CONFIRMATION OF THE TRENDS SUSPECTED TO BE PRESENT IN THE TIDE OF THE BAY OF FUNDY

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Abstract

An additional set of data on the water level at Saint John, New Brunswick, covering the years 1932 to 1946, was exploited to reassess more closely the existence of rapid changes suspected to exist in the tide of the Bay of Fundy. The possibility of trends in the M_2 tide had been suggested by DOODSON as early as in 1924 from a scrutiny of earlier data. Enough samples are now available to state unequivocally that the M_2 tide is increasing at an accelerating rate and that S_2 is diminishing. The characteristics of the trends are interpreted in terms of nonlinear frictional interactions.

AVAILABILITY OF ADDITIONAL DIGITIZED DATA FOR THE HARBOUR OF SAINT JOHN

It is not possible to use recent data for studies on the tide at Saint John, because of the poor maintenance of the instrument over the last decades. On the other hand, records of adequate quality are available over the interval 1894 to 1979. This presents us with an alternative which is to seek historical files. Consequently, the Marine Environmental Data Service in Ottawa, undertook to digitize a set of records for this harbour, that remained in tabulated form, covering the years 1932 to 1946. Gaps existed for the years 1935-38 and 1940; by chance it turns out that the missing years do not correspond with crucial intervals for the evolution of the local tide. We therefore have at our disposal some 50 years of nearly continuous recordings for Saint John, a fully adequate supply, to double check on the trends which had been first noted from the data covering 1947-1980 (GODIN and GUTIERREZ, 1986; GODIN, 1988; GODIN, 1992a,b). The scrutiny was helped as well by a reevaluation of DOODSON's findings (DOODSON, 1924).

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Changes in the local tide

The rapid rise in sea level since the last glaciation implies a radical change in the bathymetry of the coastal seas. Since tides are the dynamic response of the oceans to the applied tidal forces, they must have evolved considerably over the last 10,000 years. The time scale of scientific activity being much shorter, one can hope to get a glimpse at these changes by seeking an area of large tides where trends on a geological time scale could be detectable over a relatively short time span. Here in Canada, we are fortunate enough to have the Bay of Fundy in our midst as well as dedicated scientists, such as W.B. DAWSON, who, in the 1890's, set up a gauge at Saint John. As soon as records became available, he undertook their scrutiny and became increasingly disturbed by their irregularity. DOODSON (1924) provided most valuable insights into the problem, then proceeded to elaborate a formula that could reproduce the local modulations at Saint John. DAWSON, on the other hand, would have preferred to do away completely with such correction formulas. In perspective he was right: DOODSON's formula for the M₂ tide at Saint John, turns out to be wrong for later years.

From a practical as well as scientific point of view, the tide reaching Saint John is therefore of interest. The diurnal part of the signal is small and is perfectly regular; all the problems lie in the semidiurnals, whose major members are M_2 , N_2 and S₂. M_2 has a period of $\frac{1}{2}$ a lunar day and represents the average contribution of the moon to the tidal forces; S, is its equivalent for the sun. We shall discuss the physical origin of N₂ later on, in the paragraph dedicated to its scrutiny. M₂ is by far the largest (\approx 303cm), but the preponderance of N₂ (\approx 62cm) over S₂ (\approx 49.5cm) makes N_2 the component of central interest. It should normally be less than half the size of S_2 (41.2% more precisely, according to the force components they stand for), while at Saint John, it exceeds S₂ by 25%, making it three times larger than it ought to be (\approx 20cm). Consequently, the largest tides in the Bay of Fundy occur when the moon is at perigee (nearest to the earth), and not when the moon, the earth and the sun are in line (syzygy \leftrightarrow spring tides), as it is the case for the rest of the Atlantic. This feature was noted by the members of the expedition sent to Burntcoat Head by the British Admiralty in the 1860's, to observe the phenomenal local tides; the correspondence between perigee and large tides, is also known as a matter of course by the local population. This indicates that the resonant period of the Bay of Fundy must lie close to the N₂ periodicity (GODIN 1988, 1993).

The trends in the amplitude of M_2

DOODSON (1924) had concluded that M_2 was diminishing from the records he had at his disposal. The accumulation of additional data gives us a better perspective: it emerges that the exact opposite is true (GODIN 1992a,b). The difficulty in reaching any definite conclusion about the behaviour of M_2 is that it is most irregular, besides being affected by modulations which do not obey the usual rules. Recourse was made first (GODIN, 1992) to running means in order to eliminate the unwelcomed modulations but this reduced drastically the number of samples available. One needs at least two or three nineteen-year cycles before discerning the trends through the oscillations, without any preliminary processing of the material. The data recently compiled by the Marine Environmental Data Service allows to do just this. We show in Fig. 1 annual samples of the amplitude of M_2 (without nodal corrections naturally) over the years of digitized records now available. The maximum and minimum samples were fitted by trends. The upward increase in the amplitude of M_2 at Saint John, is now unmistakable. Note that the rate of increase of the smallest amplitudes differs markedly from that of the peak amplitudes. We will return to this feature later; all we wish to say for the moment is that it is caused by nonlinear effect.



FIG. 1.- Annual samples of the amplitude of M_2 over the interval 1932-1980. Trends in the sample M_2 amplitudes: overall: 13.7 cm/century, peak amplitudes: 10.9 cm/century, least amplitudes: 14.7 cm/century.

If we calculate the trends in the samples, we end up with the following results:

Overall rate of increase of M₂: 13.7 cm/century Rate of increase of the peak values: 10.9 cm/century Rate of increase of minimum values: 14.7 cm/century

Table 1 in GODIN (1992a) contains a trend of 14.5 cm/century from the running means.

Strictly speaking, we do not need more samples to confirm that M_2 is indeed increasing in the Bay of Fundy, but since the data from 1894 to 1917 is available from DOODSON (1924), there is no harm in tacking it on to verify what we get. Figure 2 shows all the sample amplitudes now available to us, along with the trends for the peak and least values; we are left with a blank of some 15 years. The trends obtained are:





FIG. 2.- Available sample amplitudes of M₂ since 1894.

Table 1 in (GODIN, 1992a) gives 10.4 cm/century from the average of DOODSON's data and that of the samples for 1947-1980, slightly less. We had suggested, from an examination of the running means, that the increase seems to be accelerating: the new data confirms that this should indeed be the case.

Nodal modulations of M₂

Such modulations are created by the presence of unresolvable components in its vicinity; the more important of these has a relative amplitude of 3.7% and differs in frequency by 1 cycle/19 years. In standard tidal practice, the modulation is removed by correcting the annual sample by its assumed contribution to the amplitude and phase of their resultant, denoted by f and u. The procedure is successful in general, but not at Saint John (GODIN, 1986, GODIN and GUTIERREZ 1986).

Since the time series at this site is non stationary, it has no mean nor standard deviation, from a statistical point of view. Yet we may try to assess the modulation at M_2 by calculating its fluctuations in amplitude between two successive extrema in relation to its mean value over the same interval. It turns out that the observed extreme do not necessarily coincide with the year of peak disturbance from the parasitic component. For the digitized records, we encounter the following situation:

Maximum amplitude Minimum amplitude 310.6 cm in 1941 293.2 cm in 1949 Maximum f in 1941 Minimum f in 1950 Mean amplitude over the interval 302.9 cm Effective modulation ± 2.9% 312.3 cm in 1961 Maximum f in 1959-60 Mean amplitude over the interval 302.1 cm Effective modulation ±3.2% 296.3 cm in 1970 Minimum f in 1969 Mean amplitude over the interval 302.8 cm Effective modulation ±2.6% 313.2 cm in 1977 Maximum f in 1978 Mean amplitude over the interval 303.6 cm Effective modulation ±2.8%

If we return to the data revised by DOODSON around the turn of the century, we have:

305.1 cm in 1903294.6 cm in 1894Maximum f in 1904Minimum f in 1894Mean amplitude over the interval 299.4 cmEffective modulation $\pm 1.8\%$ Minimum f in 1913288.2 cm in 1913Mean amplitude over the interval 296.3Effective modulation $\pm 2.9\%$

We see that the estimates of the actual modulations of M_2 at Saint John are quite erratic, since they must be evaluated from a mean which may not be accurate, the intervals varying in duration, this affecting its value. The only safe thing we can infer from the numbers we just obtained is that the modulations of M_2 at Saint John, are smaller than those assumed from the equilibrium hypothesis (±3.7%), this being due to the effects of quadratic friction. On the other hand, we do not know what they are; from the evidence presented, they must vary continuously. KU, GREENBERG, GARRETT and DOBSON, in a paper sponsored by the Bedford Institute of Oceanography and published by SCIENCE in 1985, state in their Table 1 that the records for Saint John demonstrate effective modulations for M_2 of ±2.30% for the interval 1947-1971 and ±2.40% for the interval 1894-1916. We illustrate in Figures 3 and 4, the fit of the M_2 amplitudes by the sinusoid A[1+R'cos(Δ t- δ')] implied in:

 $A[1+R'\cos(\Delta t-\delta')]\cos(\omega t-\Theta+u').$

The formula, on p. 69 of the paper, is meant to represent the profile in the annual samples in amplitude and phase lag of the component. R' stands for the effective f of the local modulation. We draw as well the curves for $R'=\pm 2\%$ and $\pm 3\%$, that cover a much wider range, to underline the difficulty with the fit. The authors compute a value of 13.3 hours for the resonant period of the Bay of Fundy, introducing an assumed range of $\pm 2.40\%$ in a formula derived from the theory of damped resonators (GODIN, 1993). As said previously the native population around its shores is quite aware of the importance of the moon's perigean position to the local tides; this, translated in terms of resonant period, locates this important parameter around 12.6 hours, the period of N₂, not 13.3 hours. A most elaborate and



FIG. 3.- Fit of the amplitudes of M₂ over the interval 1947-1971 by sinusoids.



FIG. 4.- Fit of the amplitudes of M₂ over the interval 1894-1917 by sinusoids.

complete numerical model of the Bay of Fundy and Gulf of Maine (GREENBERG, 1979) generates a resonant period around 12.5 hours, remarkably close to the one suggested by the tidal observations.

Disparities in the trends for large and small M₂ amplitudes

We return to this feature which is evident in Figures 1 and 2. The correction factor f for the amplitude, mentioned initially, reflects the increase or decrease in the actual intensity of the M_2 component over a given year. Since the M_2 tide recorded at Saint John does not correlate well with the amplitude correction factor f, we may use it to calculate:

$$\frac{Amplitude of M_2}{f}$$

We show in Figure 5, samples of this parameter for the digitized data: dots mark the ratios when f<1, crosses when f>1. In spite of the ever present irregularities, the parameter is predominantly large for small amplitudes, and small for large amplitudes. This is fully consistent with the effects of nonlinear friction. A large M_2 implies stronger currents, therefore more effective friction while the reverse holds for a small M_2 . Under the present conditions, if the tide keeps on growing larger, the discrepancy between large and small M_2 's will keep on diminishing (\leftrightarrow more friction in the Bay of Fundy \Rightarrow an even smaller range in the nodal modulations).



FIG. 5.- Amplitude of M_2 divided by the correction f for the year. The points indicate years for which f<1; the crosses mark years for which f>1.

The S₂ component

In general, S_2 is the least troublesome of all components. It requires no nodal corrections and is constant, year after year, for most stations; not so for Saint John. Figure 6 shows the annual samples available between 1932 and 1980. The plot exhibits two characteristics: quite untypical fluctuations plus a net downward trend. The downward trend had already been detected in the running means, now the unprocessed data show it too. The running means in Table 1 of (GODIN, 1992a), gave a trend of -4.6 cm/century; the unprocessed samples yield -4.7 cm/century, a fully consistent estimate. The running means do remove the scatter around the trend; allowing it to remain produces essentially the same result.



FIG. 6.- Trend present in the sample amplitudes of S₂.

GODIN and GUTIERREZ (1986) indicated that the fluctuations in S_2 are induced by its nonlinear interaction with M_2 : a large M_2 induces a small S_2 and vice versa. When quadratic friction is applied to a pair of harmonic components, the larger component will be less damped than the minor one. So if M_2 is larger, S_2 is damped more effectively. We illustrate the negative correlation in Figure 7 in which we superpose plots of S_2 and M_2 . Five hundred centimeters were subtracted from the M_2 amplitude and the result made positive in order to direct its changes in the same direction as those of S_2 . The latter has additional shorter term cycles which as yet are unexplainable.

The running means did indicate a slow increase in the phase lag of S_2 ; in Figure 8, we show the trend passing through the actual samples. The runnings means gave +2.1°/century while the samples suggest +3.5°/century. Because of the very considerable scatter in the sample phases, all we can conclude is that S_2 is reaching the site later by some 4 to 7 minutes per century: hardly noticeable.

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FIG. 7.- Negative correlation between S_2 and M_2 .





The N₂ component

 N_2 lies very close in frequency to M_2 . Physically it reflects the fluctuations in the moon's distance during the course of its orbit around the earth. The tidal force exerted by the moon amounts to M₂ on the average, but it is modulated by its trajectory between perigee (nearest point) and apogee (most distant point). The average time necessary for a complete orbit is 27.55 days, so the modulation in the moon's tidal force, is expressed by the superposition of M_2 of an additional harmonic component, very close in phase, which modulates it between its maximum and minimum values, over the duration of an anomalistic month, namely N₂. When they are in phase, they add their effects, generating M_2+N_2 , reflecting the moon's force at perigee; when their relative phase is such that M_2 - N_2 results, the moon is in apogee. The contribution of each component can be expressed as a linear displacement, called "equilibrium value", which would represent the distortion of the earth's surface if it responded directly to this force. That of M_2 is 63.2 cm while N_2 corresponds to 12.1 cm; this means that the effective equilibrium displacement during the course of a month varies between 51.1 cm at apogee and 75.3 cm at perigee. The actual ratio between N_2 and M_2 in the tide at Saint John is 0.2045, above the 0.1915 supplied by the tidal forces associated with them. We recall that the ratio of S_2 and N_2 as forces, is 2.430 while their responses at the site stand as 0.798, exceeding it by a factor of 3. Therefore the pair M_2 and N_2 is unusually favored by the Bay of Fundy, N_2 a shade more so than M₂. This implies that the resonant period of the Bay of Fundy must fall close to their periodicities, on the side of or beyond N_2 : this makes N_2 the component of central interest. Figure 9 shows the N_2 samples available. We have entered in the Figure the amplitudes supplies in DOODSON (1924) as well as those available in digitized form. The two series seem to refer to two distinct variables, their profiles differing so profoundly. The analyses available to DOODSON had been performed by ROBERTS (1870, 1871), whose method was a trade secret. In perspective it seems to have supplied fair estimates for M_2 but little else. The one common characteristic of the two series is the oscillation in amplitude, having a duration of approximately 9 years; DOODSON correctly identified it as a third order contribution to N_2 .

We elaborate on what is a third order effect. The field of gravity present at a point on the earth's surface is determined not only by the mass of the earth but by the other celestial bodies in its neighbourhood. When one starts calculating the gravity field created by the neighbours, one encounters square roots. A square root of the form:

$$\sqrt{1+\epsilon} \approx 1+\frac{\epsilon}{2} - \frac{\epsilon^2}{8} + \dots$$

can be approximated by the expression on the right if $\varepsilon < 1$. In the case of the sun, one needs to go only as far as $\varepsilon/2$, the next term amounting to essentially nothing. On the other hand, the moon, being so close to the earth, disturbs its field more significantly, so that the terms in ε^2 needs to be retained. If we view g as the first order contribution to the field of gravity, the neighbour creates an additional contribution proportional to $\frac{1}{2}\varepsilon$, which is of second order. For the moon, the next



FIG. 9.- Sample amplitudes of N₂ available since 1894.

term in ε_2 , although exceedingly small (but not negligible), constitutes the third order contribution. Third order effects cannot be reduced by nodal corrections: they need to be assessed from the observations.

That much we can say about the N_2 time series. No clear trend emerges even though the running means did suggest a slight decay. A decrease in N_2 would be acceptable if we attribute the evolution of M_2 towards higher values to a change in the response characteristics of the Bay of Fundy. If M_2 is favored to the detriment of N_2 , the resonant period of the system must be shifting towards shorter values. Even though S_2 lies at a large distance from M_2 on the scale of frequency, it should be growing too but at a lesser rate. Figure 6 tells us the opposite. The negative correlation between them, due to their nonlinear interaction, helps us understand why this is so. Even though S_2 should grow, a larger M_2 damps it away more effectively.

The interaction between N_2 and M_2 is of much more fundamental interest. The only piece of evidence we could extract from the data covering 1947-1980, is that even during our era, the resonant period of the body of water made up of the Bay of Fundy and the Gulf of Maine, oscillates slightly between intervals of large and small M_2 amplitudes (GODIN, 1988). A large M_2 implies a longer period, favouring N_2 ; a smaller M_2 pulls it slightly toward shorter values. Some kind of 9 year periodicity appears in M_2 , which is not predicted by theory and which could be attributed to N_2 but the noise in the data is too high to extract it with any certainty. Obviously numerical experiments with a model of the Bay of Fundy could help investigate further the relation between N_2 and M_2 . The component L₂

 L_2 in the Bay of Fundy, is not the conventional gravitational L_2 . The standard development of the tidal potential does produce an L_2 which is a composite of second and third order contributions, but the whole resultant has a small amplitude and exhibits peculiar 4 ½ year modulations. The L_2 of the Bay of Fundy does no such thing: it becomes even more destabilized if one tries to apply to it the nodal corrections for the official L_2 . We show its amplitude in Figure 10 against a background of N_2 amplitudes. Some data for N_2 is missing between 1932 and 1939: the profile of the later years suggests there should be a peak somewhere in there.



FIG. 10.- Amplitude of L₂ over a background of N₂.

The correlation between L_2 and N_2 is obvious and it should come as no surprise. An inspection of the quadratic friction term (GODIN, 1991) indicates that compound harmonics of frequency $2\sigma_1+\sigma_2$ are created by the nonlinear interaction between the fundamental harmonics σ_1 and σ_2 . If these represent M_2 and N_2 , we obtain the new frequencies $2MN_2$ and 2MN6. The latter appears as a matter of course in all records where shallow water effects exist; $2MN_2$, on the other hand, falls in the tidal band and coincides exactly in frequency, with L_2 . Therefore most of the L_2 detected at Saint John arises from the nonlinear interaction between N_2 and M_2 , it being $2MN_2$ and not the L_2 suggested by the development of the tidal potential. There are other stations upstream of Saint John which were occupied by W.I. FARQUHARSON in 1959 and 1960, who conducted the last systematic observations on the tide in that area. The gauges available could not cope with the large ranges encountered so the data is not too reliable. In any case, L_2 becomes even larger beyond Saint John. We list in the following, some major harmonic constants, including L_2 , for some sites which were occupied during the current survey of 1965.

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N ₂	M ₂	L ₂	S ₂
Margaretsville 45°03'N	J 65°04"W		
106.9 cm 306°	386.4 337°	55.5 23°	41.7 23°
Ile Haute 45°15'N 64°	58′W		
74.4 cm 307°	415.2 343°	45.5 358°	53.0 26°
Diligent River 45°24'N	I 64°25′W		
103.2 cm 328°	487.7 357°	45.8 37°	71.9 36°
Five Islands 45°23'N 6	4°04′W		
121.3 cm 352°	542.1 9°	54.6 37°	78.0 54°

From a distance, the constants for Margaretsville seem a bit off. Nevertheless we encounter an L_2 having magnitude 45-55 cm in that area, revealing the intensity of the interactions between M_2 and N_2 around Minas Channel.

The component 2N₂

We show in Figure 11 the samples available: this time the gap 1933-1938 makes us miss a possible peak. A sharp 4 $\frac{1}{2}$ year periodicity is present in the remaining data; the gravitational 2N₂ should exhibit a 19 year periodicity, not a 4 $\frac{1}{2}$ year one. We recall that the gravitational L₂ contains a preponderant 4 $\frac{1}{2}$ year periodicity, while in fact it has a 9 year periodicity which it should not have. Looking at the possible interactions between the fundamental harmonics which could create a clone of 2N₂, we find two candidates: 2NM₂ and 2MK₂. K₂ has a small amplitude of 13 \rightarrow 14 cm; however the compound harmonic 2MK₂ selected as a candidate, should be approximately equal to 2NM₂. Their size is determined by the square of the amplitude of the harmonic used twice, multiplied by the amplitude of the other. 2NM₂ should therefore be proportional to:

$$62^2 \times 303 = 1.2 \times 10^6$$

while 2MK₂ is proportional to:

$303^2 \times 13.6 = 1.2 \times 10^6$

 $2NM_2$ coincides exactly in frequency with $2N_2$ while $2MK_2$ differs from it by 1 cycle in 4 ½ years. The two compound harmonics will therefore interfere with each other over a period of 4 ½ years, exactly as seen in Figure 12. The gravitational $2N_2$ does also contribute to this frequency. Since their resultant amounts to some 8cm, we have little hope of disentangling the contributions of each one of them.

CONCLUSIONS

What had been judged initially as tantalizing glimpses into the evolution of the tide in the Bay of Fundy, is now fully confirmed by the addition of the data covering 1932-1946: a most satisfactory outcome. The tide of the Bay of Fundy is changing, and it is doing so quite rapidly, on a geological time scale. This underlines the importance of accumulating good quality geodetic records on a large term basis and of making them accessible to modern data processing by digitization.



FIG. 11.- Sample amplitudes of 2N₂.

A geological exploration of the shores of Minas and Cumberland basins, should confirm these trends by following the position of the lines of high water and low water through recent history.

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