TEST RESULTS OF DIA: A REAL-TIME ADAPTIVE INTEGRITY MONITORING PROCEDURE, USED IN AN INTEGRATED NAVIGATION SYSTEM

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Abstract

A practical method for real-time kinematic position determination and Quality Control (QC) in (integrated) navigation systems is presented as a combination of an extended iterated KALMAN Filter (KF) and the Detection, Identification and Adaptation (DIA) testing procedure for integrity monitoring as developed by the Delft University of Technology. DIA is a real-time recursive QC tool which can be used on multi-sensor integration. There will be no degradation in the number of sensors used by the navigation system, when applying the DIA theory to possible arising errors.

Test results are presented of the KF&DIA procedure, which was implemented in the software of the survey vessel HNIMS BUYSKES of the Royal Netherlands Navy. The results of DIA are evaluated by comparing the position quality (precision and reliability) of the KF&DIA procedure with the solution of a standard integrated Least Squares (LS) position with F-test and w-test (DataSnooping, DS) as QC-tools. This analysis shows that the use of a KALMAN Filter in combination with DIA gives more precise results (factor $\approx$ 1½) when compared to the Least Squares method with F-test and w-test. The reliability also increases, especially in cases where multiple errors in observations at one epoch occur. In general the quality of the KF&DIA solution is less influenced by errors than the LS&DS solution.

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1. INTRODUCTION

Hydrographic surveys performed by the Hydrographic Service of the Royal Netherlands Navy do not only require automatic data processing, but also need automatic real-time quality control of the position information, since any measurement process will contain errors. A KALMAN Filter (KF) in combination with the Detection Identification Adaptation (DIA) testing procedure meets these conditions. A KF makes optimal use of all available position information, which means besides positioning sensors, also dead reckoning sensors, and can filter out high frequency movements and irregularities. In this paper both a dynamic and a measurement model are presented, followed by the extended KF equations. The quality control procedure DIA uses results from the KALMAN filter for the performance analyses. This analyses procedure consists of three steps: the detection step, in which model misspecifications (including observation errors) are detected, the identification step in which the error type is identified and the adaptation step, in which the bias in the state vector caused by the eventual (model) error is eliminated. If after the first two steps (DI) errors are still present, these steps are repeated iteratively. When all errors are detected and identified, the adaptation step is performed with a matrix of the appropriate dimensions.

During sea trials, input to the system were Hyper-Fix readings, DGPS position, speed log and gyro readings. The KF&DIA solution is compared to the standard Least Squares (LS) & Data Snooping (DS) solution with respect to track position stability, precision and reliability.

In this paper the equations, formulae and test statistics are given as an algorithm, they are not derived. For a complete theoretical background and the derivations one is referred to TEUNISSEN [1,2,3,4] and SALZMANN [5].

2. THE KALMAN FILTER

A KF can generate a real-time estimate of the state of a ship, or moving vehicle in general, using the information from a combination of several sensors. It is a recursive algorithm, based on linear systems. Since most systems in navigation are non-linear, discrete linearized forms of the measurements models are used in the extended KALMAN Filter theory. Iterations are performed to obtain an optimal estimation of the defined state-vector. A KALMAN Filter includes a measurement model, dynamic model and stochastic model, which are presented here first.

2.1 - The models

The models used in the sea trials are summarized in matrix form in the Appendix. In this section they will be described in formula form.

2.1.1 - The dynamic model

In the test case a state vector x of 6 elements is used to describe the two dimensional motions of the ship. The motions are described in an earth based coordinate system. The elements of the state vector, the unknowns, are position in two directions (Easting and Northing), decoupled velocity in two directions (east and north) and biases of gyro and log:

$$\mathbf{x} = (E \quad N \quad V_E \quad V_N \quad gyro_{bias} \quad log_{bias})^T \tag{1}$$

The dynamic model in use is a model of constant velocity vector and constant biases in log and gyro. This model is inserted in the transition matrix Φ.

Disturbances and model errors are taken into account by using a disturbance model Q_d as white noise with zero mean. As a disturbance model for the position and velocity elements, acceleration is taken as a random (no constant) function (see [4]). The standard deviations can be estimated as one third or one half of the estimated maximum values. In a turn the acceleration a is maximum and can be estimated from:

$$a = \frac{v_g^2}{R} \tag{2}$$

where v_g is the ship's ground velocity and R the radius of the turn. The standard deviation of the acceleration is set at 0.25m/s². The maximum gyro-bias is estimated as [6]:

$$gyro_{bias} = 2\arctan(\frac{v_{dr}}{v_p}) * \frac{v_g}{\pi R} \tag{3}$$

Here v_{dr} is the drift velocity caused by wind and current, v_p the vessel velocity caused by its propulsion equipment. The standard deviation of the gyro bias is chosen to be 0.7 degrees.

The systematic speed log measurement error in a turn is according to [6]:

$$log_{bias} = 2 \, v_c * (\frac{v_g}{\pi R}) \tag{4}$$

Where v_c is the absolute current velocity, the standard deviations for the log bias is chosen to be 0.02 m/s². Correlation between the different disturbances is neglected, since these are very small and difficult to estimate. Correlation in time is not taken into account, since these disturbances are mostly due to imperfect modelling of the system dynamics [5]. Since good modelling is assumed, accounting for white noise will be sufficient.

2.1.2 - The measurement model

In this study the observables which may serve as input to the system can be range readings (of earth based systems), hyperbolic readings, (D)GPS positions, log and gyro readings. These are stored in the measurement vector y. The observation equations and their linearized equivalents are given here. The observation equations are linearized with respect to the elements of state vector x.

The observation equation for hyperbolic readings is:

$$I = \frac{r_{ps} + r_p - r_s}{\lambda} + k \tag{5}$$

where I is the lane reading, r_{ps} is the distance from the primary to the secondary station, r_p the distance from the unknown position to the primary station, r_s the distance from the unknown position to a secondary station, λ the wavelength and k is a value to obtain a zero minimum lanecount (fixed pattern offset).

The linearized observation is as follows:

$$\Delta I = (\frac{E^0 - E_p}{r_p^0} - \frac{E^0 - E_s}{r_s^0})\Delta E + (\frac{N^0 - N_p}{r_p^0} - \frac{N^0 - N_s}{r_s^0})\Delta N \tag{6}$$

where E^0, N^0 are provisional coordinates of the unknown point and E_p, N_p, E_s, N_s are Eastings and Northings of the primary and secondary station respectively.

The used covariance model of Hyper-Fix readings is described in [7]. It is based on the number of paths the signal is travelling. If the standard deviation of a (reversed) primary pattern (the two transmitters are directly phase locked) is σ, then the standard deviation of a secondary pattern (the two transmitters are indirectly phase locked) is 4/3 σ. The correlation corresponds with the number of paths the patterns have in common: for 2 primary patterns 2 of the six paths coincide so a correlation factor of 2/6 = 1/3 is used and the covariance is 1/3 * σ * σ.

From earlier calibrations σ is estimated to be .017 lane during full daylight. The correlation is positive if the lane count of both patterns increases or if the lane count for both patterns decreases towards their common station. The correlation is negative if the lane count of one pattern increases towards their common station while the lane count of the other pattern decreases towards the common station.

Since the coordinates of the DGPS receiver are used as input for the system, the observation equation is trivial.

Whereas the used DGPS receiver does not give separate variance values for the coordinates, the covariance model of the DGPS is derived by analysing a huge number of observables of the static monitoring site of the Survey Department of the Ministry of Transport, Public Works and Water Management. The position

observations are analyzed using the theory of maximum likelihood in a robust sense [8,9]. This method iterates towards an optimal covariance estimation by taking into account weights to the coordinate observations. Therefore observation outliers will hardly influence the final covariance values. The result [10] is an estimation of the variance of 0.6^2 metres^2 for the northing coordinate and a variance of 0.4^2 metres^2 for the easting coordinate. No correlation between these coordinates is found. These values are assumed valid for the southern part of the North Sea, the area where the vessels of the Hydrographic Service normally operate. The static variance values for the DGPS-coordinates are multiplied by 3^2, because they are used under kinematic conditions.

For range readings r the observation equation reads:

$$r = \sqrt{(E_s - E)^2 + (N_s - N)^2} \; * \; SF \tag{7}$$

where E_s, N_s are coordinates of a range station and E and N the unknown coordinates; SF represents a scalefactor, needed to transform the computed range in the projection to an earth based range.

Linearized:

$$\Delta r = \frac{E^0 - E_s}{r_s^0}\Delta E + \frac{N^0 - N_s}{r_s}\Delta N \tag{8}$$

The variance of the range-readings depends on the distance from the vessel to the stationary stations and therefore of the system used. For instance, a variance value of 1^2 m^2 is chosen in case of the short range system Micro-Fix while for Syledis (medium range) a variance of 3^2 m^2 is chosen.

A speed log measures the velocity v of the vessel. The observation equation for the longitudinal velocity is:

$$v = V_E \sin\phi + V_N \cos\phi - \log_{bias} \tag{9}$$

If the log measures the velocity through the water the $\log_{bias}$ is partially due to the current drift and partially to the systematic error of the log itself. If the log measures the velocity relative to the ground, the $\log_{bias}$ only represents a systematic error of the log.

The linearized observation equation:

$$\Delta V = \begin{aligned} & (\sin\phi^0 + \frac{V_N^0}{V_E^{0^2} + V_N^{0^2}} V_E^0 \cos\phi^0 - \frac{V_N^0}{V_E^{0^2} + V_N^{0^2}} V_N^0 \sin\phi^0) \Delta V_E \; + \\[4pt] & (\cos\phi^0 + \frac{V_E^0}{V_E^{0^2} + V_N^{0^2}} V_N^0 \sin\phi^0 - \frac{V_E^0}{V_E^{0^2} + V_N^{0^2}} V_E^0 \cos\phi^0) \Delta V_N \; + \\[4pt] & (V_N^0 \sin\phi^0 - V_E^0 \cos\phi^0) \Delta \text{gyro}_{\text{bias}} - \Delta \log_{\text{bias}} \end{aligned} \qquad (10)$$

where ϕ^0 represents a provisional value of the true heading, V_E^0, V_N^0 are provisional velocities of the ship in eastern and northern direction.

The variance value of log readings is set to a constant value of $0.5^2 \; m^2/s^2$, obtained by analysing several logging tapes.

For gyro readings the observation equation reads:

$$\phi = \arctan\frac{V_E}{V_N} - \text{gyro}_{\text{bias}} \qquad (11)$$

The $\text{gyro}_{\text{bias}}$ is the drift between the ground course and the measured heading. It contains the effects of wind drift, current drift, waves and coriolis force. In equation 11 it is assumed that the gyro-northern direction coincides with the true northern direction that is, an eventual systematic error is corrected in advance (calibrated) or does not exist. The linearized observation equation reads:

$$\Delta\phi = \frac{V_N^0}{V_E^{0^2} + V_N^{0^2}} \Delta V_E^0 - \frac{V_N^0}{V_E^{0^2} + V_N^{0^2}} \Delta V_N^0 - \Delta \text{gyro}_{\text{bias}} \qquad (12)$$

The variance value of the gyro is set to 1.5^2 degrees^2.

2.2 - Initialization

To start the KF, an estimate of the first state and its corresponding covariance matrix is needed. This is done by using the Least Squares method at the first two epochs, taking all the observables of the available position sensors as input. At the third fix, the first epoch the KF is used, the log-reading is introduced as an observable, since two computations are needed to make an estimation of the log bias. The gyro reading is introduced at the 5th fix, because it is best to compute the gyro bias from two filtered positions.

The formulas of the Least Squares are briefly discussed, a derivation is not given. The position of the first two fixes is computed from the so-called known design matrix A and the covariance Q_y of the observations. The design matrix contains the relation between the vector of observables y, containing m observables and the vector of unknowns x, which contains n unknown parameters.

The observables are assumed to be normally distributed.

The elements of the matrices are derived from the above described measurement model.

The Least Squares solution reads:

$$\hat{x}_t = (A_t^T \, Q_{y_t}^{-1} \, A_t)^{-1} \, A_t^T \, Q_{y_t}^{-1} \, y_t \qquad\qquad t = 1, 2 \qquad\qquad (13)$$

The corresponding covariance matrix of the position is estimated as follows:

$$Q_{\hat{x}_t} = (A_t^T \, Q_{y_t}^{-1} A_t)^{-1} \qquad\qquad (14)$$

Which yields standard ellipse and $2d_{RMS}$.

The solution for the least squares residuals $\hat{e}$ with its covariance matrix $Q_{\hat{e}_t}$ reads:

$$\hat{e}_t = y_t - A_t \hat{x}_t, \quad Q_{\hat{e}_t} = Q_{y_t} - A_t Q_{\hat{x}_t} A_t^T \qquad\qquad (15)$$

In our test, the observations used can be any (D)GPS position and all the available hyperbolic and range observables of earth based systems. Obviously log or gyro readings can not be processed with the standard Least Squares method, since this is a computation method for static positioning. In the case of non linear observation equations, like with hyperbolic readings, the matrix A contains the *linearized* observation equations and the vector y the *(calculated - observed)* observations. The final solution (state vector) is obtained through an iteration process.

The Delft testing approach (see for instance [11]) is followed to check for outlying observables. If the F-test value computed from the Least Squares residuals $\hat{e}$ and covariance matrix Q_y, exceeds a critical value corresponding to a chosen significant level, the w-test values are computed for each observation, to detect the cause of the rejection. The number of redundant observables must at least be 2 for a meaningful computation of the w-test. The w-test statistic has a normal distribution with a standard deviation of 1. If the largest w-test value exceeds the critical value belonging to the value of the level of significance *a* of the normal distribution, the corresponding observation is rejected and the computation is repeated. This process, which is called DataSnooping (DS), is repeated until the F-test value is accepted or only three observations are left. In this test the critical value is chosen to be 0.1% (recommended by Delft University of Technology), a choice this small to be sure not to reject any correct observables.

The formula for the F-test reads:

$$F_t = \frac{\hat{e}_t^T Q_{y_t}^{-1} \hat{e}_t}{(m-n)} \quad (m-n) \text{ is the number of redundant observations} \qquad (16)$$

The F-test value has the F-distribution and its critical value depends on the number of degrees of freedom and the level of significance α; e.g. if $\alpha = 0.1\%$, 3 degrees of freedom yield a critical value of 5.42.

The formula for the w-test reads:

$$w_i = \frac{-c_i^T Q_{y_i}^{-1} \hat{e}_t}{\sqrt{c_i^T Q_{y_i}^{-1} (Q_{y_i} - A_t Q_{\hat{x}_t} A_t^T) Q_{y_i}^{-1} c_i}} \tag{17}$$

with

$$c_i = (0 \ . \ . \ 1 \ . \ . \ . \ 0)^T \tag{18}$$
$$ i$$

(From now on the abbreviation "LS&DS" will be used for this method.)

From the position covariance matrix (14) resulting from the Least Squares method, the initial covariance matrix of the state vector for the KALMAN Filter process is derived. For the Easting, Northing and their correlation the corresponding values of covariance matrix of the LS method (the 2x2 matrix $Q_{\hat{x}}$) are taken. For the variances of the velocity in east and north direction, initial values of 1^2 m^2/s^4 are chosen. For the variances of the log-bias and gyro-bias the values for the variances are taken the same as the gyro and log readings as stated in the section 'The measurement model'. No correlation between the unknowns is assumed, except the one between the Easting and Northing coordinates as derived from the Least Squares solution.

2.3 - Time-update

In the first step of the KALMAN filter, the state vector $\hat{x}_{k,k-1}$ and its covariance matrix $Q_{\hat{x}_{k,k-1}}$ for time k are predicted from the state vector $\hat{x}_{k-1,k-1}$ and covariance matrix $Q_{\hat{x}_{k-1,k-1}}$ at time k-1, that is without the use of the observations y_k or any manoeuvring model. For a treatment of a dynamic prediction model based on ship manoeuvring characteristics, see [12].

Here the prediction is only computed with a dynamic model, which entails the relation between two subsequent state vectors.

Hence the estimator of the predicted state vector is obtained:

$$\hat{x}_{k,k-1} = \Phi_{k,k-1} \, \hat{x}_{k-1,k-1} + d_k \tag{19}$$

As stated before, the mean value of the disturbance d_k is taken zero.

The covariance matrix of the estimator of the predicted state vector is obtained by applying the error law of propagation:

$$Q_{\hat{x}_{k,k-1}} = \Phi_{k,k-1} \, Q_{\hat{x}_{k-1,k-1}} \Phi_{k,k-1}^{T} + Q_{d_k} \qquad (20)$$

The first part in this equation accounts for the transition, the latter for inaccuracies in the prediction due to model errors and external disturbances such as wind and current. The values are described in the section 'dynamic model'. Although the transition matrix $\Phi_{k,k-1}$ holds for straight line movement, curve movements are accounted for by the values in the Q_{d_k}, the noise on the dynamic model.

2.4 - Measurement update

In the next step of the KALMAN filter, the observables y from epoch k are processed; it is called the measurement update. First the predicted residuals v_k (or innovations) are computed. These are defined as the differences between readings and predicted readings. The predicted readings are computed by substituting the predicted elements of the state vector, as calculated in the time-update step of the KF, in the observation equations. Since the LS cannot contain observations of gyro and log, the design matrix A for the KF has a larger dimension than the A-matrix of the LS of formula 13. The predicted residuals:

$$v_k = y_k - A_k \, (\hat{x}_{k,k-1}) \qquad (21)$$

The covariance matrix of the predicted residuals is:

$$Q_{v_k} = Q_{y_k} + A_k \, Q_{\hat{x}_{k,k-1}} A_k^{T} \qquad (22)$$

These predicted residuals and their covariance matrix will be used in the DIA procedure for quality control.

Next the so-called KALMAN gain matrix is computed:

$$K_k = Q_{\hat{x}_{k,k-1}} A_k^{T} \, (Q_{y_k} + A_k \, Q_{\hat{x}_{k,k-1}} A_k^{T})^{-1} \qquad (23)$$

The gain matrix is the weighting factor for the difference between the readings and the predicted readings.

The next step in the KALMAN filter procedure is the actual filterstep in which the K_k weighted difference is added to the predicted state vector in order to get the optimal state vector:

$$\hat{x}_{k,k} = \hat{x}_{k,k-1} + K_k \, v_k \qquad (24)$$

The covariance matrix of the filtered state vector is:

$$Q_{\hat{x}_{k,k}} = (I - K_k A_k)Q_{\hat{x}_{k,k-1}} \tag{25}$$

Where I represents the unity matrix. As hyperbolic lines of position, ranges, gyro and log all produce non-linear observation equations, linearized observation models are used. Consequently the differences between estimated provisional values and the optimal estimators of the state vector are computed. It denotes the measurement update has to be repeated with new provisional values, so the $\hat{x}_{k,k}$ from equation (24) is used as input to equation (21) and $Q_{\hat{x}_{k,k}}$ from equation (25) is used for formula (22), until the defined stopcriterion is reached. The complete iteration process is shown in Figure 1.

Initialization with
least squares
compute vector x

datasnooping with
test statistic t

Time update:
predict the state vector estimators: x
and its covariance matrix: Qx

Measurement update:
compute:
the predicted residuals: v
their covariance matrix: Qv
the Kalman gain matrix: K
the optimal estimators of
the state vector: x

diff.<
no   stopcriterion

yes

compute the covariance matrix
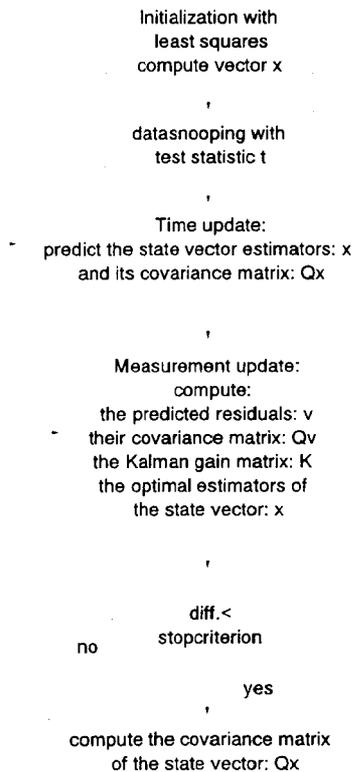of the state vector: Qx

FIG. 1.- KALMAN Filter process.

3. DIA QUALITY CONTROL:
DETECTION, IDENTIFICATION AND ADAPTATION

The DIA testing procedure for real-time validation of integrated navigation systems consists of the following three steps:

1. Detection: In this step the validity of the working hypothesis is tested. An overall model test is performed to check if an unspecified model error has occurred.

2. Identification: After a model error has been detected the cause has to be found. This implies a search for the most likely type of model error has to be performed.

3. Adaptation: After identification of a model error adaptation of the navigation filter is needed to eliminate the bias in the state vector.

The DIA procedure is an extension of the quality control for static positioning, see equations (16) and (17). The detection step can be compared with the F-test and the identification step can be compared with the w-test. But whereas observables with errors are rejected in the LS&DS approach, the adaptation step of the DIA accounts for errors. The QC tools of the LS use the residuals as input, whereas the DIA procedure operates with predicted residuals. Under the working hypothesis, the predicted residuals are Gaussian distributed with mean zero and covariance matrix Q_v. Besides the advantage of the predicted residuals and their covariance matrix being already computed in the KALMAN filter, the DIA procedure can be implemented recursively in a very efficient manner.

3.1 - Detection

The overall model tests can either be global or local. Global tests consider several epochs simultaneously, and are capable of detecting errors which are present during a certain time span. Local tests relate to a single epoch and thus are genuinely real-time. For this reason and the computational burden which arises when using global tests, only local tests are used in the performed test procedure. A special note will be made on how to deal with global errors. The overall model test does not only test for errors in the observations but also tests the validity of the mathematical models. During the design phase the underlying mathematical models have been tested thoroughly, therefore a detected error is assumed to be an observational error. The test statistic T for epoch k and normalized for the number of observations m_k is computed from the predicted residuals v_k and their covariance matrix Q_{v_k}, both already available from the KALMAN filter, and reads:

$$T^k = \frac{v_k^T \ Q_{v_k}^{-1} \ v_k}{m_k} \tag{26}$$

This test statistic is now used to perform a Local Overall Model (LOM) test: an observational error is considered present at epoch k if the test statistic T^k has a larger value than the upper a probability point of the central F-distribution with m_k, ∞ degrees of freedom. A choice of a = 0.1 % with 7 observations, (7, ∞ degrees of freedom) gives a critical value of 11.7. If T $\geq$ critical value, there is (in this case with 99.9% probability) an error which can be identified in the next step.

3.2.1 - Single Model Error Identification

With restriction to the measurement model the test statistic t at epoch k for each observation i reads:

$$t_i^k = \frac{c_i^T \ Q_{v_k}^{-1} v_k}{\sqrt{c_i^T \ Q_{v_k}^{-1} \ c_i}} \tag{27}$$

when only specifying single local outliers as alternative hypotheses, c_i is defined as: $(0 \ldots 1 \ldots 0)^T$, the 1 corresponding with the observation to be tested. Local errors can suddenly arise by for instance signal reflection or a change in physical circumstances.

Identification of a single local model error proceeds by calculating t^k for each observation. The observation for which the absolute value of t^k is at a maximum is considered the one containing the error. Once the most likely error has been found, its likelihood needs to be tested. This is done by comparing the absolute value of t^k with the critical value in a double sided normal distribution (the critical value is 3.27 when using a probability of 99.9%). If no further errors are expected or searched for, adaptation of the state vector can now be performed.

3.2.2 - Multiple Model Error Identification

If several observational errors can occur simultaneously, the identification needs to be repeated [13]. The strategy is to remove, step by step, the most likely model errors and test after each step the likelihood of the remaining model errors. The first step in this procedure is the one described above for single model error identification. The next step is to recompute the LOM-test statistic disregarding the observation j containing the error. Instead of explicitly updating the vector of residuals and its covariance matrix, the updated LOM-test statistic can be computed with the formula:

$$T_j^k = T^k - \frac{(t_j^k)^2 - T^k}{m_k - 1} \tag{28}$$

where T^k is first computed with (26).

The test statistic for the remaining observations becomes:

$$t_{i(j)}^k = \frac{c_{i(j)}^T \, Q_{v_k}^{-1} \, v_k}{\sqrt{c_{i(j)}^T \, Q_{v_k}^{-1} \, c_{i(j)}}} \qquad i \neq j \tag{29}$$

Note the jth observation is excluded, $i = 1, \ldots ,(j-1),(j+1), \ldots ,m_k$ and

$$c_{i(j)} = (I - c_j (c_j^T Q_{v_k}^{-1} c_j)^{-1} c_j^T Q_{v_k}^{-1}) * c_i \tag{30}$$

```
                    ┌──────────────────┐
                    │  Kalman Filter   │────────────────────────┐
                    └──────────────────┘                        │
                    ┌──────────────────────┐                    │
                    │  set cycle count p=1 ,                     │
                    │  set nr. of error in cycle p; j(p)=0       │
                    │  compute local overall model               │
                    │  test statistic T  with (26)               │
                    └──────────────────────┘                    │
                                                        no       │
                         ◇ T>=critical value ◇────────────────── │
                              │ yes                              │
              ┌───────────────┴──────┐    ┌──────────────────┐   │
              │ if p = 1:            │    │ If p > 1:        │   │
              │ compute maximum test │    │ comp. maximum test   │
              │ statistic |t| with (27)   │ statistic |t| with (29) for │
              │ for identification of errors   identification of errors for │
              └──────────────────────┘    │ all obs. except j(1),..j(p-1). │
                                          └──────────────────┘   │
                                                   no            │
                         ◇ |t|max >= crit.value ◇────────────────│
                              │ yes                              │
   ┌────────┐                                       no           │
   │ p:= p+1│                  ◇ p < m ◇─────────────────────────│
   └────────┘                      │ yes                         │
                    ┌──────────────────────────┐                 │
                    │ j(p):= nr. of observation with error.       │
                    │ update local overall model test             │
                    │ statistic: Tj(p) with (28)                  │
                    └──────────────────────────┘                 │
                                                   no            │
                         ◇ Tj(p) >= crit.value ◇─────────────────│
                              │ yes
                    ┌──────────────────────────┐
                    │ compute ci(j) with (30)  │
                    │ i = 1,...,(j-1),(j+1),...m │
                    └──────────────────────────┘
```
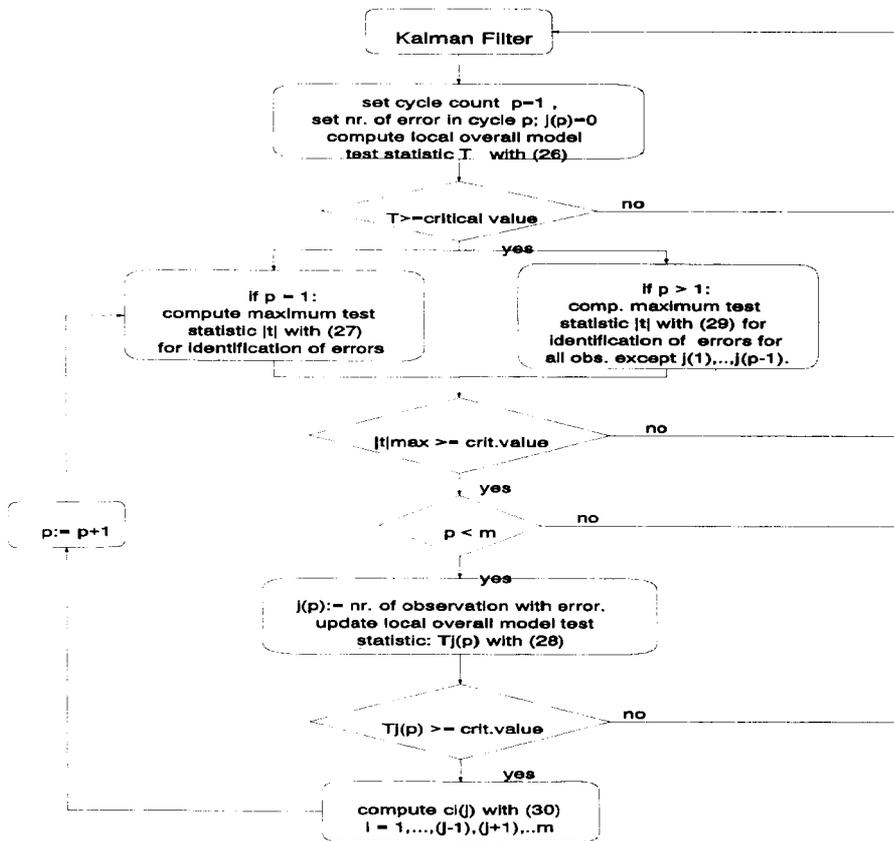
FIG. 2.- Recursive Detection and Identification.

This recursive form of local detection and identification of multiple model errors is shown in Figure 2.

3.3 - Adaptation

After identification of the most likely alternative hypothesis, step three of the DIA procedure, adaptation of the recursive KALMAN filter is performed to eliminate biases in the filtered state vector of the navigation system and update the corresponding covariance matrix.

To eliminate biases an estimate of the identified model error $\hat{\nabla}_k$ is needed. In the case of local identification only, $\hat{\nabla}_k$ can be computed directly from the predicted residuals:

$$\hat{\nabla}_k = (C_k^T \ Q_{v_k}^{-1} C_k)^{-1} (C_k^T \ Q_{v_k}^{-1} \ v_k) \tag{31}$$

where C_k is the matrix constructed from the vectors $c_{i(j)}$, computed earlier (30), which corresponds with the identified model errors. When only single model errors occur, the matrix C_k can be replaced with the vector c_i from equation (27). The state vector of the KALMAN filter can now be adapted:

$$\hat{x}_{k,k}^a = \hat{x}_{k,k}^0 - K_k \ C_k \ \hat{\nabla}_k \tag{32}$$

where $\hat{x}_{k,k}^a$ is the corrected state vector and $\hat{x}_{k,k}^0$ is the biased state vector output by the KALMAN filter. Stated in a different way, $\hat{x}_{k,k}^0$ is the state vector under the null hypothesis and $\hat{x}_{k,k}^a$ is the state vector under the alternative hypothesis at time k.

The adaptation of the covariance matrix of the state vector follows from error propagation as:

$$Q_{\hat{x}_{k,k}^a} = Q_{\hat{x}_{k,k}^0} + K_k \ C_k \ Q_{\hat{\nabla}_k} \ K_k^T \ C_k^T \tag{33}$$

with

$$Q_{\hat{\nabla}_k} = (C_k^T Q_{v_k}^{-1} C_k)^{-1} \tag{34}$$

The corrected state vector can now be used as input for the KALMAN filter for the next fix.

4. THE LANE SLIP PROBLEM,
OR DEALING WITH GLOBAL ERRORS

Slip-type model errors are either errors that build up slowly in time (could be due to a filter in the positioning system or a deteriorating clock), or errors which contaminate an observation continuously over a period of time. They are best identified and adapted by a global test procedure. When using only a local test procedure, special care has to be taken to prevent the state estimator from accumulation of biases. This could occur if a slip is identified as an 'endless' sequence of outliers, thus possibly leading to filter divergence. Due to the adaptation of covariance, the covariance matrix $Q_{\hat{x}}$ of the state vector $\hat{x}$ will be too large to detect errors.

Here two options are considered to deal with laneslips. On the one hand there is *temporary rejection* and on the other hand *observation correction*.

The scheme for *temporary rejection* with hyperbolic systems is as follows: if in the identification step of the test procedure a hyperbolic line of position error is identified and consequently adapted for, the reading of this hyperbolic pattern is tested *before* the next fix, which means before input in the KALMAN filter. The reading is rejected before entering the KALMAN filter if it differs more than a number of times its standard deviation from the predicted position. By rejecting an observation the quality of the position fix deteriorates, but possible divergence of the KALMAN filter, which is unacceptable, is avoided.

The strategy for *observation correction* is as follows: after adaptation for lane errors, the corresponding estimated model errors $\hat{\nabla}$ are applied as corrections to the hyperbolic lines of position and also the covariance matrix Q_y is corrected with the corresponding elements of $Q_{\hat{\nabla}}$.

The error estimates and their corresponding variances are accumulated and stored in so called slipbuffers:

$$\text{slipbuffer}_y = \text{slipbuffer}_y - C_k\hat{\nabla}_k$$
$$\text{slipbuffer}_{Q_y} = \text{slipbuffer}_{Q_y} + C_k Q_{\hat{\nabla}_k} C_k^T \tag{35}$$

And are applied as follows:

$$y_i = y_i + \text{slipbuffer}_{y_i}$$
$$Q_{y_i} = Q_{y_i} + \text{slipbuffer}_{Q_{y_i}} \tag{36}$$

If at some point in time the lane slips are reset manually on the receiver the slipbuffers has to be reset to zero. The slipbuffers and corresponding covariance matrices are reset if for observable i the following equation holds:

$$\frac{|\text{slipbuffer}_{y,i}|}{\sqrt{\sigma_{\hat{y}_i} + \sigma_{\hat{y}_{i-1}}}} < \text{critical value Normal Distribution} \qquad (37)$$

In our example the critical value is (the same as the identification test) 3.27. The advantage of using the observation correction method over the temporary rejection method is no deterioration of the quality, however it takes more computer time, this is why during sea trials temporary rejection was used. An overview of the scheme for observation correction is shown in Figure 3.

adaptation procedure

adaptation for   no
slips?

yes

Update slipbuffers

no   slips reset
to zero?

yes

reset slipbuffers to zero

Apply slipbuffers and
their variances to
the observations and
their variances.

next fix
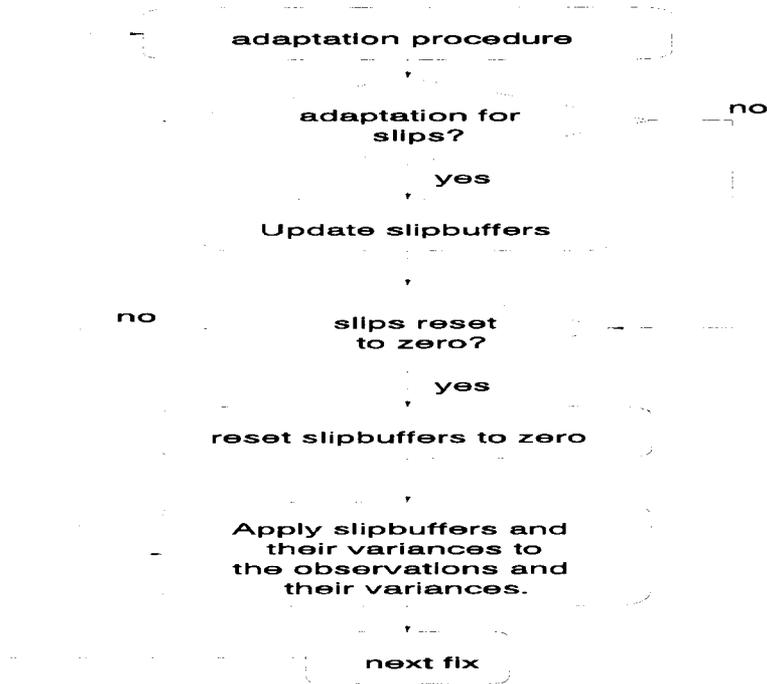
FIG. 3.- Observation correction.

5. SEA TRIALS

In November 1994 a test version of the KALMAN filter and DIA procedure was implemented as the Fortran-77 program POSCOM in the survey software of the vessel HNIMS BUYSKES of the Hydrographic Service of the Royal Netherlands Navy. The program contains the theory as described here, and can also handle the sudden breakdown of a system, by redimensioning the matrices concerned. Aboard the ship, the navigation page of the screen display of this test version showed: the position in UTM-coordinates, the Local Overall Model-value, together with its critical value, the $2d_{RMS}$ (distance Root Mean Square, an indication of the precision of the position), identified errors, type of error and an advised correction for laneslips.

For positioning and navigation the BUYSKES uses a Hyper-Fix receiver, a DGPS receiver, a gyro-compass and an EM log. The readings y of these systems are integrated in the KALMAN Filter. Seven observables are used: 3 Hyper-Fix readings, DGPS position (2 coordinates), gyro reading and log reading. The statevector x consists of 6 states: easting, northing, velocity in north direction, velocity in east direction, gyro-bias, log-bias. The fix interval is one second. The test area was the Texelstroom. In Figure 4 the test area is depicted as area 1, the used Hyper-Fix stations and DGPS station positions are also shown. When using Hyper-Fix, global errors (laneslips) may occur. Due to a filter in the receiver-software, the lane-slip errors slowly build up in time. The filter settings can be changed by changing the so called time-constant on the receiver.
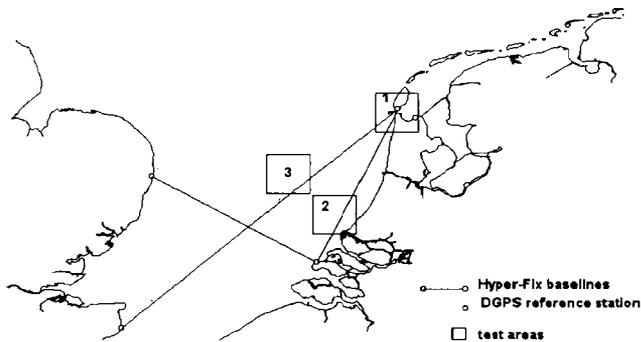
FIG. 4.- Test areas.

For a period of three days the software was tested under various circumstances. The tests were divided into two categories: sensor tests and dynamic tests. Furthermore the program was tested off-line using several datasets of the HNIMS BUYSKES or HNIMS BLOMMENDAL, a sistership.

The sensor tests consisted of: laneslips (manually applied), increasing the minimum elevation of satellites, deselection of a number of satellites, log and gyro readings selecting and deselecting alternatively, selecting and deselecting Hyper-Fix patterns, turning off the differential link of the DGPS receiver and finally varying the time constant on the Hyper-Fix receiver.

The dynamics tests consisted of sailing different manoeuvres with different speeds: straight tracks, curved tracks and accelerations.

The performance of the KALMAN filter and DIA procedure was evaluated on-line and afterwards by visually checking the trackplots of the individual systems and the integrated system. It was needed for the test area to reduce a bias which exists between DGPS and Hyper-Fix. This bias is caused by phase lag in the coastal area, and in some areas exceeds 40 metres. The results presented here are computed off-line by compensating for this bias.

Positions and precision were also computed from the also logged raw observables using integrated Least Squares (LS). Datasnooping (DS) (F-test, equation (16) and w-test, equation (17)) are used to reject observables containing errors. (The same method of positioning is used for the initialisation of the KALMAN filter.) The observables, stored in the measurement vector y, were three patterns of Hyper-Fix and the DGPS position. The parameters to be computed are the two position coordinates. A disadvantage of the LS&DS method, compared to the standard KF&DIA method, is that it cannot use the log and gyro as observables nor can it use a dynamic model. So in this case, the LS method uses two obervables less than the KF.

To show the performance of the KF&DIA approach, a comparison of the KF&DIA with the LS&DS method is made. The following three items are compared: the position/the track, the quality of the position that is the precision by means of the $2d_{RMS}$ (distance Root Mean Square) and the reliability of the position by means of the MDB (Minimal Detectable Bias).

The precision is defined as a measure of an area in which the true position lies with a certain confidence. A measure for the precision of the position is $2d_{RMS}$. The $2d_{RMS}$ -circle is defined as the region with a confidence between 95.4% and 98.1% (dependent on the ratio of the precision of the two individual coordinates) in which the position lies. The value of $2d_{RMS}$ is dependent on the configuration of the positioning system and the a-priori variance values of the measured observables. The $2d_{RMS}$ is computed from the covariance matrix of the state vector:

$$2d_{RMS} = 2\sqrt{\sigma_E^2 + \sigma_N^2} \tag{38}$$

in which σ_E^2 and σ_N^2 are the variance of the Easting coordinate and the variance of the Northing coordinate respectively. These can be found on the first two diagonal places of the computed $Q_{\hat{x}}$ matrix. The $2d_{RMS}$ value is not a measure for the actual errors. The value only represents a measure for random errors, which means it only indicates the precision under the null hypothesis. The value changes if the configuration (geometry) of the position sensors changes.

A measure for the internal reliability of observables is the Minimal Detectable Bias (MDB). This is defined as the minium error in observables which can be detected with a certain probability, and can hence be seen as an instrument to interpret how well model errors can be detected. In this case, a probability (or power of the test) of 80% (recommended by UKOOA and TUD) is chosen. The MDB for the KF&DIA is computed according [14]:

$$MDB_i = \sqrt{\frac{\lambda_0}{c_i^T Q_v^{-1} c_i}} \tag{39}$$

in which λ_0 is the non-centrality parameter, dependent on the values of α and ß. In this case with α = 0.1% and ß = 80%, λ_0 has the value of 17.07 (dimensionless).

The reliability for the LS&DS, using the same percentages of the α and β, is computed with:

$$MDB_i = \sqrt{\frac{\lambda_0}{c_i^T Q_e^{-1} c_i}} \qquad (40)$$

In this test case only a comparison of the internal reliability is made. The external reliability (Minimal Detectable Error, MDE), the highest influence of the MDB on the position, is not presented here.

5.1 - Comparison of KALMAN Filter with Least Squares

Three relevant examples of sailed tracks are shown in this paper. Since all tracks showed the same pattern on comparing reliability and precision, only the quality results of the last track are shown here. The first track is sailed during the sea trials, the other two were sailed by HNIMS BUYSKES or HNIMS BLOMMENDAL in an earlier stage. The plotting scale for each figure may be different, but within one figure the scales of the Easting and Northing coordinates are the same. The coordinates are in the UTM projection. In all figures, the dotted line represents the DGPS track, the dashed line the Hyper-Fix line, the dotted-dashed line the LS&DS solution and the solid line the KF&DIA solution.

Figure 5 shows an example of the tracks calculated with Hyper-Fix (dashed), DGPS (dotted) and the KF&DIA solution (solid line). The track shown here, is sailed during one of the three test days. When sailing this track, there were problems with the interfacing of the log-readings, so only 6 observables are used. Since DGPS and Hyper-Fix are almost similar systems, as far as precision is concerned, the integrated track lies almost in the middle of the two measured tracks.
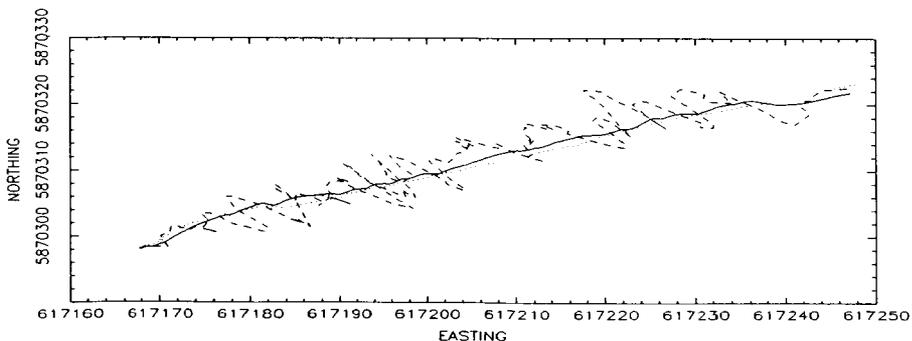

FIG. 5.- Instability track.

The Hyper-Fix track shows an instability, because the positions are just outside the area of good coverage of the Hyper-Fix chain and the time constant was set to its minimum. This instability of Hyper-Fix has little impact, as one can see, on

the positions computed by KF&DIA. The DGPS track shows on the whole a good
performance.

In Figure 6 the KF&DIA track (solid line) is shown again, together with the
integrated Least Squares solution (dot/dash line). The computed Least Squares
positions follow the Hyper-Fix instability pattern, although in a smoothed way. In the
LS&DS solution the Hyper-Fix readings are not rejected by the w-test. The
instabilities are too small to be recognised as errors. The instability of the Hyper-Fix
readings is partly compensated for by integrating Hyper-Fix readings with DGPS
positions.
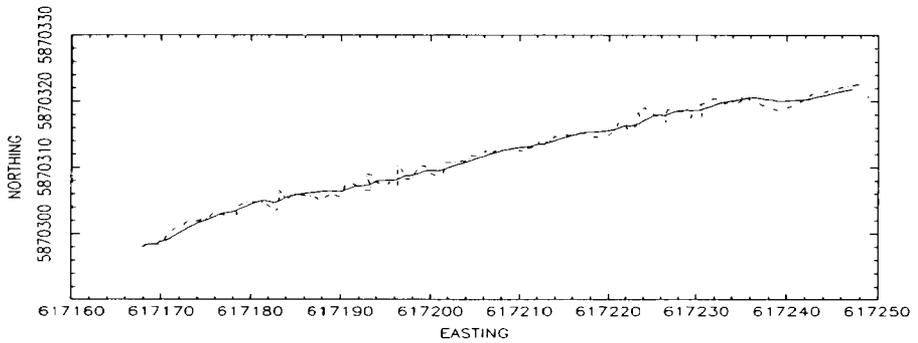
FIG. 6.- Solid line KF+DIA solution, dot-dash LS+DS solution.
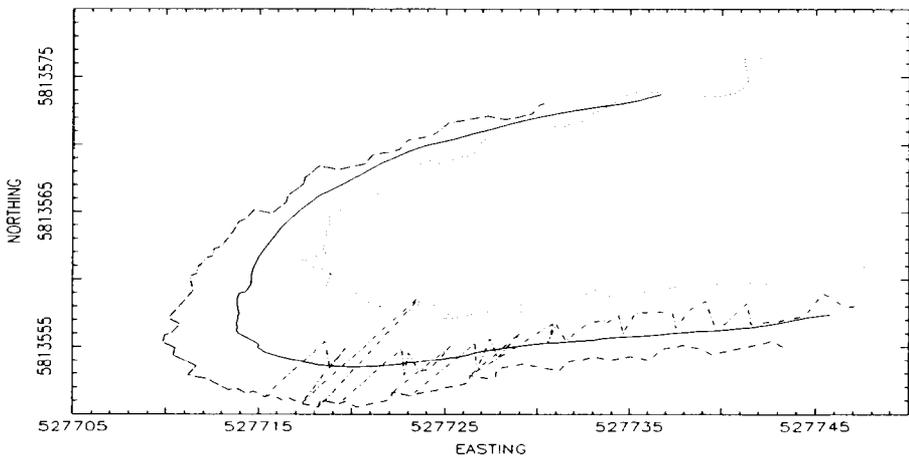
FIG. 7.- Curved track.

Figure 7 shows a part of a very narrow curved track in which Hyper-Fix is
illustrated by a dashed line, DGPS is the dotted line, the LS&DS solution is the
dashed/dotted line and the KF&DIA track is the solid line. This track is sailed in
area 2 of Figure 4. Neither the curvature nor the bias between the two solutions of
positioning systems is a problem for the KF&DIA, although the final computed

positions are influenced by the filter in the Hyper-Fix receiver and the systematic difference between the two systems.

During the turn the impact of the inbuilt filter on the Hyper-Fix solution is seen. The dashed Hyper-Fix track is taking a wider turn, compared to the DGPS track. The LS&DS solution follows the instabilities, and when the bias is too large for the LS&DS approach, the DS method rejects the DGPS position (both of the coordinates) and the LS&DS solution continues the track by following exactly the Hyper-Fix solution.

Figure 8 displays a very unstable track of Hyper-Fix (dashed) which in the end leads to 5 laneslips on the same pattern. This track was not sailed during the tests, but earlier in area 3 and was abandoned during sailing, since the Hyper-Fix lane-slip could not be handled at that moment. The slip occurred during the dusk period, so skywave could be the cause of the laneslips. To test the DIA on multiple errors, an error of 20 metres on the DGPS Easting coordinate (dotted) at fix number 111 (the track consists of a total of 140 fixes) was simulated.
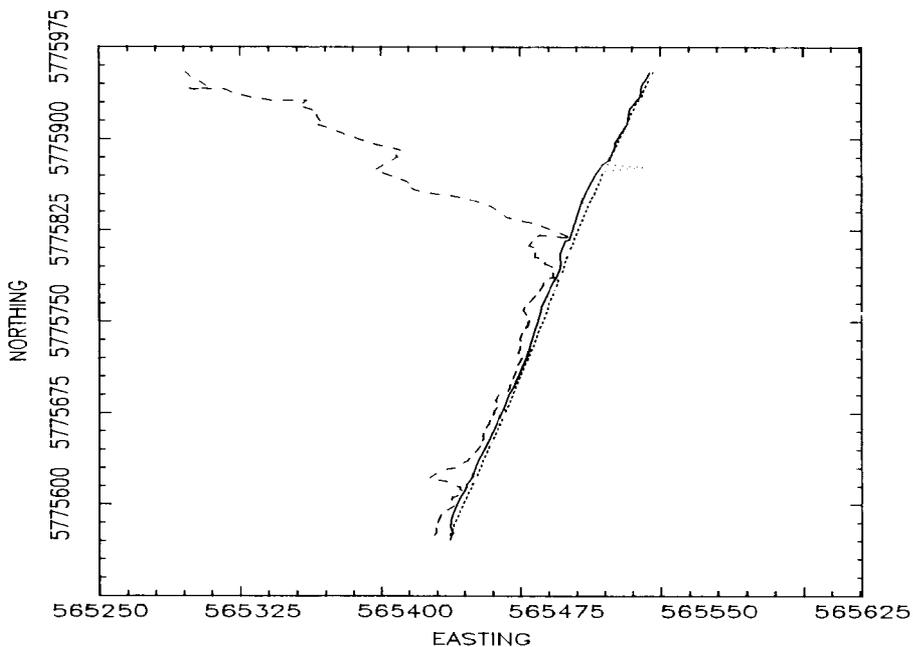

FIG. 8.- Track with Hyper-Fix laneslip.

In this particular case the first laneslip of the total of 5 laneslips was adapted and when the laneslip increased, the unstable Hyper-Fix pattern was rejected.

Here the advantage of using DIA as a quality tool is clearly illustrated. If laneslips occur when KF&DIA is implemented in the survey software, there is no need to stop surveying.

Figure 9 shows the KF&DIA solution again (solid line) together with the LS&DS solution (dashed/dotted).
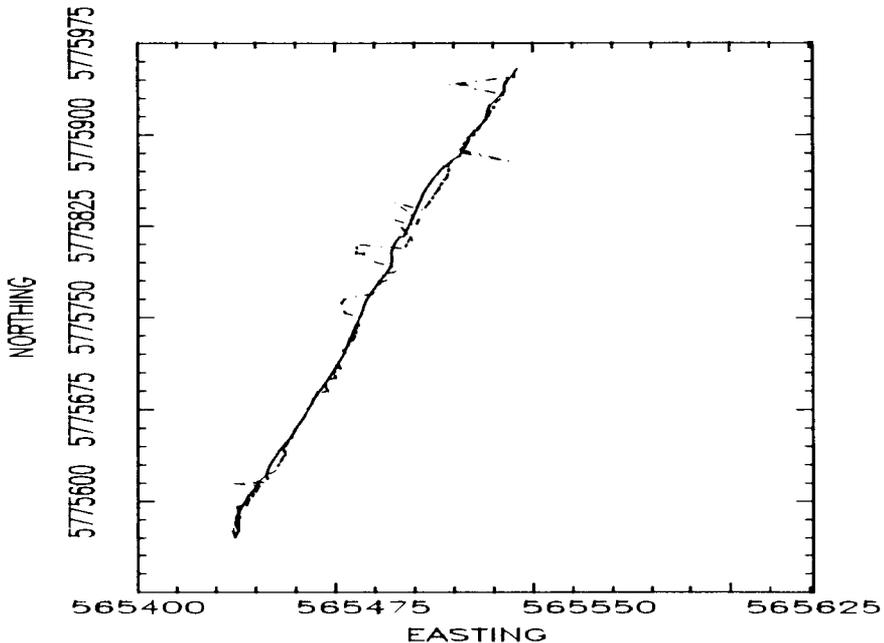

FIG. 9.- LS&DS versus KF&DIA.

The LS&DS approach rejects the Hyper-Fix pattern, containing the lane-slip. Small instabilities in the Hyper-Fix track (see Figure 8), which are not recognised as errors, are followed by the LS&DS solution. At fixnumber 111 the easting error is still visible at the LS&DS track, as a spike. The DS method is not capable of also rejecting the Easting coordinate with the simulated error, besides the also rejected Hyper-Fix pattern containing the laneslip.

When compared to the KF&DIA method, the LS&DS method does not show such a straight track.

In Figure 10 the precision, represented by the $2d_{RMS}$ (in metres) is displayed for the KF&DIA position (solid line) and the LS&DS position (dotted/dashed). For the first two positions the $2d_{RMS}$ values of the LS&DS and the KF&DIA solution are exactly the same, because they are computed the same way, by means of the LS&DS method. The entire track shows a better precision (lower values for the $2d_{RMS}$) for the KF&DIA than the LS&DS.

This trend was seen on all tracks. Furthermore, as can be seen, errors on observables have less impact on the precision of the KF track, whereas the LS solution shows higher values for the $2d_{RMS}$ when observables are rejected (change of configuration) by the datasnooping approach. The $2d_{RMS}$ value for the KF&DIA solution hardly changes when adaptation takes place.
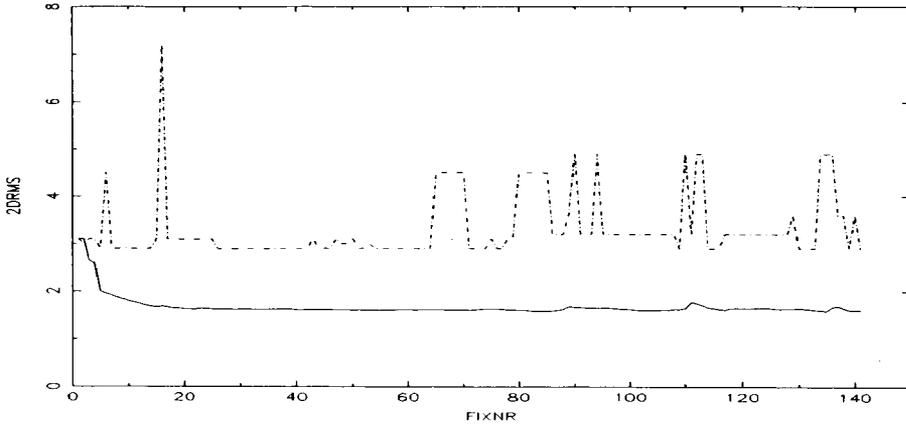
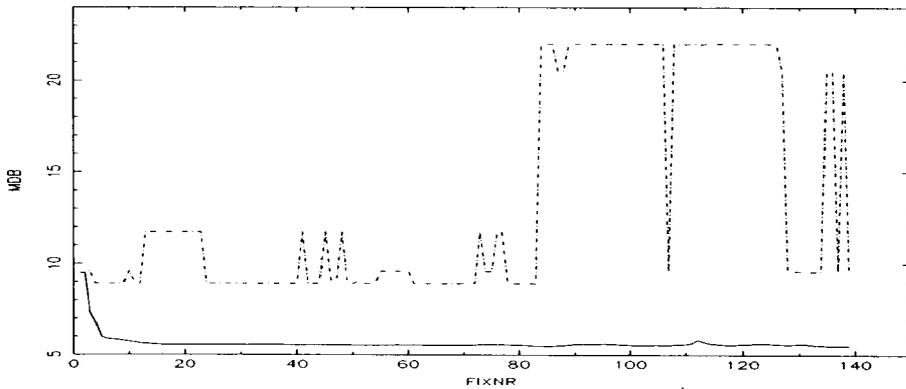FIG. 10.- 2drms LS&DS (dashed) KF&DIA (solid line).

FIG. 11.- MDB easting.

In Figure 11 the reliability by means of the MDB (in metres) of the Easting coordinate is shown. The solid line gives the MDB of the KF&DIA the dashed/dotted line gives the MDB for the LS&DS solution (the y-axis starts at the value of 5 metres). Again it can be seen that for the first two positions the MDB is the same for the two methods. On the whole the MDB values of the KF&DIA solution are lower than the MDB values of the LS&DS method. This means the level of errors which can be detected with the KF&DIA solution is higher than the errors which can be detected by the LS&DS. This last method cannot detect the small errors. Furthermore the MDB values of the KF&DIA solution, just like the $2d_{RMS}$ values, are hardly influenced by the sensor errors, while the LS&DS solution is. At the end of the track lane 3 contains a laneslip of 5 lanes. Whereas the MDB of the KF&DIA solution shows a slightly higher value, the LS&DS displays a much higher increase of the MDB value. From Figure 11 it can be seen that the MDB of the KF&DIA is small enough to detect the second error in the Easting reading. The MDB value of the LS&DS approach is much higher, around 22 meters, because the Hyper-Fix reading containing the error is already rejected. Since this MDB-value is larger than the

simulated error of 20 meters, the error in the Easting coordinate is not detected. Where the KF&DIA approach is capable of dealing with multiple errors in this case, the LS&DS method is not.

Figure 12 visualises the MDB (in lanes) of the third lane (which contains the lane-slip) in the case the laneslip is *continuously locally adapted* for. Due to the configuration of the beacons, the MDB of the KF&DIA track gives, at the beginning, only a small improvement in reliability when compared to the LS&DS. From fixnumber 87 on, there are no MDB-values of the LS&DS approach any more, because from this moment on the third lane is rejected by the Data Snooping. In this figure the danger of local adaptation of a global bias can be seen easily. Since the covariance is also adapted, it will be increasingly difficult in time to detect the global error properly. Here one sees at the end of the track, where the laneslip contains 5 lanes, the MDB is about 1 lane. This means the errors smaller than one lane will not be identified in this particular case. Global overall model test implementation will be studied in the future.
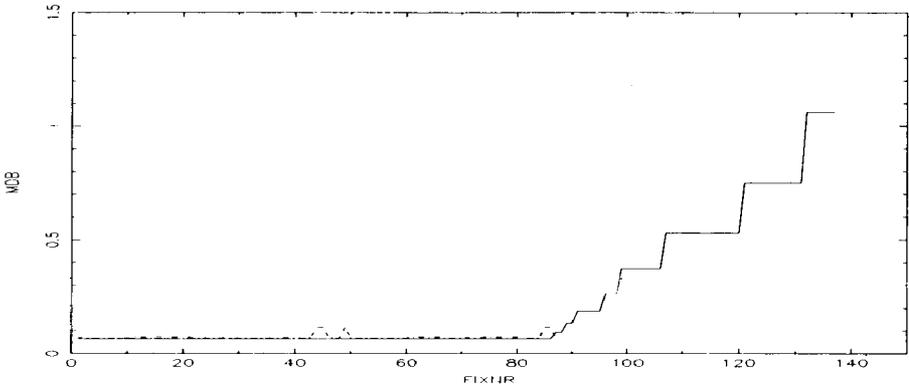

FIG. 12.- MDB lane 3.

Conclusions

The test results presented in this paper show the developed KALMAN Filter + Detection Identification Adaptation procedure performs very well, all sensor errors were eliminated. Even multiple errors are treated. Furthermore the underlying models were properly developed; the curves in several tracks were no problem for the KF&DIA procedure, although the procedure is sensible to already filtered "observables" and not applied existing biases in positioning sensors. Sometimes it could be better to choose one unbiased positioning system as input for the KF&DIA instead of integration with a system with unknown/unapplied c-o's and filtered observables.

KF&DIA gives, compared to the Least Squares method in combination with QC-tools F-test and w-test; DataSnooping (LS&DS), a much smoother track, and the

quality (precision and reliability) is better. The precision, represented by the $2d_{RMS}$ (distance Root Mean Square) value, and the reliability, by means of the Minimal Detectable Bias (MDB), of the KF&DIA solution show both on the whole lower, so better, values than the LS&DS solution. Performing adaptation, gives no, or hardly any decrease in reliability and precision as opposed to rejection with F-test and w-test.

Another advantage of the KF over the LS is that it not only gives a solution for the position, but also gives an estimate of other elements, like velocity and biases (drift) to the gyro and log observables.

In order to guarantee good performance of the KF&DIA the positioning systems must be carefully calibrated, however small biases (or computed minus observed, c-o's) are no problem.

Implementing KF&DIA can save time and money, since there is no need to abandon tracks while surveying, when errors in observations arise. Furthermore it will increase the safety when navigating, because errors are automatically accounted for (adaptation).

Recommendations

During the development and testing of the program and implemented theories, issues concerning the testprogram came to our attention. Some subjects which will be studied in the future:

- The implementation of a manoeuvre-predictor, for better state vector predictions. This is especially useful for use in submarines, using a lower fixrate or when sensors completely fail (see for instance Wulder [12]).

- Instead of using the DGPS position as two LOP's use the pseudo ranges (with their differential corrections) with satellite position models as input to the KF program.

- Implement the Global Overall Model test together with the Local Overall Model test for detection of global errors.

- Compute and make visible the Minimal Detectable Error and Bias to Noise Ratio (BNR) as a measures of external reliability.

Acknowledgement

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MATRICES

During the sea trials the following vectors and matrices were used:

Statevector x:

$$x = \begin{bmatrix} \text{Easting} \\ \text{Northing} \\ \text{Velocity}_{East} \\ \text{Velocity}_{North} \\ \text{gyro}_{bias} \\ \text{log}_{bias} \end{bmatrix} \qquad (A.1)$$

vector of observations y:

$$y = \begin{bmatrix} \text{lane1} \\ \text{lane2} \\ \text{lane3} \\ \text{E(DGPS)} \\ \text{N(DGPS)} \\ \text{gyro} \\ \text{log} \end{bmatrix} \qquad (A.2)$$

The covariance matrix of the Hyper-Fix observations depends on the patterns used.

The covariance matrix of DGPS depends on the area. For the sea trials the following matrix is used:

$$Q_y = \begin{bmatrix} 0.00043 & 0.00014 & 0 & 0 & 0 & 0 & 0 \\ 0.00014 & 0.00043 & -0.0003 & 0 & 0 & 0 & 0 \\ 0 & -0.0003 & 0.00057 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 1.44 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.24 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.00069 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.25 \end{bmatrix} \qquad (A.3)$$

The values are in observed dimensions (lanes^2, meters^2, radians^2, meters^2/sec^4).

The transition matrix with a fix interval of 1 second and constant speed and constant gyro-bias en log-bias during this interval is:

$$
\Phi =
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\tag{A.4}
$$

The used disturbance matrix reads:

$$
Q_d =
\begin{bmatrix}
\sigma_a^2
\begin{bmatrix}
\frac{1}{3} & 0 & \frac{1}{2} & 0 \\
0 & \frac{1}{3} & 0 & \frac{1}{2} \\
\frac{1}{2} & 0 & 1 & 0 \\
0 & \frac{1}{2} & 0 & 1
\end{bmatrix} & 0 & \\
0 & \sigma_{gc}^2 & 0 \\
& 0 & \sigma_{lc}
\end{bmatrix}
\tag{A.5}
$$

With acceleration disturbance σ_a= 0.25 m/s^2
gyro$_{bias}$ disturbance $\sigma_{gyro\text{-}bias}$= 0.7 degrees
log$_{bias}$ disturbance $\sigma_{log\text{-}bias}$= 0.02 m/s

The design matrix A depends on the observations used. For every element the partial derivative of the observation to the elements of the statevector must be computed:

$$A = \begin{bmatrix} \dfrac{\partial l_1}{\partial E} & \dfrac{\partial l_1}{\partial N} & \dfrac{\partial l_1}{\partial V_E} & \dfrac{\partial l_1}{\partial V_N} & \dfrac{\partial l_1}{\partial gyro_{bias}} & \dfrac{\partial l_1}{\partial log_{bias}} \\[2ex] \dfrac{\partial l_2}{\partial E} & & \cdot & & & \cdot \\[2ex] \dfrac{\partial l_3}{\partial E} & & & \cdot & & \cdot \\[2ex] \dfrac{\partial E(DGPS)}{\partial E} & & & \cdot & & \cdot \\[2ex] \dfrac{\partial N(DGPS)}{\partial E} & & & & \cdot & \cdot \\[2ex] \dfrac{\partial gyro}{\partial E} & & & & & \cdot \\[2ex] \dfrac{\partial log}{\partial E} & \cdot & \cdot & & \cdot & \dfrac{\partial log}{\partial log_{bias}} \end{bmatrix} \qquad (A.6)$$

A relevant example is:

$$A = \begin{bmatrix} 0.009 & -0.007 & 0 & 0 & 0 & 0 \\ -0.004 & -0.011 & 0 & 0 & 0 & 0 \\ 0.011 & 0.009 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.15 & -0.3 & -1 & 0 \\ 0 & 0 & 0.86 & 0.44 & -0.8 & -1 \end{bmatrix} \qquad (A.7)$$

The ones (positive or negative) and zeros in this matrix are constant, the other figures may change, according to the observations.