International Hydrographic Review, Monaco, LXXIII(2), September 1996

ON THE ESTIMATION OF STANDARD DEVIATIONS IN MULTIBEAM SOUNDINGS

by Jørgen EEG [1]

Abstract

This paper describes a simple and robust method of estimating the standard deviation of multibeam soundings. As an illustration of the method, upper and lower estimates of the standard deviations for the angular sector 0 to 60 degrees of the BCC (ELAC's BOTTOMCHART COMPACT SHALLOW WATER) and RESON's SeaBat-9001 system are calculated at various depths. Furthermore, the accuracy of the method is discussed, together with the case where the variation of the sea bed inside the footprint of the transducer beams cannot be ignored, the case in point being measurement of the size of a field stone situated at a depth of 22 metres.

INTRODUCTION

Multibeam transducers, as opposed to singlebeam transducers, send and receive their signals through the water at several different angles relative to the plumb line. Due to the physical laws governing the propagation of sound in water and the reflection of sound from the sea bed, one can expect that the variance of the depth measurements grows with increasing angles and with increasing depth. As IHO's standard for accuracy of a survey , the present [1] as well as the emerging [2] and [3], is governed by formulas which depend on the depth, it is necessary to estimate the variance of the soundings as a function of depth and beam angle in order to be able to claim that a survey adheres to this standard. The question of finding such an estimate rises in importance, when a survey is performed with the specific purpose to ascertain that a minimum depth in an area can be guaranteed, and the

[1] Royal Danish Administration of Navigation and Hydrography, P.O.Box 1919, Overgaden O.Vandet 62B, DK 1023 Copenhagen K, Denmark, farvje@unidhp.uni-c.dk.

seabed at the area in question rises to this limit. Actually, it was a problem presented in just this way which triggered the investigation below.

THE METHOD OF ESTIMATION

When the topography of the bottom is unknown - as is indeed presupposed in this paper - the variation of the sea bed may propagate into a variance estimate for the beam measurements. The parameters to be aware of in this context are the size of the footprint of the transducer beam on the sea bed, the distance between neighbouring footprints, and, relative to these entities, the long and short periodic variation of the bottom. So, in order to find a proper variance estimate, measurements carried out on a flat piece of the sea bed without any obstacles would be ideal. In practice, this requirement cannot be met. What can be done, however, is to make the procedure of estimation robust, so that the contribution from the variation of the sea bed is kept under control.

One way to attain this goal - which also is the one pursued in this paper - is to eliminate as far as possible the shape of the sea bed, while at the same time maintaining a large amount of uncorrelated estimates of the variance. This is accomplished as follows.

For each beam angle, the observations in a track are split up into pairs of neighbouring measurements, each observation member of at most one pair. For each pair $(X_1,X_2)_i$, i=1,2...f , the following statistical model is adopted:

X_1 and X_2 are stochastically independent measurements with mean μ_i and variance σ^2

The error committed by adopting this model amounts to that of regarding the sea bed as level and homogeneous in the neighbourhood of the footprints of each pair of observations.

The difference of the paired observations $X_1 - X_2$ has zero mean

$$E(X_1 - X_2)_i = 0 \qquad i = 1,2,...,f$$

and a variance, which is twice the variance of the beam measurements

$$\mathrm{var}(X_1 - X_2)_i = 2\sigma^2 \qquad i = 1,2,...,f.$$

As the pairs are disjoint, the differences are independent and we can estimate their variance by the average of the sum of their squares. The estimate s^2 of the variance σ^2 of the beam measurements is then found by dividing this entity by two, i.e. if

$$SSD = \sum_{i=0}^{f} (X_1 - X_2)_i^2$$

then the estimate ***s*** of the standard deviation ***σ*** becomes

$$s = \sqrt{\frac{1}{2f} SSD}$$

SOME EXAMPLES OF ESTIMATION OF THE STANDARD DEVIATION

Below some data sampled by Allied Signal-Elac's Bottomchart Compact (BCC) and Reson's SeaBat-9001 are used to illustrate the above method. These two swath bathymetric systems are, by construction, sufficiently different to enable the reader to apply the method on his own system.

ESTIMATION OF THE STANDARD DEVIATION OF THE 56 BEAMS ON BCC

The two identical 180Khz transducers of the BCC system are mounted at the stern of the survey vessel, one to port and the other to starboard, together covering an angular sector of 120° across the ships track. By convention, the beam angles between -60° and 0° (beam 0 - 27) belong to the starboard transducer, and those between 0° and 60° (beam 28 - 55) to the port transducer. During operation 4 pairs of roll corrected beams are sent out in rapid succession, the first pair starting at the angles -60° and 0° (beams 0 and 28), and then, shifting 15° between each succeeding pair, the remaining ones are sent out in -45°,15° ; -30°,30° ; -15°,45° by the beams 7,35 ; 14,42 and 21,49. The BCC then waits for the return signal for a preset period of time before sending out the next 4 pairs of roll corrected beams, adding 20/9° to the angles and 1 to the corresponding beam numbers, thus completing a full cycle in 7 steps. Consequently, the beams of a survey vessel moving along at a constant speed over a flat area of the sea bed, traces a pattern on the sea bed similar to the graph of the function $\tan\theta$, $|\theta| < \pi/3$.

The measurements which are used to estimate the standard deviations of the BCC system stem from a survey on a test site situated off the north coast of Zealand at a deph of 22 m of water. As the beams are corrected for roll, each beam occupies its own angular sector during the survey, and the data is accordingly split up into 56 subsets, one for each beam, each providing an ***SSD*** and an estimate ***s*** of the standard deviation . The result is depicted as a continuous curve in Figure 1.

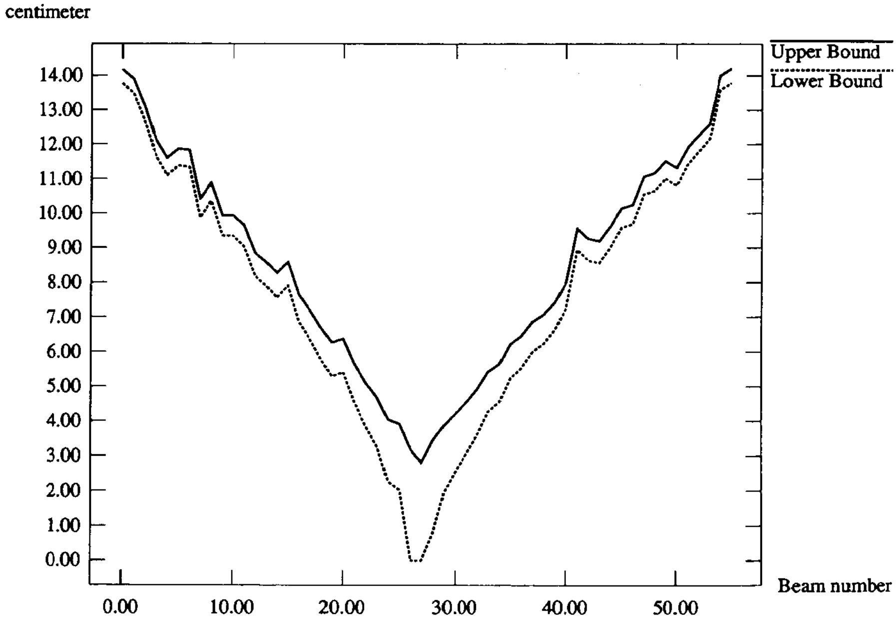

FIG. 1.- Standard Deviations for BCC at 22m.

From Figure 1 follows that the standard deviation attains its minimum value for the nadir beams 27 and 28, and grows with increasing numerical value of the beam angle to a maximum for the beams 0 and 55 at ±60°. Now, as mentioned above, the continuous curve in Figure 1. depicts an estimate of the combined effect from the variation of the sea bed and the transducer, i.e. an upper estimate of the standard deviation of the transducer. If the sea bed at the footprint of the beams is modeled as the realization of a stationary random process stochastically independent of the measurements by the transducers, then a lower estimate is found by subtracting, from the upper bound, the variance of the sea bed as estimated by the nadir beams. This lower estimate of the standard deviation of the transducer is depicted in Figure 1 as a broken curve.

ESTIMATION OF STANDARD DEVIATION IN THE ANGULAR SECTOR 0° - 60° FOR RESON'S SEABAT-9001

The 455Khz transducer was mounted at the stern of the survey vessel, with the sonar head rotated from the vertical by 30° to provide a total coverage from -15° to 75°. During a survey swathes of 60 beams, each 1.5°×1.5°, are sent in rapid succession. As there is no compensation for the vessels motion, a track plot of a survey with this transducer over a flat section of sea bed reflects the roll and pitch components of the motion. In order to cope with this, the angular sector 0° - 60° below the vessel is divided into 40 subsections, each 1.5° wide, and for each swathe the correspondence between beam numbers and subsections is established. Then the pairing of observations is accomplished by sampling observations from the same sector in neighbouring swathes.

Again, the measurements used to estimate the standard deviation of the SeaBat-9001 stem from off the north coast of Zealand, but this time from a very flat part of the sea bed situated at a depth of 20 meters. The resulting upper and lower estimates of the standard deviations are depicted in Figure 2.

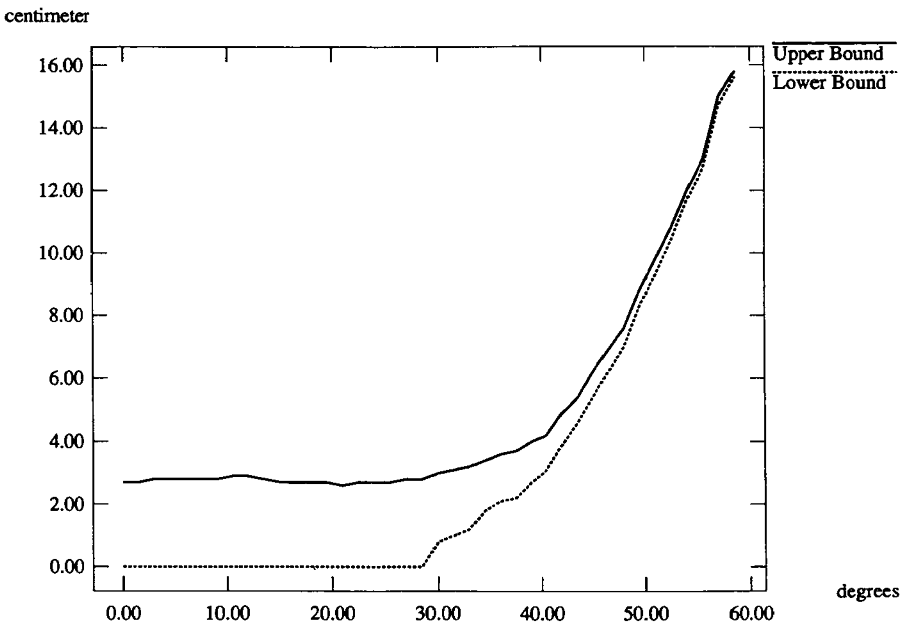

FIG. 2.- Standard Deviations for SeaBat 9001 at 20m.

DISCUSSION OF THE ESTIMATES

From Figure 2 it is obvious that the variance of the beams are hidden by the variance of the sea bed for angles less than 30°. This is in contrast to Figure 1 where the variance of the beams dominates that of the sea bed even at nadir. As pointed out to me by Dr. Lloyd HUFF [3], the variance should, other things being equal, be a linear function of the area of the footprint, so this behaviour may be caused by the difference in the size of the footprint which for the SeaBat is 1.5°×1.5° compared with BCC's 3.6°×6°.

Another difference which meets the eye is that the graphs in Figure 2 are smoother than those in Figure 1. Whether this indicates irregularities in the performance of the transducers or uncertainty in the estimate of the standard deviation is a question of interest, as it is a fringe benefit of being able to estimate the standard deviation of a transducer, that its performance can be monitored, by comparing it against itself at different time periods, or against another of the same brand.

ACCURACY

So, the question is: how accurate is the estimate $\boldsymbol{s}$ of the standard deviation $\boldsymbol{\sigma}$? Suppose, for the time being, that the paired observations follow normal distributions $N(\mu_i,\sigma^2)$, i=1,2,...,f. Then it is well known, that ***SSD*** follows a χ^2-distribution with f degrees of freedom, scale parameter $2\sigma^2$, mean

$$E(\boldsymbol{SSD}) = 2f\sigma^2$$

and variance

$$\mathrm{var}(\boldsymbol{SSD}) = 2f(2\sigma^2)^2.$$

From this it follows, that

$$var(s^2) = \frac{2f}{4f^2}(2\sigma^2)^2 = \frac{2}{f}\sigma^4$$

Now, in order to find the variance of $\boldsymbol{s}$, we expand the square root operator in a Taylor series at $E(\boldsymbol{s}^2) = \sigma^2$:

$$\sqrt{s^2} = \sqrt{E(s^2)} + \frac{1}{2\sqrt{E(s^2)}}(s^2 - E(s^2)) + \ldots$$

or,

$$s = \sigma + \frac{1}{2\sigma}(s^2 - \sigma^2) + \ldots$$

Applying the variance operator at both sides of the equality sign and ignoring terms of higher order we get

$$var(s) \approx \frac{1}{4\sigma^2}\frac{2}{f}\sigma^4$$

so that the standard deviation of $\boldsymbol{s}$ becomes

$$s.d.\ (s) \approx \frac{\sigma}{\sqrt{2f}} \approx \frac{s}{\sqrt{2f}}$$

In Figure 2, 1230 pairs of observations have been used in order to estimate the standard deviation of each 1.5° sector, whereas 862 pairs have been used to estimate each beam in Figure 1. For example, for **s** = 14 cm the standard deviation of **s** is found by the above formula to be 0.28 cm and 0.34 cm respectively. Consequently, the roughness of the graph in Figure 1 can be explained by the uncertainty inherent in the method of estimation, and it stands to reason that, given more observations, the graph will smoothe out, and, more specifically becomes concave as the graph in Figure 2, the standard deviation increasing as the beams pass through more water. As mentioned in the introduction this behaviour of the standard errors is to be expected, and is further demonstrated in Figure 3, where the standard deviations for the SeaBat-9001 is found for a range of depths, starting at 5m and ending at 14m, in intervals of 3m. This data is sampled by NOAA, and the calculation of the estimates is made by Dr. Lloyd HUFF [4]. All the estimates in Figure 3 are upper estimates, and the entire angular sector 0° to 75° is included.

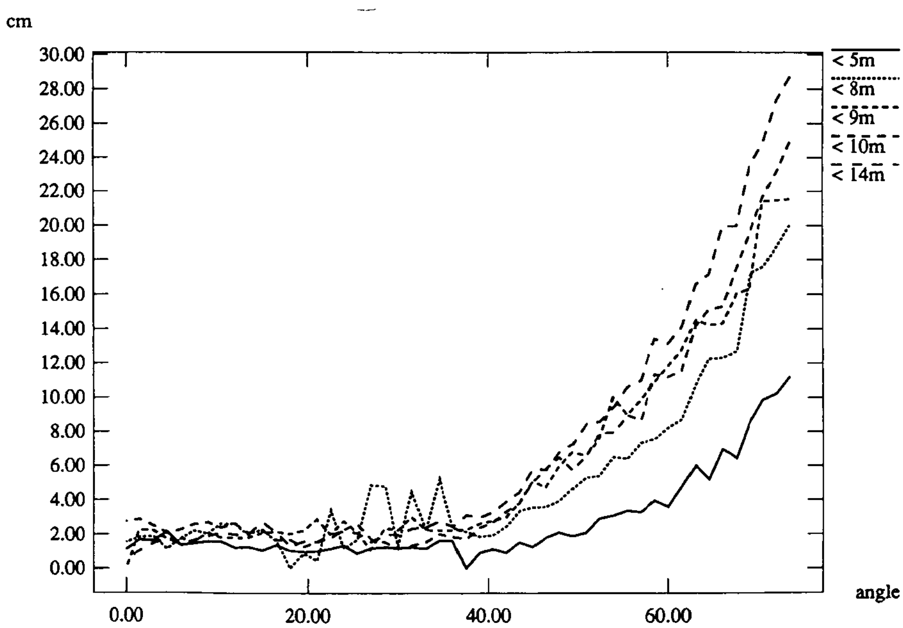

FIG. 3.- Standard Deviations for SeaBat 9001.

RELIABILITY

An important question which remains to be answered is: how reliable is this method of estimation - or, put differently - what happens when the statistical model breaks down? As to that, it is of crucial importance that the observations are uncorrelated, as the variance of the difference of two observations only is the sum of their variances when the covariance is zero. If, for example, the bottom detection algorithm uses information from the neighbouring beams in a swathe to detect the response from the sea bed in the return signal, or the conversion from raw data to the data format used in postprocessing includes a smoothing of the observations, then the above method, at best, yields an overly optimistic estimate of the standard deviation.

One way to get on the safe side of this pitfall is to study the performance of the swath bathymetric system while passing a small object of known size, placed on a flat part of the sea floor.

From a hydrographer's point of view, the ideal swathe system returns, for each beam, the least depth of any solid object inside its footprint. For such a system it would be possible, using an object with support much smaller than the footprint of the beams, to procure sets of measurements, where only one beam hits the object while the others cover its neighbourhood. By comparing the known height of the object with estimates resulting from the measurements, a test for independence can be carried out, using the standard deviation of the beams in question as found by the above method.

In practice, however, swath bathymetric systems do not live up to this ideal, and the size of the support of an object influences the measurement. If it comes to the worst then, in order to ascertain that the height of an object is measured correctly, the footprint of the beam which hits it has to be a subset of the support of the object which again has to be flat on the top. Furthermore, a 2-dimentional version of Shannon's sampling theorem states that the size of the object has to be twice that of the beam to be tested in order to ascertain that the object is hit by the beam in this way, even when measuring with full bottom coverage.

In the following, a concrete mooring anchor shaped as a cube with a support of size 0.8m×0.8m and a height of 0.9m placed on a level section of the sea bottom at a depth of 20m water, is used to test the independence of the SeaBat-9001 beams. During the test the survey vessel did 6 knots, which with the repeating rate of the SeaBat-9000 of 7 swathes a second corresponds with a movement of the sonar head of 0.4m between each swathe. Consequently, the concrete anchor will effect the measurements of at least 2 neighbouring swathes, irrespective of the size of the footprints of the beams, which depend on the distance from the sonar head to the object, going in this case from 0.5m×0.5m at nadir to about 1m×2m at 60°.

The tables below show extracts of the swathes of two parallel survey lines as they cross the mooring anchor. This is only an illustration of the technique to be used, and an investigation ultimately includes all beams and depends on the hardware and software (version number, etc.) of the data aquisition and convertion system.

Table 1

beam nr	2	3	4	5	6
swathe 1	20.01 m	20.01 m	19.97 m	19.96 m	20.04 m
swathe 2	19.91 m	20.01 m	19.05 m	19.08 m	20.00 m
swathe 3	19.95 m	20.05 m	19.09 m	19.18 m	19.99 m
swathe 4	19.98 m	19.06 m	19.10 m	20.02 m	20.00 m
swathe 5	20.00 m	19.95 m	20.02 m	19.97 m	20.02 m

As mentioned above, the sonar head was tilted during the data aquisition, so that, disregarding roll and pitch, beam nr. 10 pointed towards nadir, beam 0 15° towards port and beam 45 52.5° towards starboard. From Figure 2 it follows, that the standard deviation of the beams in Table 1 is less than 3 cm, and therefore, assuming independence, the standard deviation of the difference of two measurements is less than $\sqrt{18} \approx 4$ cm. The test now consists in checking the size of the mooring anchor against estimates procured by taking differences between *adjacent* beams in the *same* swath or between beams in the *same* angular sector but from *adjacent* swathes.

The reader may satisfy himself, that beam 3, 4 and 5 perform as should be expected if uncorrelated, as the measurement by beam nr. 5 in swath 3 can be explained as misalignment of the footprint and the anchor.

In Table 2 the footprint of beam 45 is 0.8m×1.3m, that is a little greater than the anchor. Therefore, taking the vessels speed into account, a more blurred transition between the swathes is to be expected, and inside the swathe some degradation of the size of the anchor may occur, even when the anchor is a subset of the footprint, as almost seems to be the case for beam nr. 45 in swath 4.

Table 2

beam nr.	43	44	45	46	47
swathe 1	20.19 m	20.13 m	19.89 m	20.11 m	19.98 m
swathe 2	20.06 m	20.09 m	19.68 m	20.07 m	19.90 m
swathe 3	20.42 m	19.96 m	19.32 m	20.11 m	20.34 m
swathe 4	20.24 m	19.86 m	19.12 m	20.05 m	20.03 m
swathe 5	20.23 m	20.04 m	20.13 m	19.94 m	20.17 m

At 52.5° the upper estimate of the standard deviation is 10.8cm corresponding to a standard deviation of 15cm for a difference. Again, judging from

the measurements of beam 44, 45 and 46 in swath 4 and from swath 4 and swath 5, beam 45 performs as if no correlation is present.

The hypothesis in this test is that the beam measurements are uncorrelated, and this hypothesis cannot be proved by the test, only rejected at some level of significance.

ESTIMATION OF THE EFFECT OF SEA FLOOR VARIATION INSIDE THE FOOTPRINT OF THE BEAM

From the point of view of navigational chart products, the ideal echo sounder is one which registers the top of that part of the sea floor which is covered by its footprint, together with its position. The ability to do this rises in importance with increasing footprint size of the beams, as the size of the objects which may hide inside the footprint grows proportional. Now, the length of the return signal for a beam depends on the duration of the signal sent by the transducer and the angle at which the solid angle of the signal hits the sea floor. Even if the bottom detection algorithm succeeds in separating the return signal from the background noise, the correspondence between an event in the signal and a generating subset in the footprint is not unique, so it is to be expected that the positioned depths which result from swath bathymetric measurements represent some kind of compromise with respect to the ideal case, and that the effect of this compromise will depend on the size of the footprint.

As chance would have it a field stone was left by the ice at a depth of 22m on an otherwise flat part of the sea floor north of Hundested. In June 1995 it was visited by a team of divers and described as being 4.5m long, 1.5m wide and 1.5m high, topped by a 10 cm lump at one end and covered by 25 cm high sea anemones. Placed north-south, it rises rather abruptly - at a height of 60cm the circumference was measured to 10.85m - from the surrounding sea floor which consists of sand and rubble covered by a thin layer of very fine grained mud. During the measurements the mud was stirred up which affected the precision. Later measurements with a SOUNDING 2000 single beam 210Khz echo sounder, however, establishes the height of the field stone to 1.35m.

Below the height of the field stone is estimated from a set of different runs, each performed with 100% sea floor coverage. For each run, the depth of the sea floor was estimated separately and the lowest depth registered on the top of the field stone was subtracted to yield the height.

At the top of the field stone the footprint for the beams in the above tables vary between 0.5×0.5 m^2 to 1.0×1.8 m^2 for the SeaBat 9001 and between 2.1×1.3 m^2 and 2.2×1.4 m^2 for ELAC BCC, which is nicely reflected in the two estimated standard deviations of its position. As predicted, the measured height of the field stone is biased, being 10 cm too small for the SeaBat 9001 and 30 cm too small for the ELAC BCC. The estimated standard deviations of the heights are, naturally, bigger than those estimated from Figures 1 and 2, which is to be expected, as the footprint of the beams involved in the calculation have covered different parts of the field stone only with the top in the intersection.

Table 3
Hight of field stone estimated by SeaBat 9001 on SKA16, August 1995

speed (kn)	Northing (m)	Easting (m)	beam nr.	height (m)
4	6 222 648.32	676 704.73	12	1.23
4	6 222 649.02	676 703.94	36	0.82
3	6 222 648.61	767 704.76	13	1.27
3	6 222 648.27	676 704.95	9	1.24
7	6 222 648.23	676 705.05	14	1.19
7	6 222 647.98	676 704.41	10	1.33
4	6 222 647.67	676 704.76	38	1.46
4	6 222 648.42	676 704.07	34	1.32
6	6 222 648.32	676 704.38	41	1.28
6	6 222 648.46	676 704.37	5	1.35
6	6 222 648.47	676 704.59	15	1.30
6	6 222 646.95	676 703.80	34	1.29
6	6 222 647.01	676 703.65	48	1.42
mean (m)	6 222 648.13	676 704.42		1.27
std. dev. (m)	0.60	0.44		0.15

Table 4
Height of field stone estimated by ELAC BCC on GRIBBEN, September 22, 1995

speed (kn)	Northing	Easting	beam nr.	height (m)
7	6 222 646.84	676 705.07	35	1.08
7	6 222 650.33	676 704.18	25	1.22
7	6 222 648.42	676 705.47	27	0.80
7	6 222 649.35	676 703.43	18	0.89
7	6 222 647.24	676 705.30	22	1.10
7	6 222 647.59	676 701.99	35	1.12
7	6 222 647.44	676 707.31	24	1.10
7	6 222 648.55	676 701.91	33	1.16
7	6 222 649.36	676 705.55	23	1.23
7	6 222 646.33	676 702.14	30	1.11
7	6 222647.92	676 706.11	23	1.03
mean (m)	6 222 648.12	676 704.41		1.08
std. dev. (m)	1.21	1.82		0.13

ADHERING TO IHO'S STANDARD

IHO's present standard for hydrographic surveys [1] requires for measured depths below 30 meters that 90% of the errors of the measurements must be within ±30 cm. In order to make sense in this context one has to refer the above statement to a distribution. An obvious candidate is the normal distribution $N(0,\sigma)$ with mean zero and standard deviation σ, where σ has to be determined so that 90% of the errors are within the above limit. This means that 10% of the errors may exceed this limit, or, as the normal distribution is symmetric, that 5% of the errors may be greater than 30 cm, and 5% less than -30 cm. From a table of the cumulative normal distribution function one finds, that the integral of $N(0,1)$ between -1.65 to 1.65 is 0.90. It follows, that σ then is determined by

$$\sigma = 30/1.65 \text{ cm} \approx 18 \text{ cm}$$

As IHO's standard for measured depths refers to the end product of a set of measurements which involve determination of the position of the transducer and the path through the water of the signal, we must allow for errors in the determination of the velocity profile and long periodic (relative to the sampling frequency) deviations in roll, heave and pitch , which for example may occur due to incorrect time tagging of the sensors. For a complete error budget for a multibeam system see [5].

If, for example, the standard deviation of these errors is set to 10 cm, it follows, that in order to live up to IHO's present standard, measurements by beams/angles with standard deviation exceeding

$$s = 15 \text{ cm}$$

must be discarded.

Conclusion

A simple and robust method to estimate the standard deviation of a multibeam echosounder is described. Although the results are sharper for a flat sea bed, it is the authors experience from daily use of the method for more than 2 years that most data sampled in Danish waters can be used, if only one remembers to stratify it in, say, 2m intervals. Thus it is feasible to monitor the accuracy of surveys by using data sampled on location as opposed to having to rely on the special conditions (structure and reflectivity of the sea floor) at a test site.

References

[1] IHO Standards for Hydrographic Surveys. *Special Publication No 44, 3rd Edition 1987.*

[2] Guidance on the Treatment of Data Quality on Navigation Chart Products including the Electronic Navigation Chart. *IHO Special Publication No. 52,* Appendix XXX, *1st Edition, March 1995.*

[3] IHO Standards for Hydrographic Surveys. *Special Publication No.* 44, *Fourth Edition. (Draft for circulation to Member States), 1996.*

[4] Dr. Lloyd. Huff, NOAA. private correspondance.

[5] HARE, Rob: Depth and Position Error Budgets for Multibeam Echosounding. *International Hydrographic Review, vol* LXXII No. 2, *September* 1995.