International Hydrographic Review, Monaco, LXXII(2), September 1995

A TECHNIQUE FOR QUALITY CONTROL AND SELECTION OF TIDAL HARMONIC CONSTITUENTS

by William R. CRAWFORD [1]

Abstract

This paper outlines a procedure to select an optimal set of tidal harmonic constants for use in the prediction of tidal heights. We begin with 18 sets of harmonic constants, each computed from one of 18 consecutive years of hourly tidal heights at a port using a standard tidal analysis program. These constituents are input to a spreadsheet in which are computed average amplitude and phase, and standard deviations of amplitude over the 18 years. As well, graphs of the amplitude and phase deviations for all constituents are presented to allow a user to identify any trend or cyclical variation in the amplitudes or phases, and to identify years where the quality of measurement is poor. The standard error of the average amplitude is computed and may be used as an accept/reject criterion for inclusion of tidal constituents in subsequent predictions. Although nominally intended for 18 years of data, the program will operate on a smaller number of years, and can be adapted to include more years.

Many tidal constituents of astronomical origin have frequencies close to or identical with shallow water constituents generated by the interaction of astronomical constituents. If the shallow water constituent interferes with the astronomical one, it will modulate the amplitude and phase of the astronomical constituent, often with periods near 8 and/or 19 years. This modulation can be identified in the time series plots of deviations of amplitudes and phases.

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INTRODUCTION

It is traditional in tidal analysis to fit a series of cosine functions to the observed time series of hourly heights at a tidal station. The periods of these functions are determined by astronomical factors such as the rate of rotation of the earth about its axis, the orbital periods of the earth around the sun and the moon around the earth, and the times of closest approach of the moon and sun to the earth, plus a few other slowly varying factors. These periods are known to great accuracy. As well, we know the relative amplitudes of the astronomical tidal forces at each of these frequencies.

What is not known without measurement is how the ocean responds to these time varying forces. To determine the ocean response at a port, hydrographers traditionally measure hourly heights for a year. They then predict tides many years into the future using parameters arising from an analysis of that single year of observations.

This paper addresses several concerns:

- At some ports we have many years of tidal measurements. How can we use these measurements to improve our predictions?
- Do the tidal constituents at a port change over a period of time? Tides can change over a period of years as a harbour silts up or an entrance to a harbour is constricted or dredged.
- Are we predicting the frictional effects properly? In shallow waters the tide at a particular period will be influenced by frictional interaction of tides at other periods.

The FOREMAN (1977) programs are now one of the standard tools for the analysis and prediction of tides. The analysis program follows the tradition set by DOODSON (1921) and GODIN (1972) in specifying only those constituents whose frequencies differ by at least one cycle per year, and accounts for slower variations in these constituents by varying their amplitude and phase according to the modulation of the astronomical potential. Although it is best applied to one year of data, at most ports there are many years of data available. To take advantage of such longer time series, FOREMAN and NEUFELF (1991) devised a program that analyses 18.6 years (or more) of data to compute amplitudes and phases of constituents with distinct first five DOODSON numbers. More recently, FRANCO (1995) devised a program to compute 10 consecutive FOURIER analyses of 2^{14} hourly heights, from which accurate tidal constituents are computed. However, neither of these programs readily reveal any trends in constituents over the years, and they require special prediction programs which differ from the traditional operational routines.

This paper describes a process which uses a commercial spreadsheet software program to average tidal harmonic constants over many years, where each year of data contributes one set of constants. The behavior of constituents over many years is used to determine average amplitudes and phases, and confidence limits for these values. The graphical presentation within this program provides a degree of quality control of the data. The method can be readily applied by most hydrographic offices and is available for distribution.

DETAILS OF SPREADSHEET

This spreadsheet, which runs in Microsoft® Excel version 5.0 on a personal computer, operates on output of the FOREMAN (1977) harmonic analysis routine. The Rayleigh parameter was set to 0.9 to ensure that 68 constituents would be included in each yearly analysis. Eighteen years of data are input, one year at a time, to the harmonic analysis routine. The output consisting of amplitudes and phases of 68 constituents for each of 18 years are loaded to the spreadsheet. A sequence of 18 years was selected because it is close to the nodal cycle of 18.6 years, and the calculations and graphics can fit onto a personal computer with 16 Mbytes of RAM. This program can be readily adapted for additional years of data, and has been adapted for fewer years of data. The spreadsheet computes the following statistics for each constituent, although phase information for Z_0 is not computed, since it is unnecessary.

Vector average amplitude A and phase θ of each constituent over the 18 years.
The amplitude and phase for each year are converted to x and y components, which are averaged separately, then combined to form the average amplitude and phase.

Standard error S of the vector amplitude over 18 years.
The average and individual amplitudes and phases are converted to x and y components. The standard deviation of each component is computed; then the RMS value of these average deviations is computed. The *sample* standard deviation σ, rather than the *population* standard deviation is computed. Finally, the standard error $S = \sigma/18^{1/2}$ of the sample mean is computed and printed.

Ratio r formed by dividing the standard error S by the mean A.
This ratio may be used to determine an accept/reject criterion for using a selected constituent in tidal prediction programs.

Deviations of individual amplitudes ΔA_i and phases $\Delta\theta_i$ from the 18-year averages.

Largest positive and negative deviations of individual amplitudes $\Delta A_{i(ma)}$, $\Delta A_{i(mi)}$ and phases $\Delta\theta_{i(ma)}$, $\Delta\theta_{i(mi)}$ from the 18-year averages.

Once computed, the spreadsheet plots annual deviations of amplitude ΔA_i and phase $\Delta\theta_i$ for the 68 constituents. Scales of these plots are automatically selected such that the largest plotted deviation will be at the full range. Table 1 presents the summary statistics for Point Atkinson, British Columbia (in the Strait of Georgia at 49° 20'N, 123° 15'W). Plotted deviations of amplitude and phase are presented in Figure 1.

Constit.	Point Atkinson, version 1.1 16 Jun 1995				minimum	maximum
	Average				(mm)	(mm)
Z0	**amp (m)**	**3.08418**	**st dev**	**0.03416**	**-53.38**	**74.22**
	phase	*0.00*	*error ratio*	*0.0030*	*0.00*	*0.00*
SA	**amp (m)**	**0.04785**	**st dev**	**0.04097**	**-26.75**	**46.45**
	phase	*334.13*	*error ratio*	*0.2288*	*-132.20*	*41.23*
MF	**amp (m)**	**0.02532**	**st dev**	**0.02076**	**-15.02**	**42.38**
	phase	*126.49*	*error ratio*	*0.2191*	*-62.65*	*35.01*
Q1	**amp (m)**	**0.07762**	**st dev**	**0.00413**	**-4.92**	**6.28**
	phase	*149.73*	*error ratio*	*0.0142*	*-5.10*	*4.16*
RHO1	**amp (m)**	**0.01570**	**st dev**	**0.00490**	**-4.50**	**8.80**
	phase	*143.93*	*error ratio*	*0.0835*	*-21.67*	*12.70*
O1	**amp (m)**	**0.47599**	**st dev**	**0.00639**	**-8.09**	**5.01**
	phase	*151.78*	*error ratio*	*0.0036*	*-0.99*	*1.09*
NO1	**amp (m)**	**0.04173**	**st dev**	**0.02089**	**-18.13**	**46.77**
	phase	*173.77*	*error ratio*	*0.1338*	*-25.97*	*42.39*
S1	**amp (m)**	**0.04176**	**st dev**	**0.01243**	**-10.96**	**14.64**
	phase	*120.67*	*error ratio*	*0.0796*	*-19.84*	*25.86*
K1	**amp (m)**	**0.86341**	**st dev**	**0.00831**	**-9.61**	**6.19**
	phase	*165.87*	*error ratio*	*0.0026*	*-0.88*	*0.87*
J1	**amp (m)**	**0.04989**	**st dev**	**0.00769**	**-10.69**	**8.51**
	phase	*196.31*	*error ratio*	*0.0412*	*-6.96*	*11.35*
SO1	**amp (m)**	**0.03019**	**st dev**	**0.00381**	**-6.59**	**3.21**
	phase	*278.28*	*error ratio*	*0.0337*	*-9.46*	*4.83*
2N2	**amp (m)**	**0.02168**	**st dev**	**0.00817**	**-7.78**	**13.82**
	phase	*110.39*	*error ratio*	*0.1008*	*-21.86*	*27.14*
NU2	**amp (m)**	**0.03858**	**st dev**	**0.00215**	**-1.98**	**2.12**
	phase	*142.64*	*error ratio*	*0.0149*	*-4.29*	*2.91*
H1	**amp (m)**	**0.00412**	**st dev**	**0.00485**	**-3.02**	**5.98**
	phase	*290.45*	*error ratio*	*0.3151*	*-96.79*	*75.90*
M2	**amp (m)**	**0.92021**	**st dev**	**0.00856**	**-8.91**	**7.89**
	phase	*159.11*	*error ratio*	*0.0025*	*-0.77*	*0.68*
L2	**amp (m)**	**0.02805**	**st dev**	**0.00553**	**-3.85**	**8.05**
	phase	*187.13*	*error ratio*	*0.0527*	*-12.85*	*20.50*
S2	**amp (m)**	**0.23213**	**st dev**	**0.00379**	**-3.03**	**2.47**
	phase	*179.47*	*error ratio*	*0.0044*	*-0.98*	*1.52*
K2	**amp (m)**	**0.06249**	**st dev**	**0.00272**	**-1.79**	**2.51**
	phase	*179.44*	*error ratio*	*0.0116*	*-3.20*	*4.55*
MK3	**amp (m)**	**0.00625**	**st dev**	**0.00362**	**-3.75**	**5.15**
	phase	*186.93*	*error ratio*	*0.1548*	*-69.64*	*12.94*
M4	**amp (m)**	**0.00724**	**st dev**	**0.00272**	**-2.54**	**2.46**
	phase	*163.15*	*error ratio*	*0.1004*	*-36.87*	*22.35*
M6	**amp (m)**	**0.00928**	**st dev**	**0.00114**	**-1.98**	**0.72**
	phase	*67.93*	*error ratio*	*0.0329*	*-5.24*	*16.17*

Table 1. Statistics of selected constituents for the port of Point Atkinson, British Columbia. Phase lags are in Pacific Standard Time (Z+8). Columns 6 and 7 present the amplitude and phase differences between the mean values and the smallest (col. 6) and the largest (col. 7) amplitude and phase.

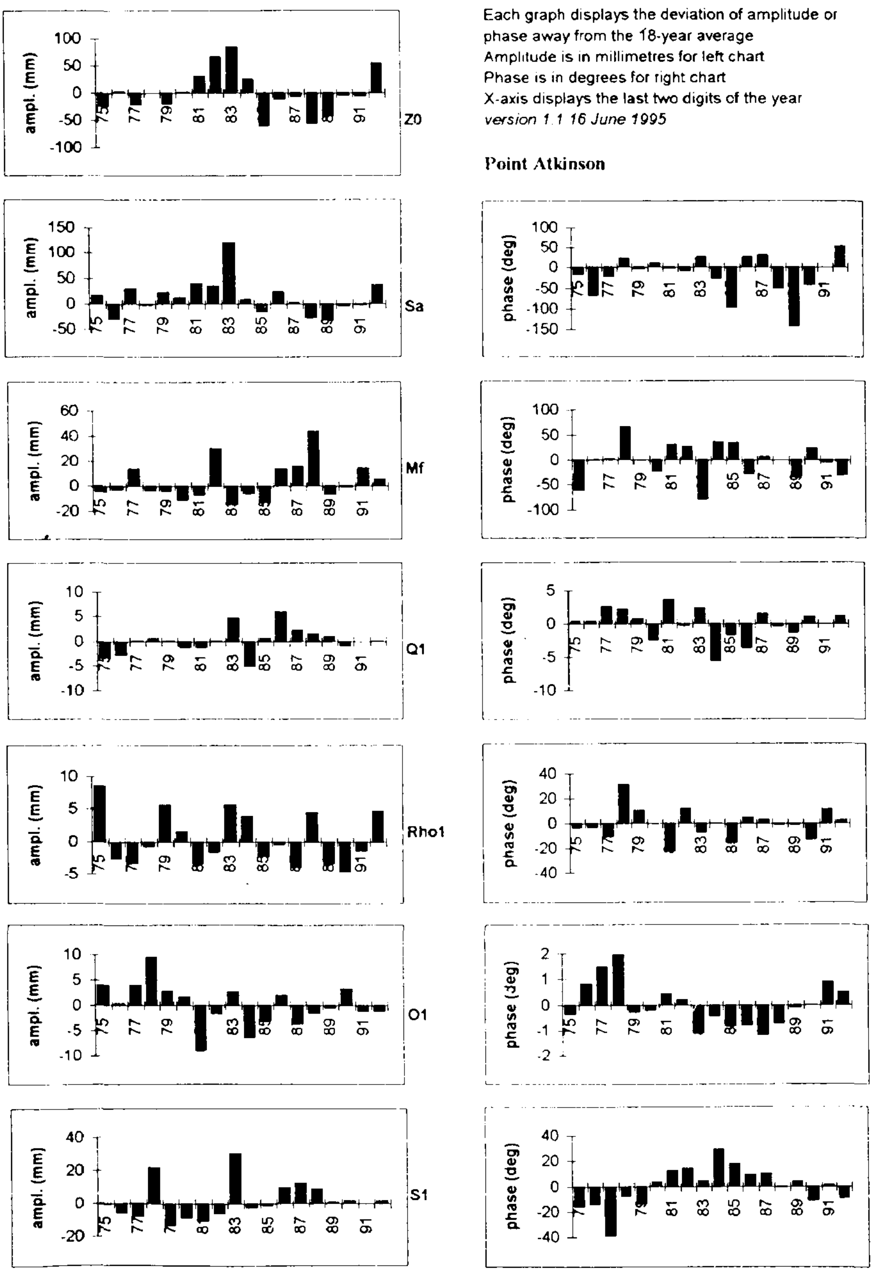

FIG. 1a.- Plots of the deviations of amplitude and phase of 21 selected tidal constituents for the port of Point Atkinson, British Columbia, Canada.

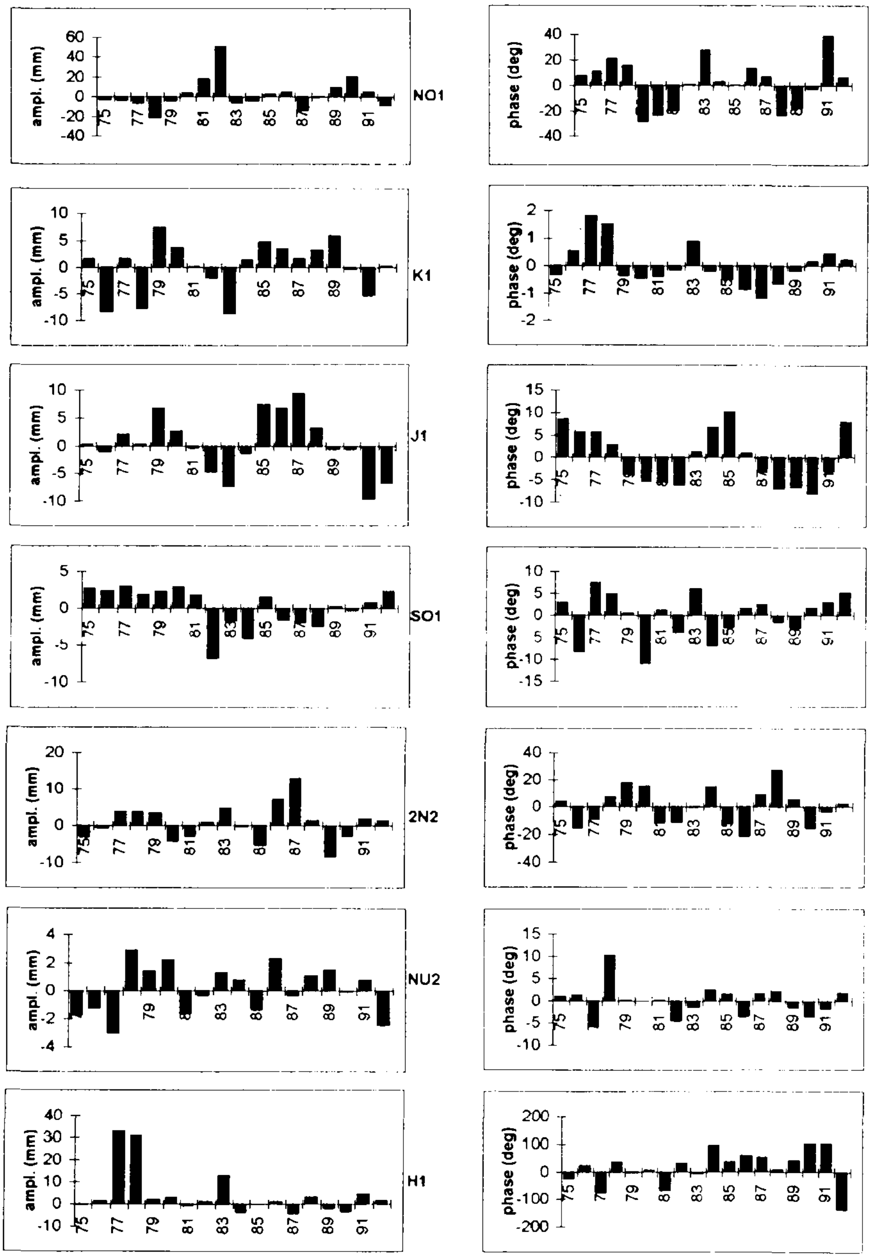

FIG. 1b.- Plots of the deviations of amplitude and phase of 21 selected tidal constituents for the port of Point Atkinson, British Columbia, Canada.

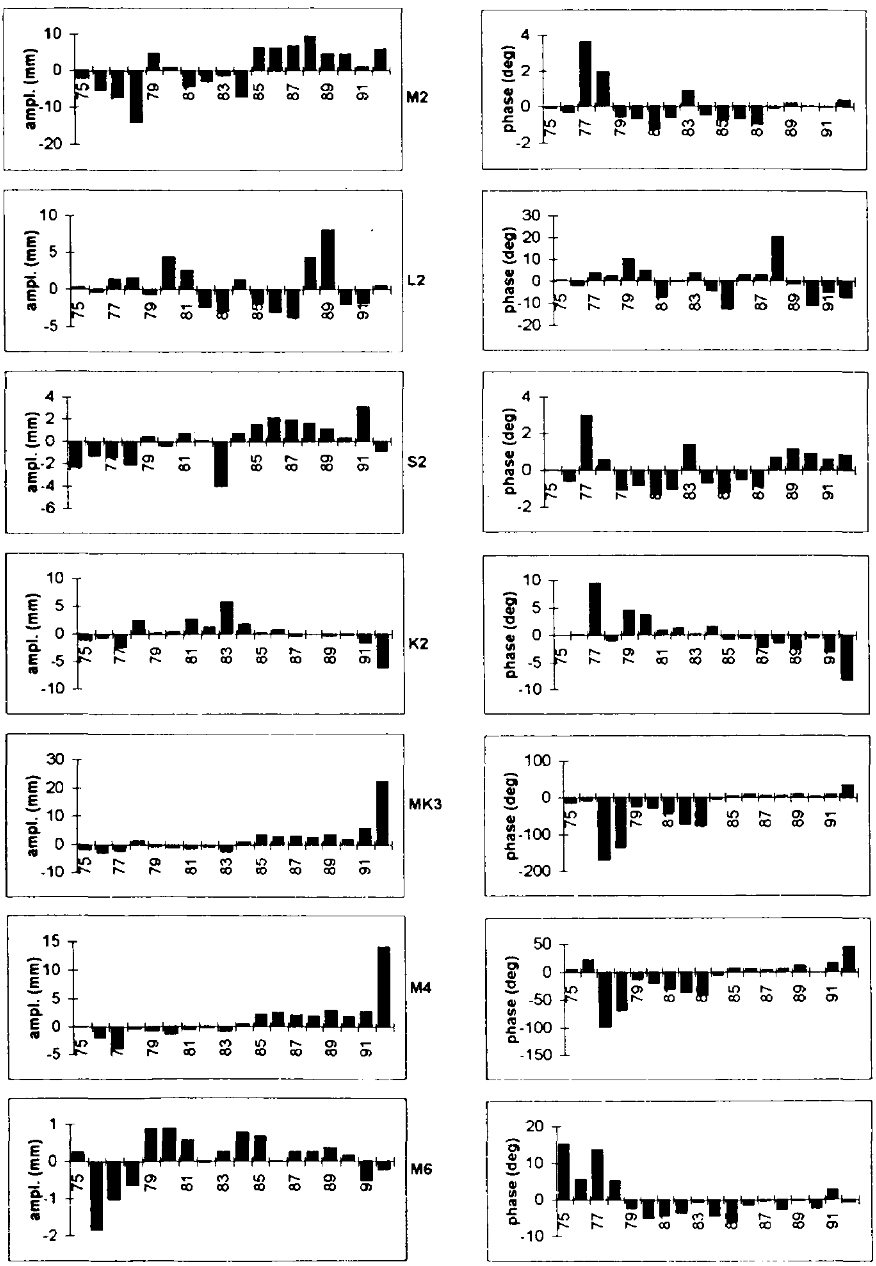

FIG. 1c.- Plots of the deviations of amplitude and phase of 21 selected tidal constituents for the port of Point Atkinson, British Columbia, Canada.

INTERPRETATION

This spreadsheet has been used to investigate the following features.

1. Trends and anomalies.

The plotted values will immediately show a long term trend in amplitude and phase of constituents and in amplitude of Z_0, the annual average sea level. The 1982-83 El Niño normally high. The highest recorded sea levels at several gauges hit in late December 1982. Consequently, the Z_0 and Sa constituents both display large amplitudes in 1983, and to a lesser extent in 1982, as indicated in Figure 1. The 1992 El Niño also increased sea level, as revealed in these Zo and Sa constituents.

One may wish to know whether to use a constituent set based on 18 years of measurements, or use a more recent year's constituents. The choice depends on the inter-annual stability of the constituents. Most British Columbia ports are in deep channels with little silting, such that frequent dredging is not required, and the tidal constituents are stable over many years. None of the British Columbia stations examined revealed a significant trend over the 18 years. For ports with a long term trend, such as Saint John, New Brunswick, Canada, as described by GODIN (1994), this program would clearly identify the nodal variation of 18.6 years in some of the constituents, and allow one to determine how to represent this tide in predictions.

This spreadsheet as presently set up cannot determine intra-annual variations, such as seasonal changes in the phase of a constituent. For example, the semi-diurnal constituents at Victoria, British Columbia, display a small summer-winter shift in amplitude and phase (FOREMAN *et al.*, 1995). To some extent the shift of M_2 will be accounted for by the inclusion of neighboring constituents H_1 and H_2 in predictions. However, other semi-diurnal constituents do not all have such close neighbours to account for their seasonal variation.

2. Quality control of observations of hourly heights.

One often finds years with incomplete records, or poor quality data. Should these years be used for subsequent predictions of tidal heights? If not, then what is an acceptable degree of missing data for tidal analyses? The analysis program outputs a numerical value called a matrix condition, which is zero if the analysis matrix is singular, and 1 if it is diagonal. Table 2 reveals that at Point Atkinson, B.C., the years 1978 and 1983 were each missing a significant period of observations, to the degree that the matrix condition dropped to a value less than 0.1. Such a low value normally indicates that at least one pair of constituents is poorly resolved by the analysis. Figure 1 indicates that the constituents H_1, M_2, and several others revealed irregularities in 1977, while Mf, S_1, NO_1, K_1, H_1, S_2, and K_2 all displayed unusual amplitudes or phases in 1983. Rather than look to determine which pairs of constituents are causing the problem, it would be appropriate here to remove these years from the spreadsheet calculation of average values and of confidence limits for the constituents.

year	number of observations	matrix condition	year	number of observations	matrix condition
1975	8675	0.808	1984	8612	0.785
1976	8783	0.821	1985	8759	0.818
1977	8759	0.818	1986	8707	0.808
1978	7301	0.068	1987	8759	0.818
1979	8759	0.818	1988	8783	0.821
1980	8783	0.821	1989	8759	0.818
1981	8759	0.818	1990	8759	0.818
1982	8759	0.818	1991	8759	0.818
1983	6433	0.001	1992	8783	0.821

Table 2. Details of input data to annual harmonic analyses for Point Atkinson, British Columbia. The matrix condition is a diagnostic output from the FOREMAN, 1977, tidal analysis program.

Several constituents in Figure 1 reveal unusual deviations for the years 1977, and 1992 although these years have full sets of observations. Records for 1978 show that the stilling well for this gauge had silted up, and it was cleared July 1978, creating a shift in constituents. Based on the anomalies seen here it is likely that the stilling well silted up in 1977 and remained so until 1978. The gauge attendant's records for 1992 indicate that the stilling well had silted up in the spring, and it was cleared in July, this time creating a shift in the high frequency constituents. The attendants have reported that the stilling well for this gauge can silt up in about two years. It appears that the clearing did not keep up with the silting in 1978 and 1992.

3. Reliability of constituents.

The process described here allows one to determine the degree of confidence one has in each constituent, based on the year-to-year variations in amplitude and phase of each constituent.

The first step is to compute the absolute value of the vector difference between the amplitude and phase of the mean and individual annual estimates of a constituent. The standard deviation of all these individual differences is σ. Given a series of n normally distributed estimates of a value A_i, the standard error of the mean is $S=\sigma/n^{1/2}$, where σ is the sample standard deviation of the estimates of A_i. Therefore, our confidence in computing the true value of an amplitude increases as the square root of the number of estimates.

There should be some accept/reject criterion based on the error ratio of $r=S/A$, which is listed in column 5 of Table 1. Certainly, if r is much smaller than one the constituent should be used for tidal predictions, while a value of r much larger than one should preclude it from a prediction scheme.

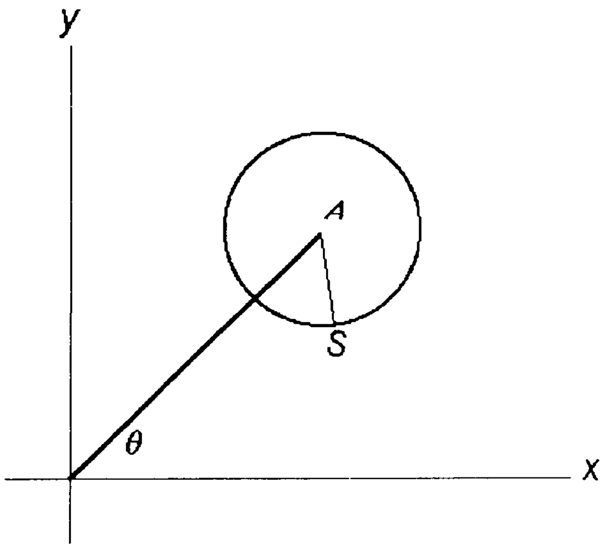

FIG. 2.- The circle of radius S represents the standard error of the sampling distribution of the mean of a constituent whose sample mean amplitude is A.

To illustrate these features, Figure 2 presents the mean amplitude A and phase θ of a tidal constituent. The standard error of the mean is presented as a circle of radius $S=\sigma/n^{1/2}$. An interesting case results when $S=A$, since at this point the standard deviation of the sampling distribution of the mean equals the mean. If the true value lies outside such a circle, one actually "inflicts damage" by including it in a tidal prediction, since its phase lag will be in error by more than 90 degrees, or its amplitude in error by more than a factor of 2; or some combination of these two errors will result in an increase in residual variance rather than a decrease. Since such a circle represents one standard deviation, it will encompass the true value about 2/3 of the time for large samples (many years). For samples with fewer than about 30 years, which includes our sample size of 18, the standard statistics textbooks recommend that Student's t be used to compute confidence limits for the amplitude.

One clearly wants the mean amplitude to lie within the circle of radius $S=A$. If we want the true mean to lie within a circle of radius A 95% of the time, then for 18 years of data the ratio $r=S/A$ should be less than 1/2.101 = 0.48. (For a very large sample size with normally distributed data the accept/reject criterion should be less than 1/1.96. = 0.51.) Table 3 lists the "two tailed" 95% confidence limits from Student's tables for sample sizes between 2 and ∞.

The application of student's t to these statistics is not straightforward. It might be more accurate to compute standard errors of the x and y components of the vector amplitude A, in which case the application of Student's t tables would be more conventional. However, the individual deviations along the x and y directions are not statistically different, and by using an RMS value of these two deviations we have a single estimator of the uncertainty of the mean amplitude and phase.

f	0.05		*f*	0.05		*f*	0.05
2	4.303		11	2.201		20	2.086
3	3.182		12	2.179		21	2.080
4	2.776		13	2.160		22	2.074
5	2.571		14	2.145		23	2.069
6	2.447		15	2.132		24	2.064
7	2.365		16	2.120		25	2.060
8	2.306		17	2.110		30	2.042
9	2.262		18	2.101		60	2.000
10	2.228		19	2.093		∞	1.960

Table 3. Values of Student's t_f (0.05) for a two tailed distribution, for various degrees of freedom f.

Values of the amplitude *A*, standard deviation σ, and the error ratio $r=S/A$ for the port of Point Atkinson, British Columbia are presented in Figures 3a to 3c. Amplitudes and phase lags for the years 1977, 1978, 1983 and 1992 were removed, after considering the anomalies in these years noted earlier. When 18 years of data are available, the error ratio $r=S/A=0.48$ is the cut-off criterion for 95% confidence. With 14 years, the cut-off is 1/2.145=0.47.

Amplitudes of the first 46 constituents are plotted in Figure 3a, and show a typical pattern for constituents on the west coast of Canada: The tide is mixed diurnal and semi-diurnal, and a few constituents dominate the plot. (The amplitudes are clipped at 0.3 metres to emphasize the smaller constituents.) Standard deviations of the amplitudes presented in Figure 3b reveal a general decrease toward higher frequencies, which continues into the 22 higher frequency constituents not plotted here. Largest errors are in the long-period bands (fortnight, month, semi-annual and annual) due to El Niño and wind and air pressure influences, which have more energy at these periods. Errors in all these long-period bands are greater than in either the K_1 or M_2 constituents. The lesson here is simple: one should be careful when including long-period constituents in predictions.

Finally, Figure 3c plots the values of the error ratio $r=S/A$. Four of the long period constituents fall outside the 95% confidence test. CRAWFORD (1982) noted that the long period constituents could be resolved better if the portion of the sea level record coherent with air pressure changes was removed from the time series. The constituents OQ_2 and R_2 are the only main tidal potential constituents in the semi-diurnal band to fail the test. Among the shallow water constituents, several higher frequency ones not plotted have error ratios above the threshold. The high error ratio of H_1 might be attributed to an erratic variation of M_2, which it serves to modulate.

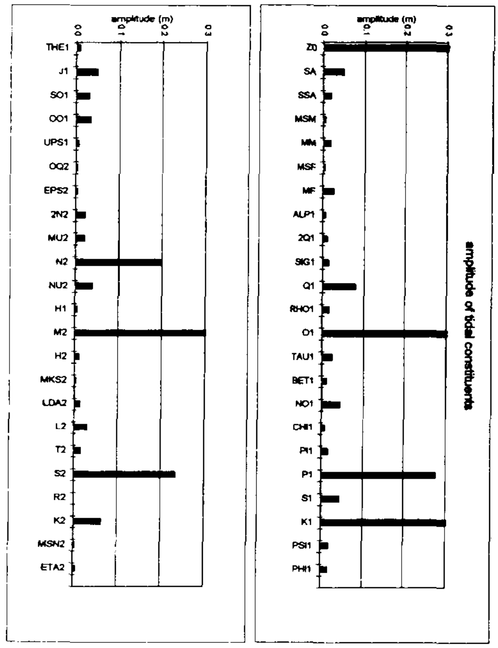

FIG. 3a.- Amplitude of tidal constituents at Point Atkinson, B.C., based on 14 years of observations between 1975 en 1991. Values are clipped at 0.3 m to emphasize smaller values.

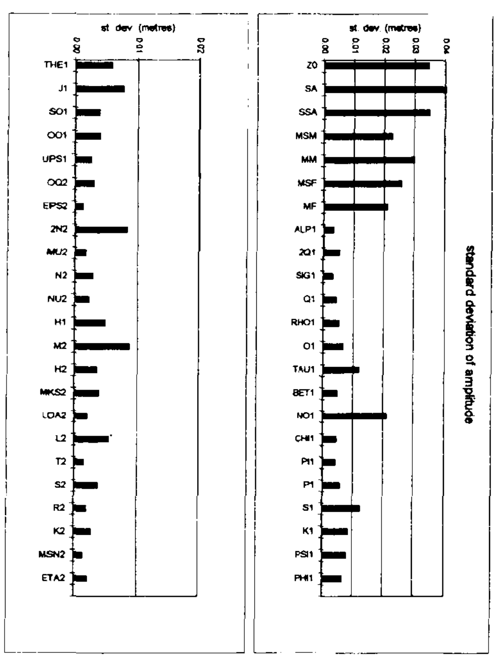

FIG. 3b.- Standard deviation of the amplitude of tidal constituents at Point Atkinson, B.C., based on 14 years of observations between 1975 and 1991.

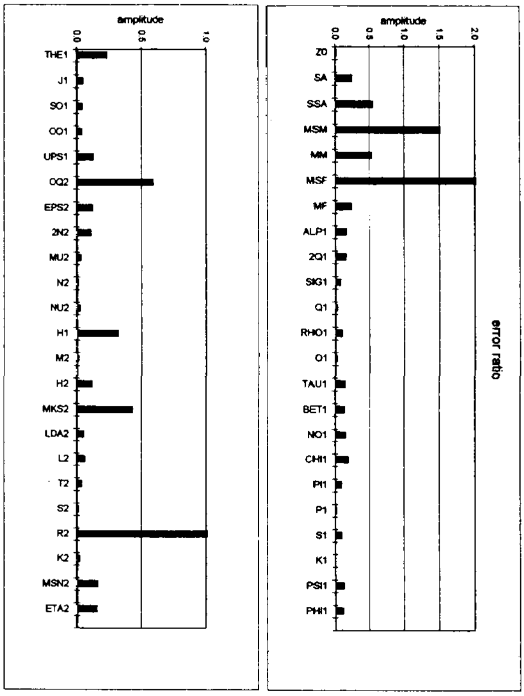

FIG. 3c.- Error ratio r: S/A of tidal constituents at Point Atkinson, B.C., based on 14 years of observations between 1975 and 1991.

4. Satellite modulation.

The standard set of 68 constituents have inter-annual variations denoted as "satellite modulation". Each (or most) of these 68 constituents is actually a cluster of constituents whose first three DOODSON numbers are identical, and whose name is assigned to the largest constituent within the cluster. The smaller constituents in the cluster modulate the larger one.

Most tidal analysis programs automatically account for the satellite modulation by assuming that the tidal response of the local oceanic basin is uniform across the constituent's frequency band. A problem arises when the satellite modulations of the shallow water and astronomical tides are not the same for constituents at the same frequency. If frictional effects at a port are significant, and the analysis program has assumed that the modulation is solely due to the astronomical potential, the resulting harmonics will vary in a cyclical manner from year to year. (Pairs of shallow water and astronomical constituents at the same frequency are listed in Table 4.)

Astronomical	Shallow water
Msm	λ_2-M_2 β_1-O_1
Mm	N_2-M_2 O_1-Q_1
Msf	S_2-M_2 O_1-P_1
Mf	K_2-M_2 K_1-O_1
Q_1	NK_1
O_1	MK_1
TAU_1 (τ_1)	MP_1
NO_1	NO_1
P_1	SK_1
K_1	MO_1
J_1	MQ_1
SO_1	SO_1

Astronomical	Shallow water
OQ_2	OQ_2
EPS_2 (ε_2)	MNS_2
$2N_2$	O_2
MU_2 (μ_2)	$2MS_2$
N_2	KQ_2
GAM_2 (γ_2)	OP_2
M_2	KO_2
L_2	$2MN_2$
S_2	KP_2
K_2	K_2
MO_3	MO_3
M_3	NK_3

Table 4. Shallow water constituents that may mask astronomical constituents. The long period constituents are listed by CRAWFORD (1982), others are from FOREMAN (1977) based on notes by GODIN (personal communication).

In addition to the complications noted above, there may be modulations due to third order tides. Several constituents such as Q_1, NO_1, J_1, $2N_2$, N_2, and L_2 have third order satellite constituents whose amplitude and phase are normally difficult to predict (GODIN, 1986).

These two effects are revealed in Figure 1 as inter-annual variations in amplitude and phase of several constituents, with regular (although small) variation observed in NO_1, L_2 and J_1 at Point Atkinson. Each of these constituents can be modulated by both of the mechanisms noted above. Although this spreadsheet is a tool to identify these variations, it provides no simple cures. If such variability is large, one should re-define the constituent as a shallow water tide, or use a full satellite analysis such as described by FOREMAN and NEUFELF (1991), or FRANCO (1995).

The largest deviation of any of the diurnal and semi-diurnal constituents is found in 1982 in the NO_1 constituent. Amplitudes of this constituent are high in 1982 along the British Columbia coast, as shown in the graphs of amplitude and phase in Figure 4 for four of these ports. This unusual behavior is likely linked to a quirk in the satellite modulation of the astronomical potential of this constituent in 1982. NO_1 is an unusual constituent, in that its satellite modulation can be large in amplitude, and although the modulation is dominated by the constituent M_1, which differs in frequency by one cycle in 4.4 years, there is also a third order constituent whose behavior may be erratic. Calculations of the theoretical satellite modulation using the standard analysis software revealed that the amplitude of the astronomical potential of this constituent at the latitude of Point Atkinson in October

1982 was only 44% of its nominal value, and its average value throughout the year was 54% of its nominal value.

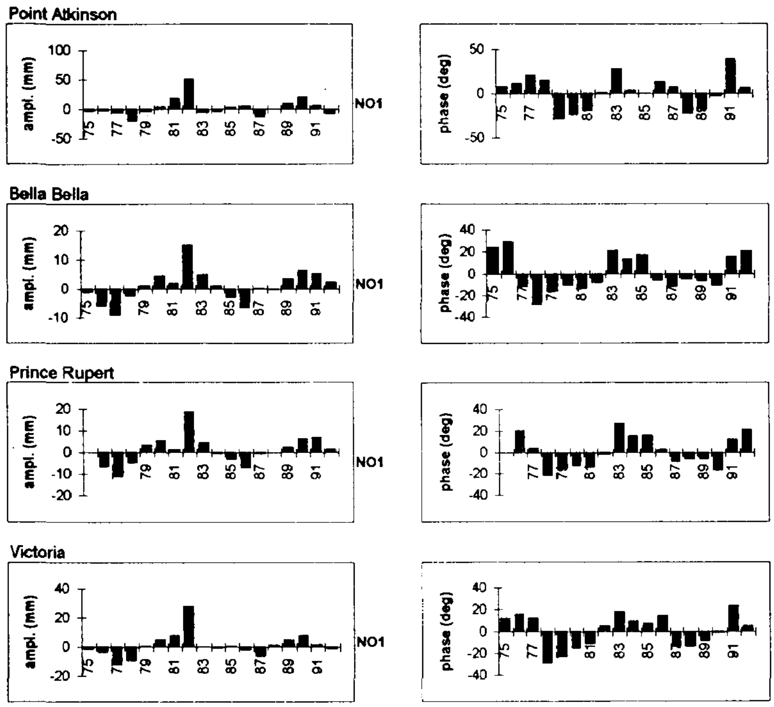

FIG. 4.- Plots of the deviation of amplitude and phase of NO_1 at four ports in British Columbia.

However, it is likely that at Point Atkinson the satellite constituents of NO_1 do not follow the potential behavior. The analysis of thirty-eight years of tidal data at Victoria, British Columbia by FOREMAN and NEUFELF (1991), revealed that the third order satellite constituent of NO_1 differs significantly from the expected behavior. The actual satellite constituent amplitudes and phases determined by this thirty-eight year analysis show that the amplitude of NO_1 reduced to 64% of its nominal value in November 1982 (M. FOREMAN, personal communication). The difference of 20% between these ratios of satellite modulation could explain the anomalous amplitudes at Victoria in 1982, and other British Columbia ports such as Point Atkinson.

CONCLUSION

A procedure is developed to use multiple years of harmonic analyses to improve tidal prediction accuracy at a port where many years of observations are available. This technique enables one to readily determine

- whether a tidal constituent should be included in subsequent tidal predictions at the port,
- if a particular year of data is of poor quality for tidal analysis,
- if an inter-annual trend in amplitude and phase is present in any constituent,
- if shallow water tides are co-existing with main tidal constituents,
- a master set of tidal constituents for a particular port.

Acknowledgments

I appreciate valuable comments from M. FOREMAN, R. HARE and M. WOODWARD. The tidal analyses were processed by A. MA of the Canadian Hydrographic Service.

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