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RAPID TIDAL ANALYSES, FROM A 3 3/4 JULIAN YEARS SPAN UP TO A NODAL CYCLE SPAN, WITH A PC

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Abstract

It will be shown how analyses of tidal records of $n*2^{14}$ hourly heights, with n from 2 up to more than 10, can be quickly performed by using a PC. The method on which such analysis is based is the one developed by FRANCO and HARARI (1988). If the analysis of a $10*2^{14}$ hourly heights span, very nearly equal to a nodal cycle, is worked out, then the main constituents and the respective satellites, a total of 1014, can be separated in 6 minutes and 25 seconds if a PC 386 DX (40 megahertz), with coprocessor, is available.

1 - INTRODUCTION

It is well known that Doodson's method of tidal analysis is based on the separation of the species, with special filters (daily process) followed by the separation of small groups of constituents (monthly process) and finally the separation of the constituents themselves.

In our basic method of tidal analysis (FRANCO-ROCK, 1972a), the separation process is worked out through the Fourier analysis of the time series, with the FFT (Fast Fourier Transform) algorithm. Then two systems of linear equations are built up, for each species, in order to separate the unknowns *Rcosr* and *Rsinr*, where *R* and *r* are the amplitude and the phase at the time origin respectively, for each constituent. Some years later, this author developed the refined method of tidal analysis (FRANCO, 1978), where the tidal heights are previously weighted with a cosine taper (a hanning filter in the time domain) in order to reduce the lateral contamination of small groups of constituents. Such procedure allowed the reduction

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of the number of unknowns in each system. In fact, with a span of 2^{14} hours, the largest system of 31 semidiurnal constituents is split into several small ones, the largest one having only 4 unknowns, corresponding to the constituents T_2, S_2, R_2 and K_2, which have close frequencies. As far as the diurnal constituents are concerned, the largest system is formed by the constituents Π_1, P_1, S_1, K_1, ψ_1 and ϕ_1.

Another characteristic of the refined method is that the matrices of the systems in *Rcosr* and *Rsinr* are the same. Thus, only one matrix inversion is necessary to solve both systems.

On the other hand, when considering the analysis of 19 years of data, the system corresponding to Π_1, P_1, S_1, K_1, ψ_1 and ϕ_1, and the respective satellites, will have 21 unknowns. In fact, the largest system to be solved has 28 unknowns (FRANCO and HARARI, 1988). The total number of the main astronomical constituents (41) and their satellites, according to CARTWRIGHT-TAYLER (1971) and CARTWRIGHT-EDDEN (1973) developments, plus the main shallow-water constituents (143) and the respective satellites, according to FRANCO (1988), is equal to 1014. One can see that it would be very difficult to handle a single system with 1014 unknowns when analysis can be successively solved very rapidly.

2 - THE STEPS OF THE ANALYSIS AND SOME RESULTS

The first step of the analysis of a 18.69 Julian years span is the working out of 10 Fourier analyses of 2^{14} hourly data each, sequentially filed with no interruption. It will be very convenient that each row of the file contains 12 heights, since fewer heights in each row require much more computer memory. The results of these analyses are randomly filed in the hard disk or, preferably in the RAMSDK, in order to be read in the same way.

The next step of the analysis is to read iteratively a file of data of the constituents which belong to each system to be built up. Table 2-I is an example.

The first two figures of the file are the number of constituents (25) and the approximate number of cycles per day (2), of the species to which the group of constituents belongs. The first column of the table shows the symbols of the constituents of the system to be formed. These symbols are followed by figures between parenthesis; such figures, from the largest negative to the higher positive, indicate the satellites, in order of increasing frequencies. Thus, the negative figures correspond to satellites with frequencies lesser than the one of the main constituent and the positive to those of higher frequency.

The six integers following the symbol are the usual Doodson's numbers, corresponding to coefficients of τ, s, h, p, N' and p', where:

τ ≡ moon's hour angle
s ≡ mean longitude of the moon
h ≡ mean longitude of the sun
p ≡ mean longitude of the moon's perigee

$N' \equiv 360°$ - mean longitude of the ascending node
$p' \equiv$ mean longitude of the perihelion

Table 2-I

25 2							
"OP2(-2)"	2	0	-2	0	-2	0	0
"OP2(-1)"	2	0	-2	0	-1	0	2
"OP2"	2	0	-2	0	0	0	2
"OP2(1)"	2	0	-2	2	0	0	0
"OP2(2)"	2	0	-2	2	1	0	0
"MTS2(-1)"	2	0	-1	0	-1	1	2
"MTS2"	2	0	-1	0	0	1	0
"MTS2(1)"	2	0	-1	0	1	1	0
"KO2(-3)"	2	0	0	-2	-1	0	0
"M2(-2)"	2	0	0	0	-2	0	0
"M2(-1)"	2	0	0	0	-1	0	2
"M2"	2	0	0	0	0	0	0
"KO2(1)"	2	0	0	0	1	0	0
"KO2(2)"	2	0	0	0	2	0	2
"M2(1)"	2	0	0	2	1	0	0
"KO2(2)"	2	0	0	2	-1	0	0
"M2((2)"	2	0	0	2	0	0	0
"MST2(-1)"	2	0	1	0	-1	-1	2
"MST2"	2	0	1	0	0	-1	0
"MKS2(-2)"	2	0	2	0	-2	0	0
"MKS2(-1)"	2	0	2	0	-1	0	2
"MKS2"	2	0	2	0	0	0	0
"MKS2(1)"	2	0	2	0	1	0	0
"MKS2(2)"	2	0	2	0	2	0	0
"MKS2(3)"	2	0	2	2	0	0	0

The seventh integer is the factor c to be multiplied by 90° in order to adjust the constituent's phase, according to the harmonic development. These astronomical elements are computed for the initial time of the series, to form the vector $(\tau\ s\ h\ p\ N'\ p'\ c)^T$, where T means transpose. The product of the matrix of Table 2-I with that vector gives the astronomical arguments of the constituents. The product of the same matrix (considering c=0) with the vector formed by the angular frequencies of the elements of the vector, corresponds to the frequencies of the constituents included in the first column. The extreme values of these frequencies define the frequency band into which are the constituents of Table 2-I, giving the subscripts of the Fourier coefficients filed in random form, in the hard disk. Then, these coefficients, which correspond to each one of the 10 analyses, will be combined to form the known terms of the redundant systems.

That procedure will become clearer if we transcribe the basic equations from the original text:

$$a_n\ (k) + a_n\ (2m-k) = \sum_j 2A_{nj} \cos\ [\sigma_j\ (k-m)]\ R_j\ cosr_j \tag{2a}$$

$$b_n\ (k) + b_n\ (2m-k) = \sum_j 2A_{nj} \sin\ [\sigma_j\ (k-m)]\ R_j\ cosr_j \tag{2b}$$

$$b_n\ (k) + b_n\ (2m-k) = \sum_j 2A_{nj} \cos\ [\sigma_j\ (k-m)]\ R_j\ sinr_j \tag{2c}$$

$$-a_n\ (k) + a_n\ (2m-k) = \sum_j 2A_{nj} \sin\ [\sigma_j\ (k-m)]\ R_j\ sinr_j \tag{2d}$$

where a_n and b_n are the Fourier coefficients of the k^{th} and the $(2m\text{-}k)^{th}$ analyses, picked up from the file in the hard disk (or the RAMDSK); the matrix element A_{nj} corresponds to the n^{th} harmonic and the j^{th} constituent and σ_j is the virtual frequency of the j^{th} constituent in degrees per 16384 (2^{14}) hours (FRANCO and HARARI, 1988). The redundant system corresponding to the selected group of constituents is then built up by programming expressions (2a) to (2d).

The next step is the building up of the normal system matrix and its inversion. Then, the unknowns are found as usually. The further steps are the same as in any harmonic analysis of tides (FRANCO, 1988).

If the number n of series of 2^{14} data is equal to or larger than 7 the necessary inputs to run the program appear sequentially in the micro computer screen and include two options: the analysis of any number of species according to their increasing frequencies or the analysis of a single group of constituents. The screen display corresponding to the first option, when n≥7, is as follows:

File name of the recorded data:	CANANEIA
Diskette (A,B,C....) where are these data:	E
Diskette (A,B,C....) with the constants for analyses:	E
Diskette (A,B,C....) for filing the FFT analyses:	F
Probability for the rejection of small constituents:	0.99
Number of series of 16384 (2^14) hours:	10
Number of species or system (1) to be analyzed:	13
Diskette (A,B,C....) for filing the whole results:	E
File name of the recorded analysis:	UUUU
File name of the recorded harmonic constants:	VVVV
Diskette (A,B,C....) for filing these constants:	E

In this case the analysis will cover the 13 species, from 0 to 12 cycles per day. The screen display for the second option is:

File name of the tidal data:	CANANEIA
Diskette (A,B,C....) where are these data:	E
Diskette (A,B,C....) with the constants for analyses:	E
Diskette (A,B,C....) for filing the FFT analyses:	F
Probability for the rejection of small constituents:	0.99
Number of series of 16384 (2^14) hours:	10

Number of species or system (1) to be analyzed: 1
Species to which belongs the system: 2

After this last input the screen display will show:

13. 2MN2S2
14. 2NS2, 3M2S2
15. OQ2, MNS2
16. 2N2', 2N2, MU2, 2(NU)M2
17. SNK2, N2, NU2, 2KN2S2
18. OP2, MTS2, M2, MST2, MKS2
19. LAMBDA2, L2
20. 2SK2, T2, S2, R2, K2
21. MSN2, KJ2
22. 2SM2, 2MS2N2

CHOSE ONE OF THE ABOVE NUMBERS: 18

Diskette (A,B,C....) for filing the results: E
File name: WWWW

The results shown in Table 2-II were obtained with the second option and correspond to the analysis worked out for Cananéia harbour (25°01'S and 44°56'W).

The figures preceeding each constituent in the full analysis are 202 to 225. Since the shallow-water constituents result from the interaction of the main astronomical constituents, it is obvious that the former will also have satellites, due to the interaction of the satellites of the latter with the main astronomical constituents, as well as from the mutual interaction of satellites. Thus, it is not possible to ignore the satellites of the shallow water constituents in the analysis of a nodal cycle span. Then, this author built up an extensive table of shallow-water constituents and the respective satellites, through a computer program, before working out the analysis of the long series. Table 2-II shows how many satellites appear in the analysis of a group with only 5 main constituents. In fact, the shallow-water constituent $K0_2$ (K_1+O_1) has exactly the same frequence as M_2 and cannot be separated by the analysis; however, its satellites must be included in the computations.

The constituents of Table 2-II flagged with an asterisk did not resist the statistical test, based on the confidence interval (+/-) derived from a Student's T distribution (FRANCO-ROCK, 1972b). It can be seen that the amplitudes of the flagged constituents are lesser than their confidence intervals. In such cases it is not possible to compute the confidence intervals for the phase lags. The constituents which resist the statistical selection are filed in the chosen diskette with the Doodson's numbers, in order to make the prediction easier. In this case, considering the full analysis with all the species included, 163 constituents were recorded in the hard disk.

Since each term of the left hand side of equations (2a) to (2d) is the result of the combination of two Fourier coefficients, then the standard deviation of the known terms of these equations is equal to $\sigma\sqrt{2}$, where σ is the standard deviation of the Fourier coefficients.

Many conclusions can be taken about the micro structure of the tidal phenomenon from this long series analysis (FRANCO and HARARI, 1991, 1993). The first option can be followed from 7×2^{14} onwards. For shorter series, the satellites of the 176 main constituents have to be ignored and the perinodals factor f and angle u must be restored. However, it is necessary to use their mean values for the series (FRANCO, 1988). In fact, it is obvious that the perinodal corrections for the central time of the series would have no meaning. Furthermore the option "How many species will be analyzed?" is not available and all constituents analyzed will be listed.

Table 2-II

No.	Symbol	Deg./h	H cm.	+/-	G Deg.	K Deg.	GW Deg.	+/-
1	*OP2(-2)	28.8975541	0.08	0.24	266.63	257.48	353.33	*****
2	*OP2(-1)	28.8997605	0.16	0.24	14.41	5.26	101.11	*****
3	OP2	28.9019669	0.36	0.24	17.35	8.21	104.06	41.70
4	*OP2(1)	28.9112506	0.08	0.24	324.28	315.17	51.02	*****
5	*OP2(2)	28.9134570	0.08	0.24	84.21	75.10	170.95	*****
6	*MTS(-1)	28.9408311	0.21	0.24	86.61	77.58	173.43	*****
7	MTS2	28.9430375	0.65	0.24	109.42	100.40	196.25	22.06
8	*MTS2(1)	28.9452440	0.09	0.25	86.90	77.88	173.73	*****
9	*KO2(-3)	28.9726142	0.14	0.31	5.31	356.38	92.23	*****
10	M2(-2)	28.9796914	0.26	0.24	79.96	71.04	166.89	68.08
11	M2(-1)	28.9818978	1.34	0.24	102.11	93.20	189.05	10.45
12	M2	28.9841042	35.78	0.24	94.66	85.76	181.61	0.39
13	*KO2(1)	28.9863106	0.22	0.24	107.18	98.29	194.14	*****
14	*KO2(2)	28.9885170	0.18	0.25	98.29	89.40	185.25	*****
15	*M2(1)	28.9955943	0.17	0.28	48.26	39.39	135.24	*****
16	*KO2(2)	28.9911815	0.17	0.25	97.52	88.65	184.50	*****
17	*M2(2)	28.9933879	0.10	0.27	267.11	258.24	354.09	*****
18	*MST2(-1)	29.0229645	0.01	0.24	259.88	251.10	346.95	*****
19	MST2	29.0251709	0.38	0.24	232.67	223.89	319.74	40.50
20	*MKS2(-2)	29.0618287	0.07	0.24	120.48	111.82	207.67	*****
21	*MKS2(-1)	29.0640351	0.07	0.24	298.24	289.59	25.44	*****
22	MKS2	29.0662415	0.47	0.24	170.64	161.99	257.84	30.44
23	*MKS2(1)	29.0684479	0.13	0.24	198.32	189.68	285.53	*****
24	*MKS2(2)	29.0706543	0.08	0.24	300.53	291.89	27.74	*****
25	*MKS2(3)	29.0755252	0.07	0.24	195.94	187.31	283.16	*****

Table 2-III shows part of the results obtained from an analysis of $5x2^{14}$ hourly heights. Constituents with phase differences of 360° after one revolution of the lunar perigee can be separated. e.g., M_1', $2N_2'$, N_2'; also the third diurnal F_3, which depends on the fourth power of the lunar parallax, separated in the analysis of the 10 series (FRANCO and HARARI, 1991), is clearly detected through the analysis of a $5x2^{14}$ span. The enhance of the third diurnal constituent in the Brazilian southern Coast was explained by HUTHNANCE (1980) as a kind of organ pipe resonance.

Table 2-III

No.	Symbol	Deg./h	H cm.	+/-	G Deg.	K Deg.	GW Deg.	+/-
1	Sa	0.0410686	5.69	2.76	25.71	25.84	25.84	28.97
2	*Ssa	0.0821137	1.91	2.41	7.07	7.31	7.31	*****
3	*Mm	0.5443747	1.05	2.01	229.64	231.27	231.27	*****
4	*MSf	1.0158958	0.47	2.50	34.32	37.37	37.37	*****
5	*Mf	1.0980331	0.81	2.50	232.09	235.39	235.39	*****
6	Mtm	1.6424077	2.16	2.08	305.43	310.36	310.36	74.37
7	2Q1	12.8542862	0.44	0.21	36.56	27.20	75.12	29.45
8	SIGMA1	12.9271398	0.23	0.21	65.52	56.38	104.30	69.56
9	Q1	13.3986609	2.86	0.21	63.61	55.88	103.81	4.13
10	RO1	13.4715145	0.51	0.21	62.55	55.04	102.96	24.11
11	O1	13.9430356	11.19	0.77	84.70	78.60	126.52	3.97
12	*MP1	14.0251728	0.18	0.77	358.10	352.25	40.17	*****
13	M1′	14.4920521	0.59	0.18	83.82	79.37	127.29	17.99
14	M1	14.4966940	0.40	0.18	105.93	101.49	149.42	27.51
15	*QUI1	14.5695476	0.16	0.18	129.94	125.72	173.65	*****
16	*PI1	14.9178647	0.18	0.33	170.28	167.11	215.03	*****
17	P1	14.9589314	2.28	0.33	145.96	142.91	190.83	8.37
18	S1	15.0000000	1.57	0.33	118.97	115.95	163.87	12.19
19	K1	15.0410686	6.32	0.33	145.22	142.42	190.34	3.01
20	*PSI1	15.0821353	0.04	0.33	193.65	190.97	238.89	*****
21	*FI1	15.1232059	0.09	0.33	138.38	135.82	183.75	*****
22	*TETA1	15.5125897	0.07	0.19	252.44	251.05	298.97	*****
23	J1	15.5854433	0.19	0.19	253.44	252.28	300.20	84.45
24	*2PO1	15.9748272	0.15	0.17	222.34	222.34	270.26	*****
25	SO1	16.0569644	0.19	0.17	288.10	288.34	336.27	66.72
26	OO1	16.1391017	0.23	0.17	312.25	312.75	0.67	48.52
27	*KQ1	16.6834764	0.08	0.11	62.01	64.14	112.06	*****
28	*N(MU)S2	26.4079379	0.06	0.10	108.47	91.84	187.69	*****
29	*2NS2	26.8794590	0.08	0.11	311.86	296.65	32.50	*****
30	M(MU)S2	26.9523126	0.17	0.11	123.77	108.78	204.63	40.81
31	*OQ2	27.3416965	0.19	0.21	130.62	116.80	212.65	*****
32	MNS2	27.4238337	0.27	0.21	98.45	84.87	180.72	53.31
33	*MNK2S2	27.5059710	0.13	0.21	42.05	28.72	124.57	*****
34	*2N2′	27.8907130	0.10	0.20	206.84	194.66	290.51	*****
35	2N2	27.8953548	1.84	0.20	152.55	140.38	236.23	6.12
36	MU2	27.9682084	2.13	0.19	142.62	130.68	226.53	5.23
37	*2(NU)M2	28.0410660	0.16	0.19	197.51	185.78	281.63	*****
38	*SNK2	28.3575922	0.14	0.18	296.15	285.37	21.22	*****
39	N2′	28.4350877	0.33	0.19	280.62	270.08	5.93	33.91
40	N2	28.4397295	5.79	0.19	165.32	154.79	250.64	1.84
41	NU2	28.5125851	0.87	0.18	186.51	176.20	272.05	12.22
42	*2KN2S2	28.6040041	0.03	0.18	160.44	150.41	246.26	*****
43	*OP2	28.9019670	0.28	0.61	15.13	5.98	101.83	*****
44	MTS2	28.9430375	0.79	0.61	96.13	87.11	182.96	50.86
45	M2	28.9841042	35.74	0.61	94.68	85.78	181.63	0.98
46	*MST2	29.0251709	0.49	0.61	267.04	258.27	354.12	*****

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3 - COMMENTS AND CONCLUSIONS

We believe that the software prepared to analyse long tidal series is a very useful tool to investigate many anomalies observed in the predictions for any place. However, an important question can be risen: are the predictions significantly improved with all the constituents given by the long series analysis? Such answer cannot be given straightforward. In fact, strong meteorological perturbations produce important anomalies on the predictions worked out for Cananéia harbour, for example. The 'mean' sea level oscillates very much about the long term 'mean' sea level (Fig. 1); thus, generally speaking, the times of high and low waters are fairly well predicted but not the heights, especially for days of half moon. Consequently, one can see that Cananéia harbour is not an adequate station for the evaluation of the final results of the long term analysis, as far as prediction is concerned. In addition, some satellites of the main constituents, which are included in the analysis, in order to compute the perinodal corrections, are statistically rejected and we wonder if such rejection is valid. Furthermore, our experience (FRANCO and HARARI, 1993) showed that the harmonic constants of the long period constituents obtained from the analysis of the 'mean' sea level, at the Cananéia station were not statistically reliable. In fact, these "constants" did not present the necessary stability. However an important conclusion can be taken from the analysis: the micro structure of the phenomenon can be well studied at a very low cost.

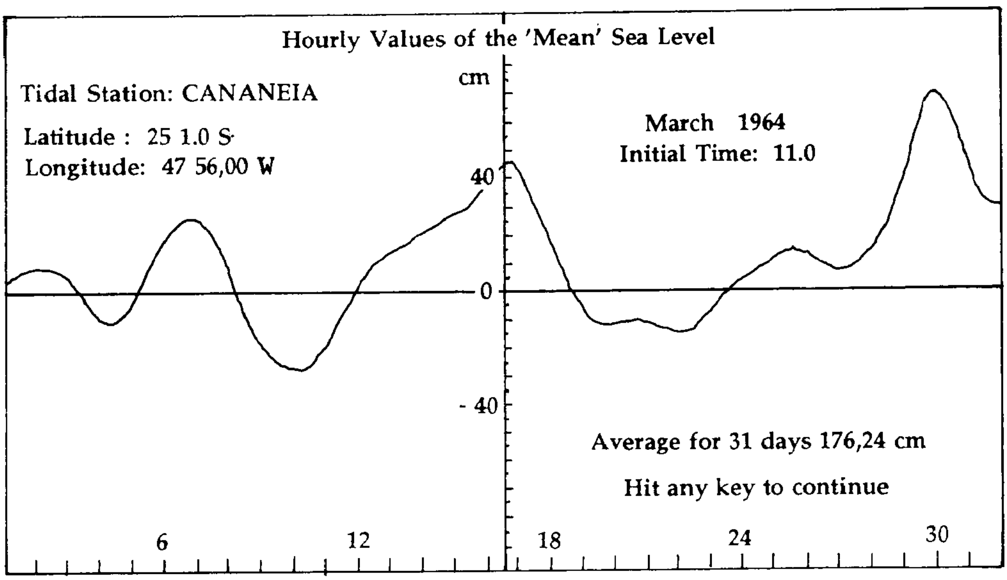

FIG. 1.- Hourly Values of the Mean Sea Level.

As it has been mentioned, analyses can also be worked out for spans shorter than $10x2^{14}$. In fact, the software menu indicates that the number of Fourier partial analyses can be larger or lesser than 10. However 7 is the least number of Fourier analyses which give fair results for the harmonic constants of the satellites. For spans shorter than 7×2^{14} hours (13.018 Julian years) the satellites must be ignored.

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