K_1, N_2 and S_2, respectively. Thus, it was concluded that the Rayleigh's rule was not applicable when tapered heights were analysed. In fact, the spectra of residual amplitudes of an analysis of a 1368 hourly heights (57 days) did show peaks on the Fourier frequencies near the ones of P_1 ,ν_2 and K_2. It was really surprising the possibility of separating the pairs (P_1, K_1), (ν_2, N_2) and (S_2, K_2) through the analysis of such a short span. However, it can be mentioned that the method devised by DOODSON (1941) for the British Admiralty, to analyse a 15 days span allowed the separation of N_2 and M_2, with a phase shift of about 180^0 during the period analysed.

Table 5-I shows some results obtained from analyses of data taken at Cananéia. One sees there that the harmonic constants for the above mentioned pairs of constituents, obtained through the analysis of a 57 days span, are very reliable. Let us investigate the reason for that.

The Hanning window considerably reduces the lateral contamination of the other constituents in a group as well as the background noise near these frequencies. Hence each group is contaminated only by the background noise and the measuring error under the spectral window. This is why it is not necessary a 360^0 phase shift of two neighbouring constituents, during the period covered by the series, in order to separate them. In fact, if no noise and measuring error existed, the separation would be possible even for small values of the phase shift. Table 5-II shows the redundant matrix for the semidiurnal constituents for a 1368 hourly tapered heights span. One can see that the largest value in each column is preceeded and followed by only four important contributions of neighbouring constituents. Values beyond these four Fourier harmonics are meaningless.

The reduction of these lateral contaminations by the hanning window allows the analysis of spans of any extension, not tied to the lunations.

Obviously, the accuracy of the analysis increases with the span length; the analysis of a 2^{14} hourly heights span is extremely accurate.

For series up to 8500 hourly heights the refined analyses are faster than the normal ones. Expression (2e) and (2f) shows that matrix A_{nj} holds for both sets of unknowns, $R_j \cos r_j$ and $R_j \sin r_j$. Thus there will be also a single normal matrix and the respective inversion for each species. For spans equal to or larger than 8500 hourly heights, the system covering each species can be split into several small ones, with the result that the analyses of these spans are much faster than those which work out lesser ones.

Table 5-I - Comparing some results of refined analysis

	18.69 years (1972-1989)				682.67 days			
	cm	cm ±	deg.	deg. ±	cm	cm ±	deg.	deg. ±
P_1	2.36	0.03	149.31	0.68	2.70	0.17	152.39	3.63
K_1	6.36	0.03	145.65	0.25	6.31	0.17	144.43	1.69
N_2	5.78	0.04	163.30	0.35	5.87	0.14	161.53	1.36
ν_2	0.77	0.04	168.93	0.27	0.57	0.14	162.45	14.28
S_2	23.68	0.06	95.96	0.15	23.69	0.22	95.25	0.54
K_2	7.47	0.06	86.73	0.48	8.07	0.22	66.63	1.88
	369 days				57 days			
P_1	2.67	0.36	153.63	7.77	2.59	1.62	171.52	38.15
K_1	6.37	0.45	145.56	4.39	5.99	1.59	141.74	17.09
N_2	5.68	0.25	163.55	2.40	6.75	1.68	158.34	14.01
ν_2	0.69	0.24	156.95	19.39	*0.67	1.68	215.78	*****
S_2	23.71	0.31	96.19	1.41	23.30	1.51	94.21	3.52
K_2	7.51	0.23	87.19	3.60	7.83	1.52	91.74	14.39

Table 5-II - Value of 1000xAnj for the semi diurnal species

Rows correspond to Fourier frequencies rows from 25.789 to 32.105 o/h
$\Delta f = 0.263$

$N\mu S_2$	$2NS_2$	MNS_2	μ_2	N_2	γ_2	M_2	L_2	S_2	K_2	MSN_2	$2SM_2$	$2SN_2$
-27	-1	0	0	0	0	0	0	0	0	0	0	0
-271	-4	0	0	0	0	0	0	0	0	0	0	0
933	-16	-3	-1	0	0	0	0	0	0	0	0	0
-739	-413	-6	-2	0	-1	0	0	0	0	0	0	0
90	991	-21	-3	0	-1	0	0	0	0	0	0	0
17	-588	-365	-7	0	-2	0	0	0	0	0	0	0
24	978	-25	-1	-4	0	0	0	0	0	0	0	0
3	5	-639	-318	-2	-3	-1	0	0	0	0	0	0
2	2	42	958	-7	-26	-2	-1	0	0	0	0	0
1	1	9	-688	-476	-273	-4	-1	0	-1	0	0	0
1	1	3	64	999	935	-15	-2	0	-1	0	0	0
0	0	2	13	-534	-736	-6	9	-2	0	0	0	0
0	0	1	5	8	88	992	-21	0	-3	0	0	0
0	0	1	2	2	16	-538	-368	1	-8	0	1	0
0	0	0	1	1	5	23	979	5	-26	-1	1	0
0	0	0	1	0	3	0	-636	-521	-297	-2	1	0
0	0	0	0	0	2	2	41	1000	949	-6	3	0
0	0	0	0	0	1	1	9	-479	-710	-470	8	1
0	0	0	0	0	1	1	3	-4	74	999	37	2
0	0	0	0	0	0	0	0	-1	14	-530	-626	5
0	0	0	0	0	0	0	1	0	5	7	982	20
0	0	0	0	0	0	0	1	0	2	2	-377	-575
0	0	0	0	0	0	0	0	0	1	1	-20	994
0	0	0	0	0	0	0	0	0	1	0	-6	-426
0	0	0	0	0	0	0	0	0	0	0	-2	-14

6 - FINAL COMMENTS

Excellent observations of tidal heights at "São Pedro and São Paulo" Rocks (Lat. 00^0 56′ S, Long. 29^0 17′ W), Brazil, with a pressure gauge, during a little more than 29 days, with a sampling interval of 0.25 hour were also analysed, for observed and interpolated data, with the normal method. The results were identical since the interpolation through smaller sampling interval is more effective.

In the interpolation technique here presented, the successive differences of the various orders are convenient approximations to the derivatives,thus it is understandable that good results are reached through the analysis of interpolated data.

As it was said before the Doodson's method of tidal analysis for a 15 days span, then used by the Hydrographic Department of the British Admiralty (1941) allowed the separation of M_2 and N_2, with a phase shift of about 180°. Thus it is not surprising that constituents with a phase shift less than 180° are separated through the refined method of analysis.

This author has prepared a set of eight programs, to be used in PCs,to check tidal observations, predict and analyse tides and tidal currents and to solve related problems. These programs are a complement to his book on tides (FRANCO, 1988), thus an English version of that software is also available (FRANCO, 1989).

Spectral analyses of interpolated random time series is also well succeeded (J. R. MARTINS, private communication). Hence, it is possible that the same improvement can be introduced in the harmonic analysis of tidal currents. However, the background noise in the current observation is usually so high that the data filtering of such spurious interference must preceed the interpolation procedure.

The above mentioned programs run in any IBM PC and the processing speed depends on the kind of used machine. If, for example, an IBM 486 is available, the normal analysis of a 8856 hours span is worked out in about 10 seconds.

It is in fact, very remarkable that so good results can be reached by using such a simple technique.

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