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HYDROGRAPHIC DATA BASES FOR TIDAL NUMERICAL MODELS

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ABSTRACT

Bathymetric data for tidal numerical models may be organised either as a list of soundings located in space, or as a table of average depths in rectangular areas which correspond to the cells used in the model. The techniques of averaging used in the second method are described, and its accuracy is examined. Details are given of its operation in the case of the author's tidal numerical models.

1. INTRODUCTION

Since the late 1960s, it has been possible to simulate the propagation of tides in shelf seas using numerical models. At first, these models were two-dimensional, and contained only $\sim 10^3$ points (HEAPS, 1969). Numerical modelling has now developed to such an extent that a specific module entitled "Modelling of the Marine Environment" has been introduced into the Marine Studies modular degree programme offered at the Institute of Marine Studies.

Tidal numerical models require a detailed specification of the shape of the sea-bed. The highly irregular hydraulic geometry is schematized by dividing the area of the model into small compartments or cells. In finite-difference models, these cells often approximate squares, whereas in finite-element models (WERNER and LYNCH, 1987), the cells are triangular.

The axes of certain finite-difference models (PINCREE & MADDOCK, 1977) are inclined to the meridional and zonal directions. This is done in order to save storage space by maximizing the number of sea-points to total points. It is more common, however, to use models whose axes are N.-S. and E.-W., so that the edges of the cells are lines of latitude \emptyset and Longitude L. In order that the cells also approximate squares, a convenient ratio between the meridional and zonal scales is chosen. For

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models of the N.W. European shelf seas, a convenient ratio is such that the compartments have sides of length 1 minute of latitude (i.e. nautical miles, abbreviated to NM) and 1.5 minutes of longitude. This ratio arises because in the latitudes concerned, $\cos a \approx 2/3$ (the latitude for which this is exact is 48°11').

The depths and drying heights on the charts are expressed relative to chart datum, whereas the tidal elevations in models are computed relative to mean sea level. The difference between the two datums, termed Z_{σ} is specified at all points in the model. It is poorly known offshore, but does not need to be known accurately, since the depths themselves are specified only to the nearest metre.

2. METHODS FOR EXTRACTING BATHYMETRIC DATA

There are currently two different approaches to extracting and storing bathymetric data for use in numerical models.

- (a) The older method is to draw the model grid on navigational charts, and to extract the average depth for each cell, by hand; this will be termed the gridding method. The techniques and accuracy of averaging are described below in Section 3, and the format for storing the resulting data in Section 4.
- (b) The newer method, termed here the **sounding method**, is conceptually much simpler. A data base consisting of thousands of soundings (in the form of latitude, longitude and depth) is compiled from primary sources (hydrographic surveys) or secondary sources (charts); and elaborate software is used to interpolate depths for a given model.

Both methods contain the same amount of intrinsic difficulty; in the gridding method it is concentrated in the averaging of the depths, and in the sounding method it is concentrated in the interpolation.

Because the averaging is done by hand, the gridding method can take isobaths into account, whereas this is not usually done in the sounding method. The gridding method also has a direct hydrographic relationship to the chart, because the cells in the model may be drawn on the chart.

The great advantage of the sounding method is that it can deal with any schematization (finite differences or finite elements) from the same data-base; whereas the gridding method requires separate averaging for different scale models of the same sea-area, and is not designed for finite-element modelling.

It is evidently valuable to have access to unpublished bathymetric data directly from hydrographic offices, such as sounding charts (for the gridding method), or data bases of soundings (for the sounding method).

3. AVERAGING IN THE GRIDDING METHOD

3.1 Techniques

Bathymetric charts are gridded with the "square" cells used in the model, and the average depth in each is recorded. The determination of the average depth is an acquired skill. Both spot-soundings and isobaths may be used. The following examples illustrate some of the techniques in common use; they are not mutually exclusive, and may be combined in a suitable cell to give a more accurate estimate.

(a) Weighted Averaging of Soundings

The ideal case would be a sea-bed of gently varying depth, having just one sounding in the centre of each cell. This ideal is rarely approached, and weighted averaging has to be used.

- (i) If the soundings are evenly spaced, unit weights may be used (Fig. 1).
- (ii) With fairly sparse soundings, weights of 2 for soundings within the cell, and 1 for soundings on its boundary have been found to be effective (Fig. 2).
- (iii) Sometimes the cell may be divided into two (or more) distinct regions with different densities of soundings; the depth in each region determined by direct averaging, and the average for each region weighted by the area of the region (Fig. 3).
- (b) Use of Isobaths
 - (i) Near the coast, it often happens that a cell is traversed by a set of nearly parallel isobaths representing a uniform slope (Fig. 4); in such a case, one can estimate which depth contour would bisect the cell, and take this as the mean depth.
 - (ii) Offshore in areas where the sea-bed is relatively flat, one sometimes finds a sinuous isobath wandering all over the cell (Fig. 5); the mean depth may be taken as that of the isobath.
- (c) Interpolation Between Cells

It sometimes happens, especially with small cells, that the density of soundings is significantly less than one per cell. In this case, numerous cells have no sounding in them, and interpolations have to be made from nearby cells which do have soundings.









= 24m

FIG. 3.- Two areas with different densities of soundings.



h -≏ 12 m







FIG. 6.- Large-scale and small-scale depiction of an isolated shoal.

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3.2 Avoidance of bias

It has to be borne in mind that whereas oceanographic charts depict the bathymetry without bias, the much more commonly used navigational charts deliberately emphasize shoals, since these may constitute potential hazards to navigation. Consider the case of a small pinnacle rock, whose summit is 8.3 metres below chart datum, rising abruptly above an otherwise flat sea floor of depth 50 metres; this will be represented by 8_3 surrounded by a small circle around it (Figs. 6a and 6b). If the rock is of small areal extent, the circle depicting it will represent an area larger than its real size, and the smaller the scale of the chart, the greater will this misrepresentation be.

Modellers have been known arbitrarily to add 2 metres to the extracted depths in order to allow for this bias. I prefer to try to allow for it in the actual averaging process.

3.3 Density of soundings and limit of resolution

The gridding method becomes difficult when the surroundings are so sparse that frequent interpolation has to be made between depths in cells (technique (c) above). This happens if the density falls lower than one sounding per cell. Common sense also suggests that one is chasing a non-existent accuracy if the number of cells exceeds the number of soundings.

Typical densities of soundings on the large-scale coastal (metric) charts produced by the British Hydrographic Department vary from about 100 per dm² in offshore areas of relatively smooth bathymetry. It follows that it is impractical to grid a chart so as to have more than about 25 cells per dm²; i.e. the cells should be at least about 20 mm in side.

Thus the practical limit of spatial resolution of the numerical models is dictated to some extent by the scale of the chart coverage of the area concerned. This gives the following resolutions for the commonest scales:

Scale of chart	20 mm represents	Mesh-size used in practice
1:200,000	4000 m	4.0 NM = 7408 m
1:150,000	3000 m	2.0 NM = 3704 m
1: 75,000	1500 m	0.8 NM ≈ 1482 m

Thus for the English Channel, which is chartered at 1:150,000, the effective limit of spatial resolution is \sim 2 NM, and this is the mesh-size of the model used by the author.

There are, however, other considerations which influence the choice of mesh-size. Until about ten years ago, it was rare to find models of shelf seas with a mesh-size of less than 5 NM, since computers were not large or fast enough.

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Nowadays, with larger and faster computers, the choice of mesh-size may be dictated by the purpose of the model. For modelling tidal residual flows, it is essential to have a mesh-size which is less than 1/8 of the tidal excursion (ABRAHAM *et al.*, 1987). Thus SALOMON and BRETON's (1991) model of the English Channel has a mesh-size of only 1 NM, and their model of the north coast of Brittany (GERREAU, 1992) a mesh-size of only 500 m. Yet it is important to realize that the implied accuracy of the bathymetry in offshore areas of these models is unreal.

An analogous case is that of models of oceanic circulation, which sometimes have mesh-sizes as small as 1 km (GREATBACH, 1992). In such models, the mesh-size needs to be smaller than the Rossby radius of deformation. The sounding method is used in such models to extract the depths at this resolution, but the accuracy of depicting the bathymetry is not thereby increased.

3.4 Accuracy of averaging in the gridding method

The twelve students in this year's modelling class were given a copy of a small portion of Admiralty Chart No. 2668, scale 1:150,000, which had been gridded into "squares" of side 2 NM. As an exercise, they were asked to determine the average depth in each cell, using the chartlet. The density of soundings in the cells varies from zero to over 250 per dm². (The upper limit is much greater than those quoted above, because this chart is old-fashioned, having soundings in fathoms and feet).

An accurate assessment of the mean depths had already been determined by gridding larger-scale 1:50,000 charts with "squares" of side 0.8 NM, and finding the mean depth in each of the small "squares"; the average depth in each 2 NM cell was then found by averaging the depths in the 25 constituent cells. These values, termed the "true" depths, were then used to check the students' estimates of the mean depths in each 2 NM cell.

The results are a measure of:

- (a) the ability of the students accurately to estimate mean depths using the techniques described above;
- (b) the suitability and accuracy of the small-scale chart as a source of information about depths for numerical modelling.

The mean and the standard deviation of the students' estimates of average depth in the 88 cells were computed. It was found that:

- (a) There was a weak correlation between the standard deviation and the density of soundings; i.e. the fewer the soundings, the greater the differences between the students' estimates.
- (b) The overall average of the students' estimates was 0.5 m shallower than the average of the "true" depths.
- (c) In 14 of the 88 cells, the following relation was satisfied:

"true" depth < mean depth - 2 x standard deviation

In these cells, therefore, there was a real and significant difference between the depth estimated from the small-scale chart and that averaged from the large-scale chart.

(b) and (c) are consistent with the argument that extracting depths from small-scale charts produces values which are shallower than they ought to be.

4. ORGANIZATION OF DATA COMPILED USING THE GRIDDING METHOD

The system used by the author is now described. Each hydrographic data base has two files, one for depths and one for masks; both files have similar formats; both are divided into blocks for ease of management. An example of each type of file is given in Figures 7a and 7b.

4.1 Blocks

It is convenient to store the data in blocks of 300 cells, 15 in latitude by 20 in longitude. If a field of 4 characters is allowed for each cell, then the block fits neatly on to a screen of a monitor, 80 columns wide and 15 rows deep. A header line is also included for each block.

When a finer-scale model is nested within a coarser-scale model, it is computationally convenient if the scales of the two are simply related, and a factor of five is often used. Most of the mesh-sizes currently in use by the author fall into two series:

		Nominal mesh size	Block size									
Series 1	Series U											
	coarse	20.0 NM	20	30	5°	lat.	x	10°	long.			
	medium	4.0 NM	4	6	1°	lat.	х	2°	long.			
	fine	0.8 NM	0.8	1.2	12′	lat.	x	24′	long.			
Series I	C											
	coarse	10.0 NM	10	15	21⁄2°	lat.	x	5°	long.			
	medium	2.0 NM	2	3	1⁄2°	lat.	х	1°	long.			
	fine	0.4 NM	0.4	0.6	6′	lat.	x	12′	long.			

Hyperfine models with mesh-sizes of 0.2 NM have also been constructed (see Fig. 10).

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	90	136	137	184	218	232	224	196	139	22	<u>5</u> •	111			LLL

EXAMPLES OF A BLOCK FROM A MASKING FILE

•	LLL	LLL	LLL	LLL	irr	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL
•	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL	LLL
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FIG. 7.- Example of a block: a) soundings; b) masks.

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The format of the header lines is as follows:

Columns	Data
1 to 6	An alphanumeric code identifying the block
7 to 10	Mesh size Δy in nautical miles
11 to 20	Latitude of N.W. point in ° ′ (F4.0, F6.2)
21 to 30	Longitude of N.W. point in ° ′ (F4.0, F6.2)
31 to 70	Name or description of the block
72	= no data available for the block
	L block is entirely land
	block is entirely sea (masking files only)

Notes: 1) Long. E. is +ve, long. W. is -ve.

- 2) Long. 0°30'W is coded as: 0-30.00
- Long. 1°30'W is coded as: -1 30.00
- Names of 1° x 2° blocks are taken in part from maps produced by the Geological Survey.

4.2 Format of fields depicting depths

Each of the 15 rows consists of 20 fields of 4 characters (20A4), each field representing the average depth in the cell below chart datum, as follows:

(a) If the depth exceeds 1000 units, the 4 characters give the depth directly.

(b) If the depth is less than 1000 units, the characters are:

1st character:	* if the cell is partly sea and partly land (as defined by the H.W. coastline on charts), but the
	amount of sea is < ½ of the cell area <i>blank</i> otherwise

2nd to 4th character: the depth

The units used are metres or decimetres.

If the sea-bed is above chart datum (drying banks), then the depth is coded as -ve.

(c) If the cell is all land, the coding LLL is used.

If the block is all land, then only the header line is included, with 'L' in column 72.

4.3 Format of fields depicting masks

The models operate using an Arakawa-C grid, in which tidal elevations, easterly flows and northerly flows are computed at different locations within the cell (Fig. 8). The purpose of the masking file is to define precisely the model coastline within such a grid. Other authors, (OZER, in WERNER & LYNCH (1988) have used numerical codes to define the large number (up to 30) of different configurations which the coastline may take locally. The system described here includes an element of redundancy, but is simple to operate.





Each of the 15 rows consists of 20 fields of 4 characters (20A4), as follows:

1st character:	blank
2nd character:	applies to the E-point in the cell
	in the sea OR L on the land
3rd character:	applies to the U-point in the cell
	flow possible OR L no flow possible
4th character: a	applies to the V-point in the cell
	flow possible OR L no flow possible

4.4 Inventory of hydrographic data bases

Four major hydrographic data bases have been compiled using the above format:

Locality	Code	Mesh-size	No. of blocks		
N.W. European shelf seas	CS	4.0 NM	138		
English Channel & Flemish Bight	MF	2.0 NM	69		
Cornish Coast	AK	0.8 NM	~ 64		
Normano-Breton Gulf	NB	0.4 NM	~ 90		

and several specific minor ones for individual models elsewhere. A list of selected models constructed under the author's direction is given in Fig. 10.

4.5 Recovery of data for use in models

There is no need for areal coverage of the hydrographic data base to be the same as that of the model, but it has to be at least as large as that of the model. It is common for several models to be constructed using the same hydrographic data base. Software written by the author enables depths and masks to be extracted from the data base for a given model, on specifying the geographical limits of the model; the model may then be plotted in the formats of Fig. 9.

5. CONCLUSION

The necessity to construct two-dimensional tidal numerical models on a regular basis, as part of a course in oceanographic modelling, has led to a rationalization and streamlining of the tedious process of schematizing the bathymetry. Although the gridding method which is employed may be less versatile than the newer sounding method, the relationship of the gridded chart to the models is more immediate, and students appreciate the problems of schematization through having to average depths themselves. The results of one such exercise confirmed that average depths taken from small-scale navigational charts are less than those taken from large-scale charts, indicating that the largest scale charts available should be used.

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FIG. 9.- Plot of a small model using the author's software.

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AREA	MESH	NO. OF	NO.	OF NO.	DATA-
	SIZE	POINTS		OF	BASE
	(NM)		5	TEPS	CODE
		•		CALCS.	
			per	tidal cycle	
N.W. European shelf seas	20.0	1764	120	0.21×10^{6}	specific
Celtic Sca and English Channel	10.0	1484	360	0.53 x 10 ⁶	specific
Celtic Sea and English Channel	4.0	8064	450	3.63 x 10 ⁶	CS
N.W. European shelf seas	4.0	52500	450	23.63×10^6	CS
Irish Sea	4.0	1800	600	1.08×10^{6}	CS
Central English Channel	2.0	2774	600	1.66 x 10 ⁶	MF
Cornish coast	2.0	2279	600	1.37 x 10 ⁶	MF
English Channel	2.0	10000	720	7.20×10^{6}	MF
Isle of Wight	2.0	529	600	0.32 x 10 ⁶	MF
Baie du Mont St. Michel	2.0	150	360	0.06 x 10 ⁶	MF
English Channel & Flemish Bight	2.0	27750	720	19.98 x 10°	MF
Eastern Irish Sea	2.0	1600	720	1.15 x 10°	specific
West Cornwall	0.8	3000	1800	5.40 x 10 ⁶	AK
Scilly to West Cornwall	0.8	6300	1800	11.34 x 10 ⁶	AK
Lizard Point to Start Point	0.8	6300	1800	11.34 x 10 ⁶	AK
Cornish coast	0.8	14700	1800	26.46 x 10 ⁶	AK
Isle of Wight	0.8	3500	1800	6.30 x 10 ⁶	specific
Bristol Channel	0.8	8550	1800	15.39 x 10 ⁶	specific
Alderney	0.4	2774	3600	9.99 x 10 ⁶	NB
Guernsey	0.4	2924	3600	10.53 x 10 ⁶	NB
Jersey	0.4	3344	3600	12.04 x 10 ⁶	NB
Baie du Mont St. Michel	0.4	1925	1800	3.47 x 10 ⁶	NB
The Wash	0.4	3025	1800	5.45 x 10 ⁶	specific
Waddenzee	0.4	3850	720	2.77 x 10 ⁶	specific
Morecambe Bay	0.4	2000	1800	3.60×10^6	specific
Western Lyme Bay	0.4	1050	1800	1.89 x 10 ⁶	specific
St. Ives Bay	0.2	900	3600	3.24 x 10 ⁶	specific
Plymouth Sound & Tamar estuary	0.2	3000	3600	7.20 x 10 ⁶	specific
St. Austell Bay	0.2	500	3600	1.80 x 10 ⁶	specific
Isles of Scilly	0.2	2700	6000	16.20 x 10 ⁶	specific

LIST OF SELECTED MODELS CONSTRUCTED BY THE AUTHOR

An idea of the time taken to run these models may be gained by noting that the effective speed of the PRIME computer system used at the University of Plymouth is 2000 calculations per second.

FIG. 10.- List of models constructed by the author.

References

- ABRAHAM, G., GERRITSEN, H. and LINDIJER, G.J.H., 1987: "Subgrid tidally induced residual circulations" Cont. Shelf. Res., 7, pp. 285-305.
- GARREAU, P. 1992: Modèle mathématique courantologique du littoral de Bretagne Nord. Paper presented to a Symposium on the English Channel, Brest, September 1992.
- GREATBATCH, R. 1992: A model of Conception Bay in Newfoundland. Paper presented to the JONSMOD '92 conference, Copenhagen, August 1992.
- HEAPS, N.S. 1969: "A two-dimensional numerical sea model." Phil. Trans. R. Soc. Lond. A265, pp. 93-137.
- PINGREE, R.D. and MADDOCK, L., 1977: "The M₄ tide in the English Channel derived from a nonlinear numerical model of the M₂ tide". *Deep Sea Res.* 25, pp. 53-63.
- SALOMON, J.C. and BRETON, M., 1991: Modélisation mathématique du transport et de la dispersion des rejets dissous sur les côtes de la Manche. Paper presented to the OSATES conference, Brest, September 1991.
- WERNER, F.E. and LYNCH, D.R., 1988: "Field verification of wave equation tidal dynamics in the English Channel and southern North Sea." *Advances in Water Resources*, 10, pp. 115-130.
- WERNER, F.E. and LYNCH, D.R. 1988: Tides in the southern North Sea/English Channel: data files and procedure for reference computations. Thayer School of Engineering, Dartmouth College, Hanover, New Hampshire, USA.