

# THE USE OF "EOF" IN THE MEAN SEA LEVEL OSCILLATIONS STUDY

## AN APPLICATION TO CADIZ

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### Abstract

Oscillation modes in mean sea level in the port of Cádiz are computed and analyzed from simultaneous water level and atmospheric pressure records. Empirical Orthogonal Function Decomposition (EOF), cross spectral and least square harmonic analysis are used.

EOF is applied in an unusual way in order to extract common oscillation modes of atmospheric pressure and Mean Sea Level (MSL) series. In spite of the short length of time series, less than four months, EOF is a very powerful tool and makes possible the isolation of astronomical and meteorological oscillation modes.

## 1. INTRODUCTION

One of the main problems in the analysis of Mean Sea Level (MSL) variations, is the determination of the parts due to astronomical and to meteorological forcing.

In many places, water level-atmospheric pressure quasi-static response oscillations of MSL shows periodicities similar to those of long period tidal waves. Also it is in these periodicities where most of the MSL variance is located.

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Isolation of both oscillations from any MSL series is usually performed in the following way:

1. Cross spectral analysis of MSL series and the meteorological variables in order to extract the oscillation of meteorological origin.
2. Harmonic analysis of MSL series to get amplitude and phase lag of long period tidal waves.

However, this procedure may be useless if one of the oscillations is not dominant.

Due to the existing energy from meteorological forcing in MSL, a direct harmonic analysis on the whole signal does not lead to good harmonic constants estimations. So, before any kind of analysis can be done, the meteorological forcing must be estimated and eliminated by a multivariate procedure such as cross spectral analysis or multiple regression (SMITH, 1986). However, the isolation of the MSL response to meteorological forcing is not perfect because of the presence of the astronomical part in the same or close frequencies.

Another constraint is the length of the data set which must be long enough to allow the estimation of the low frequency waves by harmonic analysis. Since most of data records cover only a few months this work presents a method to separate the response to both kinds of forcings in water level records of any length. The basis of our procedure is the Empirical Orthogonal Function Decomposition (EOF) (KUNDU *et al.*, 1975). In section 2 the data series and their preprocessing are described. In section three the results from cross spectral analysis are presented and in section four the results of EOF analysis are discussed.

## 2. DATA SETS AND PRE-PROCESSING

Atmospheric pressure records were provided by the Real Instituto y Observatorio de la Armada in San Fernando and by the Instituto Nacional de Meteorología at Puerta Tierra, Cádiz. The water level record, from the Integrated Tidegauge Network, was provided by the Instituto Hidrográfico de la Marina in Cádiz. The selected period for this study was from January to May of 1989 as simultaneous data of all variables were available for this period only. See Figure 1 for locations.

Water levels in the port of Cádiz and atmospheric pressure in San Fernando, taken with a sample interval of one hour, were reduced to their low frequency portion by a moving average filter  $A_{24}A_{24}A_{25}$  (GODIN, 1972), yielding a representative MSL variation time series for the port of Cádiz. Since the atmospheric pressure at Puerta Tierra was sampled at synoptic hours, a moving average filter  $A_5$  was applied to isolate the low frequency signal.

Before including the atmospheric pressure series in any analysis they were reduced to sea level to maintain the same reference level for the pressure data in the two stations.

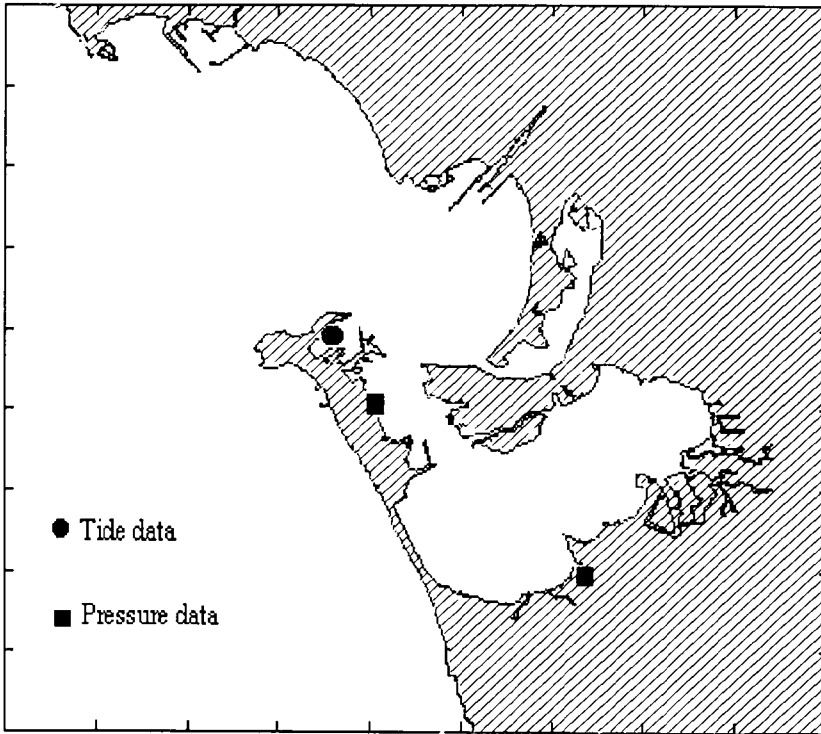


FIG.1.- Map of location of the data stations used.

### 3. SPECTRAL RESPONSE OF MSL TO ATMOSPHERIC PRESSURE IN THE PORT OF CÁDIZ

The spectral response of MSL to pressure variations has been studied in the past (FERNÁNDEZ DE CASTILLEJO, 1967 and ADMETLLA & GARCÍA, 1980). The quasi-static response behaviours of MSL to atmospheric pressure variations may be analyzed from cross spectral analysis of MSL series and an atmospheric pressure series from a nearby station. The modulus and phase diagrams of the transfer function obtained from this analysis allow us to establish that response.

Figure 2 shows both the atmospheric pressure series in San Fernando and MSL series in the port of Cádiz for five months in 1989. A visual inspection reveals common behaviours. However, some differences can be observed in results from the cross spectral analysis of these series.

The results of cross spectral analysis between MSL in Cádiz and atmospheric pressure in San Fernando are shown in Figure 3. In the frequencies from 0.10 to 0.25 cycles/day after what the energy spectrum of MSL is zero (Figure 3.b) the coherence and modulus and phase of the transfer function show an

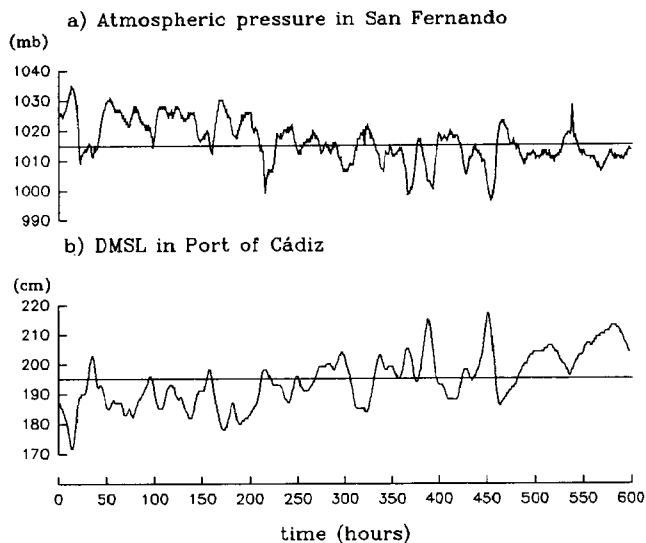


FIG. 2.- Series of atmospheric pressure in San Fernando and MSL in the port of Cádiz. Period from 1 Jan 89 to 31 May 89. MSL referred to the chart datum of the port of Cádiz.

almost inverted barometer pattern of MSL. However in the frequencies from 0. to 0.10 cycles/day clear deviations from this behaviour can be seen. The coherence is far below 1; the transfer function modulus shows values quite different from 1 and the phase shows values far away from  $180^\circ$ .

The results above indicate that quasi-static response of MSL is not evident in that frequency range. Furthermore, if we take into account that the main long period tidal waves signals are located in this frequency range, we may think that their presence in the MSL is the cause of these deviations from inverted barometer response. In the next section this idea will be investigated by EOF analysis.

#### 4. EOF ANALYSIS AND DISCUSSION

The isolation of the quasi-static response of MSL to atmospheric pressure can be done by EOF analysis. EOF is applied in an unusual way on two different variables.: MSL and atmospheric pressure. Both the atmospheric pressure series in San Fernando and at Puerta Tierra were included in the analysis.

With EOF the input series can be written as:

$$s_i(t) = \sum_{k=1}^M A_{ik} \cdot P_k(t)$$

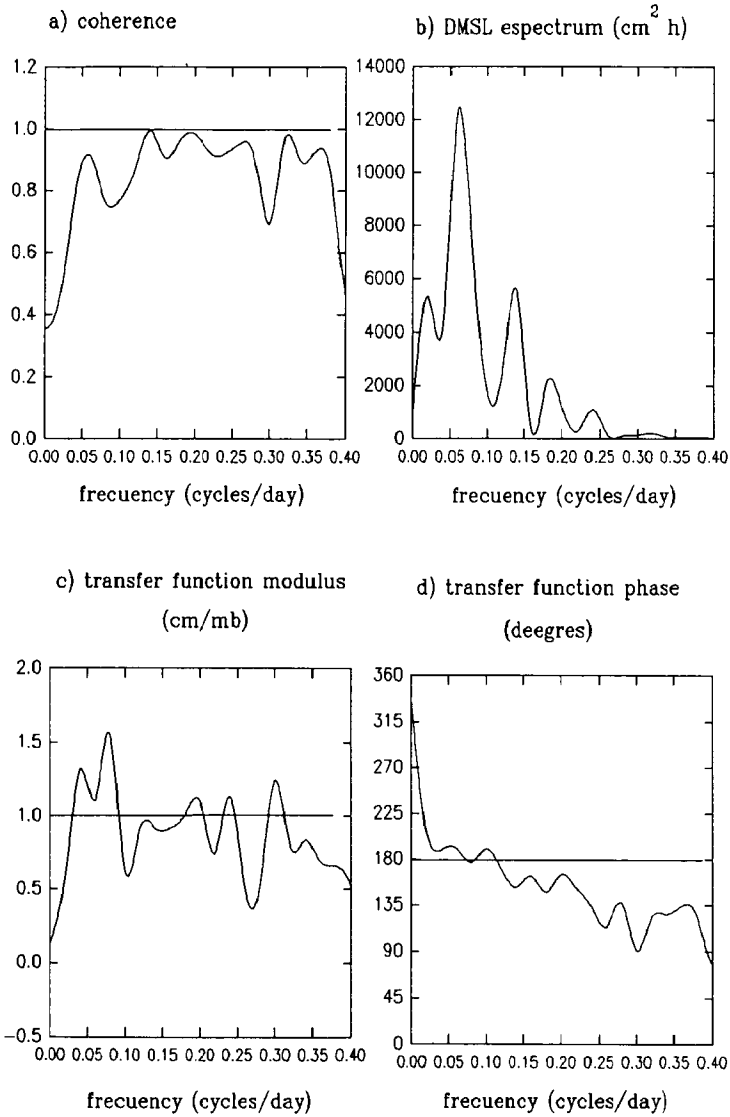


FIG. 3.- Results from cross spectral analysis between atmospheric pressure in San Fernando and MSL in Cádiz. Atmospheric pressure (mb) as the input and MSL (cm) as the output.

where  $s_i$  is the  $i$  input serie,  $P_k$  stands for the temporal weights for each common oscillation mode,  $A_{ik}$  are the eigenvectors that indicate the importance of the  $P_k$  modes in the input series variance.  $M$  stands for the number of series or number of modes. The different series have been abbreviated as follows: MSL by the letter  $\zeta$ , atmospheric pressure in San Fernando by  $P_{SF}$  and atmospheric pressure at Puerta Tierra by  $P_{PT}$ .

Prior to any analysis the pressure data were reduced to sea level using temperature data. EOF analysis was applied on the last three month only, due to errors in temperature sensors in January and February at Puerto Tierra. The sampling interval was reduced to six hours as pressure at Puerto Tierra had been sampled at that interval.

Tables 1 and 2 show the EOF analysis results. Table 1 shows the variance percentages accounted for by each mode for the three variables. Table 2 accounts for eigenvectors. Temporal weights of the two more energetic modes are plotted in Figure 4.

Table 1

VARIABLE	MODE 1	MODE 2	MODE 3
$\zeta(t)$	57.52	42.24	0.03
$P_{SF}(t)$	99.83	0.00	0.17
$P_{PT}(t)$	99.75	0.00	0.25

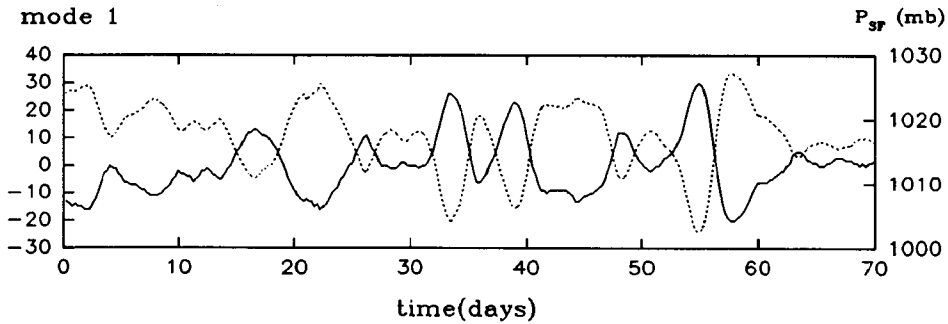
Table 2

VARIABLE $s_i(t)$	$A_{i1}$	$A_{i2}$	$A_{i3}$
$\zeta(t)$ (i=1)	0.57	-0.49	-0.013
$P_{SF}(t)$ (i=2)	0.48	0.00	-0.019
$P_{PT}(t)$ (i=3)	0.47	0.00	0.023

Mode 1 accounts for about 57% of MSL and is highly correlated with atmospheric pressure variations and may be due to the quasi-static MSL response to atmospheric pressure which is based on the behaviour of the temporal weights of the first mode and its relation, by cross spectral analysis, with  $P_{SF}$  series (Figure 5). Coherence, modulus of transfer function and phase lag plots (Figures 5a, 5b, 5c) show high coherence, a constant value in the transfer function and a phase lag in the low frequency band of about 180°. Hence this MSL mode shows an inverted barometer behaviour.

The second mode explains about 42% of MSL and does not keep a linear relationship with  $P_{SF}$ . Due to fortnightly periodicity (Figure 5b) it can be related with the low frequency astronomical tide.

a) mode 1 and atmospheric pressure in San Fernando  $P_{SF}$



b) mode 2 and predicted low frequency tide  $\zeta_M$

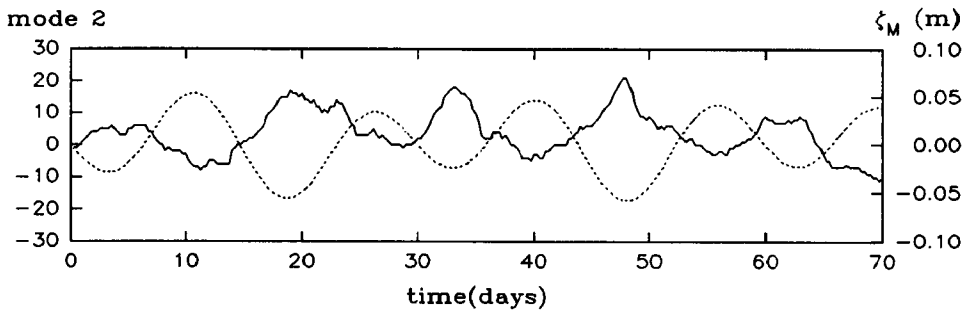


FIG. 4.- Temporal weights of the two more energetic modes obtained from EOF analysis of  $P_{SF}$ ,  $P_{PT}$  and MSL series. a) mode 1 (solid line) and  $P_{SF}$  (dashed lined). b) mode 2 (solid line) and predicted low frequency tide in Cádiz,  $\zeta_M$  (dashed line).

Table 3

CONSTITUENT	AMPLITUDE (cm)	PHASE (degrees)
MM	1.77	352.53
MSF	4.09	111.65
MF	1.30	148.03

The second oscillation mode has been validated by a three step procedure:

- a) Low frequency harmonic constants estimation of the astronomical tide from water level record after the atmospheric pressure correction.\*

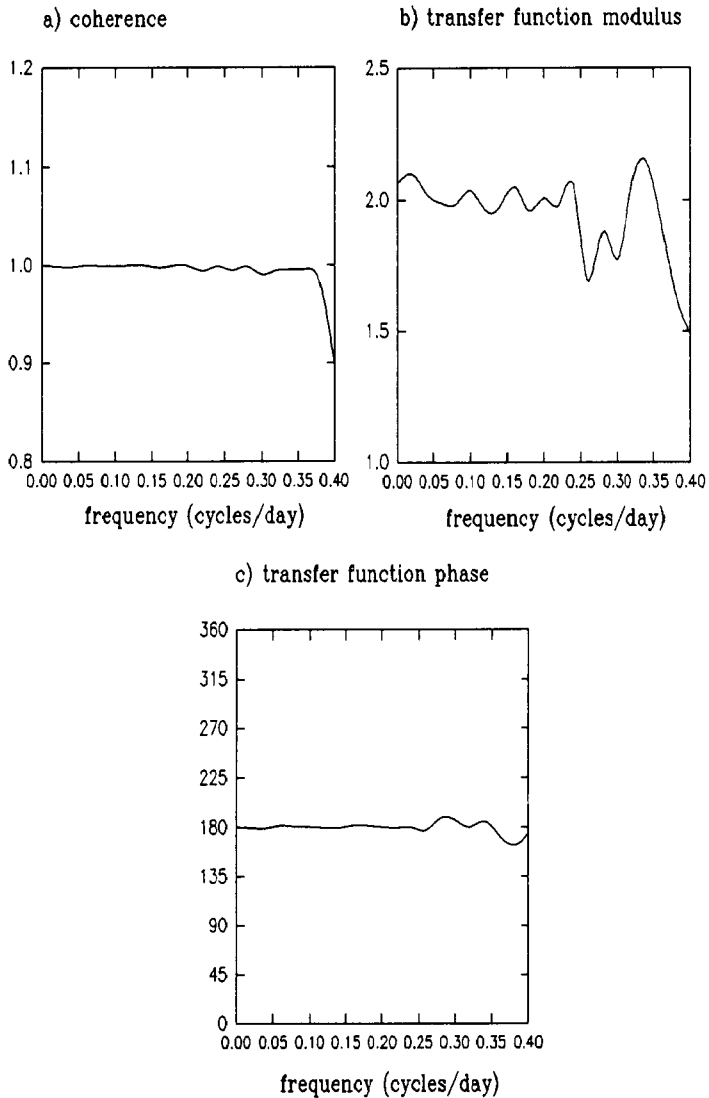


FIG. 5.- Results from cross spectral analysis between  $P_{SF}$  (input) and mode 1 (output).

In our case there was only a five month simultaneous data record period and the least square harmonic estimation (FOREMAN (1976)) was on Mf, Msf and Mm tidal waves (Table 3).

- b) Six hourly prediction of low frequency tidal signal band from previous results.
- c) Comparison of the second mode with the predicted low frequency tide.



The last step allows a visual correlation analysis. Similar behaviour of the second mode with the predicted tide suggests that the mode is related with astronomical tide (Figure 4b).

## CONCLUSIONS

From the results in sections 3 and 4, the MSL in Cádiz shows two main oscillation modes, isolated by EOF application:

- The first mode is associated to the inverted barometer behaviour MSL vs atmospheric pressure.
- The second mode is associated to the low frequency astronomical tide.

Two pressure series are introduced to improve EOF analysis in the isolation modes. The third mode is for noise and spurious waves and is not significant because of the low values of its weights.

EOF analysis makes possible direct quasi-independent data length mode isolation associated to low frequency tidal waves whose harmonic constants can be estimated only with great difficulty with a short length data record.

## ACKNOWLEDGMENTS

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