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A SIMPLE METHOD FOR TIDAL CURRENT PREDICTION IN SHORT LENGTH CONVERGENT CHANNELS

AN APPLICATION TO THE VIGO AND PONTEVEDRA ESTUARIES

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Abstract

In this paper a simple method for tidal current modelling in short length convergent and elongated channels is presented. The authors have found that for this kind of channels a very suitable approach to tidal current velocity estimation can be obtained through a very simple formula. This method has been developed to obtain reliable tidal current velocity estimations along the channel using as less information as possible. In fact, the only necessary information to apply this method are the harmonic constants of the tidal elevation at any location in the channel and a proper nautical chart.

A graphical method is also presented to inquire when the geometrical characteristics of a given channel allow the application of our method. The validation has been performed on two estuaries of the Gallega Estuarine System in Spain.

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1. INTRODUCTION

Because of the small cross dimension, compared to the longitudinal size, of most of the estuarine systems, the main features of the tidal hydrodynamics in estuaries can be efficiently reproduced and modelled using an one-dimensional approach (UNCLES, 1981; PRANDLE, 1984; GODIN, 1988). However, even the one-dimensional modelling implies a quite complex computation when bottom friction effects and the geometrical variations of breadth and effective depth of the estuary must be considered. In fact, to achieve satisfactorily this objective, it is necessary to solve the hydrodynamic equations by numerical integration. Because of this and in a general sense, the computation of the tidal current regime in any estuary is a task for physical oceanographers while for other professionals related to the marine environment this kind of information used to be difficult to obtain when needed.

In this paper, the authors present a simple method which allows, under certain assumptions, the prediction of tidal currents in short length convergent channels by using a proper nautical chart and the harmonic constants of the tidal elevation for a harbour located in the channel. Some numerical experiments on theoretical convergent channels with exponentially variant breadth and depth, similar to the one designed by PRANDLE (1984), will lead to the determination of the geometrical parameters that allow the application of the proposed method. It has been validated for two estuaries of the Gallega Estuarine System which have these characteristics.

The article is organised as follows: in Section 1, the theoretical basis for the method is developed. Section 2 is dedicated to describe the procedure of the numerical simulation. In Section 3, the validity of the method is determined by results of the previous section. In Section 4, the authors apply the method on the Vigo and Pontevedra estuaries and the results are discussed. Finally the conclusions are drawn in Section 5.

1. Theoretical Basis

The linearised tidal hydrodynamics equations, the momentum and the mass balance equations, for the one-dimensional problem are:

$$\frac{\partial u}{\partial t} = -g\frac{\partial \zeta}{\partial x} - ru \qquad (1)$$

$$\frac{\partial (Au)}{\partial x} = -b\frac{\partial \zeta}{\partial t} \qquad (2)$$

where u is the current velocity, ζ is the sea level height, A is the cross sectional area of the channel, b is the breadth, h is the effective depth which is defined as A/b, x is the along axis co-ordinate, t is the time and r is the parameter arising from the linearisation of the bottom friction term and it is defined as (TAYLOR, 1919):

$$r=\frac{8kU(x)}{3\pi h} \tag{3}$$

where k reads for the bottom friction coefficient and $U(x)$ is the amplitude of the tidal current velocity as a function of x.

The solution of the system of Equations (1) and (2) can be expressed in the general form:

$$\zeta=Z(x)\cos\left[\omega t-\delta_{\zeta}(x)\right] \tag{4}$$

$$u=U(x)\cos\left[\omega t-\delta_{u}(x)\right] \tag{5}$$

where ω is the angular speed of the considered tidal wave, $Z(x)$ is the amplitude of the tidal elevation, and $\delta_{\varsigma}(x)$ and $\delta_{u}(x)$ are the phase lags of the tidal elevation and the current velocity respectively.

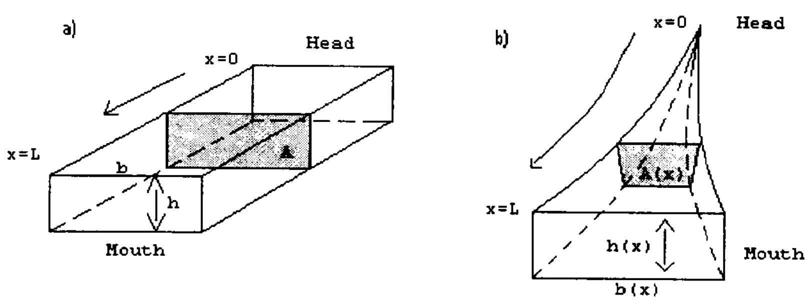

FIG. 1.- Scheme of two ideal channels closed at its inner side,
a) Constant rectangular cross section, b) variable rectangular cross section.

The analytical solution of Equations (1) and (2) in the case of a channel of a constant rectangular section closed at its inner side (see Fig. 1a) is:

$$Z(x)=Z_0\left[\cos^2(a_z x)\cosh^2(b_z x)+\sin^2(a_z x)\sinh^2(b_z x)\right]^{1/2} \tag{6}$$

$$\delta_{\zeta}(x)=\arctan\left[\frac{\sin(a_z x)\sinh(b_z x)}{\cos(a_z x)\cosh(b_z x)}\right] \tag{7}$$

$$U(x)=\frac{Z_0 g}{\omega\left(1+\frac{r^2}{\omega^2}\right)}(a_u^2(x)+b_u^2(x))^{1/2} \tag{8}$$

$$\delta_u(x)=\arctan\left(\frac{b_u(x)}{a_u(x)}\right) \tag{9}$$

where Z_0 is the amplitude of the tidal elevation at the head of the channel. The x co-ordinate is 0 at the head and L at the mouth. The r parameter has been re-defined as independent of the x co-ordinate and it is expressed as (GODIN and MARTÍNEZ, 1994):

$$r=\frac{km}{2h} \tag{10}$$

where m is a constant whose optimum value is 0.7 for the velocity range 0.2 to 1.0 $m \cdot s^{-1}$ (GODIN, 1988) and h is the depth of the channel. The other parameters of Equations (6) to (9) arising from the analytical solution are expressed as follows:

$$a_z=\frac{\omega}{\sqrt{gh}}\left(1+\frac{r^2}{\omega^2}\right)^{1/4}\cos\left(\arctan\left(\frac{r}{\omega}\right)\right) \tag{11}$$

$$b_z=\frac{\omega}{\sqrt{gh}}\left(1+\frac{r^2}{\omega^2}\right)^{1/4}\sin\left(\arctan\left(\frac{r}{\omega}\right)\right) \tag{12}$$

$$a_u(x)=\left(\frac{a_z r}{\omega}-b_z\right)\sin(a_z x)\cosh(b_z x)-\left(a_z+\frac{b_z r}{\omega}\right)\cos(a_z x)\sinh(b_z x) \tag{13}$$

$$b_u(x)=\left(\frac{b_z r}{\omega}+a_z\right)\sin(a_z x)\cosh(b_z x)+\left(\frac{a_z r}{\omega}-b_z\right)\cos(a_z x)\sinh(b_z x) \tag{14}$$

Now, this theoretical result will be used to investigate the joined capability of the depth and the length of the channel to produce variations of the amplitude and phase lag of the elevation and the current velocity associated to a tidal wave.

In order to characterise such variations some additional parameters must be defined:

a) the damping or amplification rate, R, as the ratio between the amplitudes of the tidal elevation at the head and at the mouth, obtained by the evaluation of Equation (6) at $x=L$:

$$R=\frac{Z_0}{Z(x=L)}=\left[\cos^2(a_z L)\cosh^2(b_z L)+\sin^2(a_z L)\sinh^2(b_z L)\right]^{-1/2} \tag{15}$$

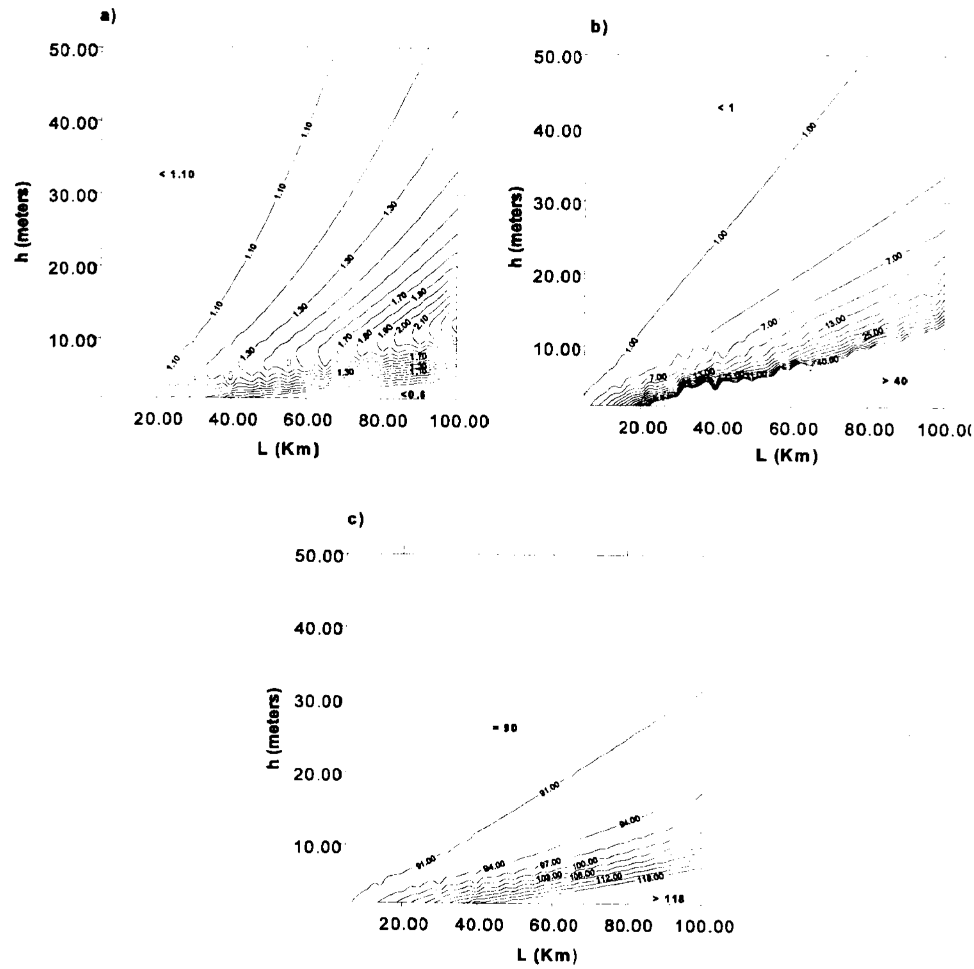

FIG. 2.- Values of the parameters; a) R, b) $\Delta\delta$ and c) $\Delta\varphi$ for the case of the channel of constant rectangular cross section along different values of h and L. Computations have been done taking the values; $\omega=\omega_{M2}=1.4\ 10^{-4}\ s^{-1}$, $K=2.5\ 10^{-3}$ and $Z_0=1$ m.

b) the phase lag of the tidal elevation at the head with respect to the tidal elevation at the mouth. Since a proper choice of the phase lag frame is to select it as 0 at the head, $\Delta\delta$ can be computed from the evaluation of the Equation (7) at $x=L$:

$$\Delta\delta = \delta_\zeta(x=0) - \delta_\zeta(x=L) = \arctan\left[\frac{-\sin(a_z L)\sinh(b_z L)}{\cos(a_z L)\cosh(b_z L)}\right] \qquad (16)$$

c) the averaged channel value of the phase lag of the current velocity with respect to the tidal elevation:

$$\Delta\varphi = \overline{\delta_u(x) - \delta_\zeta(x)} \qquad (17)$$

The evaluation of these parameters for different values of depth, h, and length, L, for the frequency of the M2 wave, $\omega = 1.4 \cdot 10^{-4}$ rad•s^{-1}, with a bottom friction coefficient of $k = 2.5 \cdot 10^{-3}$ and an amplitude for elevation at the mouth of $Z_0 = 1$m is shown in Figure 2. It can be observed that R and $\Delta\delta$ decrease when h and L increase. For a channel of length L, a_z and b_z decrease when h increases and $R \to 1$ and $\Delta\delta \to 0$ (Equations (15) and (16)). On the other hand, for a fixed value of depth, or keeping a_z and b_z constant, the decreasing of L produces the same results on R and $\Delta\delta$.

In relation to $\Delta\varphi$ (Equation (17)), it tends to be 90 degrees when h increases and L decreases. The quadrature value implies a stationary behaviour of the tidal oscillation and similar variation of the phase lag of the tidal current and elevation. Therefore, a value of $\Delta\delta$ close to zero degree with a 90-degree phase lag between tidal elevation and currents implies a small along channel phase lag of the current velocity gradient.

Some combinations of depth and length can verify the values of $R \cong 1$, $\Delta\delta \cong 0$ and $\Delta\varphi \cong 90$. Even for the shallowest case, 2 meters, these values are observed for channels of less than 20 km length. This means that for these channels it can be assumed that

$$\frac{dZ}{dx} \approx 0 \qquad (18)$$

$$\frac{d\delta_u}{dx} \approx 0 \qquad (19)$$

$$\frac{d\delta_\zeta}{dx} \approx 0 \qquad (20)$$

$$\delta_u = \delta_\zeta + 90^\circ \qquad (21)$$

Expressing the mass conservation (Equation (2)) in terms of the solutions (Equations (4) and (5)) for the case of a channel of constant rectangular cross section,

$$\frac{dU}{dx}\cos(\omega t - \delta_u) + U\frac{d\delta_u}{dx}\sin(\omega t - \delta_u) = \frac{Z}{h}\omega\sin(\omega t - \delta_\zeta) \qquad (22)$$

and assuming the conditions of the expressions given by equations(18) to (21), it is simplified to

$$\frac{dU}{dx} = \frac{Z}{h}\omega \qquad (23)$$

Considering a more general case where the depth and the breadth of the channel can change with the x co-ordinate (see Fig. 1b) , the mass balance is from Equation 2:

$$\frac{d(AU)}{dx}\cos(\omega t-\delta_u)+AU\frac{d\delta_u}{dx}\sin(\omega t-\delta_u)=bZ\omega\sin(\omega t-\delta_u) \qquad (24)$$

and with Equations (18) to (21):

$$\frac{d(AU)}{dx}=bZ\omega \qquad (25)$$

where in Equations (23) and (25), Z is assumed to be constant.

The simplification on the mass conservation equation given by Equations (23) and (25) implies that when the length of the channel allows to admit the conditions given by Equations (18) to (21), the maximum and minimum of the elevation and the current velocity can be assumed to be on phase, happening at the same time in the whole of the channel. Also the amplitude of the current velocity can be computed from the mass conservation equation being possible disregard the momentum balance equation.

If Equation (25), is integrated from the head, $x=0$, to the mouth of the channel, $x=L$, using an approximation of centred finite differences, the tidal current velocity amplitudes can be computed by:

$$U_i=\frac{A_{i-1}U_{i-1}}{A_i}+\frac{\overline{b_{i-1/2}}Z\omega\Delta x}{A_i} \qquad (26)$$

where Δx is the distance between two consecutive sections. This equation said that the amplitude of the tidal current velocity in a given cross section i can be computed from the amplitude of the current velocity at the previous section $i-1$, the cross sectional area at the previous section $i-1$, the averaged breadth between the sections $i-1$ and i, $\overline{b_{i-1/2}}$, the values of cross sectional area at section i, the amplitude of the tidal elevation and the frequency of the considered tidal wave. The computation would start with the calculus of the amplitude of the current velocity in the second section, U_1, after a value for the amplitude of the current velocity at the head, $U_0=0$, for no flux boundary condition is assumed.

So, the tidal current velocity can be computed quite approximately with very few data knowing the geometric characteristics of the channel, its breadth and depth variations from the nautical chart, and a value of the amplitude of the tidal elevation at an inner harbour. Although, it is implicit that the tidal behaviours accomplish a stationary character with slight variations of Z, δ_ζ and δ_u with x.

The question now is how shorter a channel must be and, at the same time, what characteristics must be present in depth and breadth variations to compute the tidal currents through the Equation (26). In the next section it will be presented a set of numerical experiments on convergent channels varying exponentially in depth and breadth which will allow to define the geometrical conditions for what approximation given by Equation (26) can be applied on.

2. Description of the numerical experiments

The numerical experiments consisted of the numerical integration of the system of differential equations, Equations (1) and (2), using centred finite difference schemes in several convergent channels varying exponentially in depth and breadth with the along channel co-ordinate x (PRANDLE, 1984):

$$h(x) = h_o e^{\beta x} \tag{27}$$

$$b(x) = b_o e^{\alpha x} \tag{28}$$

where x varies from $x=0$ at the head to $x=L$ at the mouth and h_0 and b_0 are the depth and breadth at the head respectively. The constants α and β mean for the rates of exponential change in depth and breadth respectively.

The different geometrical configurations correspond to channels of five lengths; L=10, 20, 30, 40 and 50 km. Such configurations were generated to different pairs of values (α_n, β_m) where $\alpha_n = n/L$, with n varying from 0 to 4 with an increment of Δn=0.2 and $\beta_m = m/L$, with m varying from 0 to 4 each Δm=0.2. The limit of 4 for n and m is established for the geometrical configurations of which value of the relations $h(x=L)/h_0$ and $b(x=L)/b_0$ are greater than e^4= 54 and they will not be considered. The most of the real convergent channels are contained under this limit.

The numerical solution to Equations (1) and (2) in each generated channel were obtained following the numerical integration scheme given by DEFANT (1961). In this technique, the forms of the solutions are firstly assumed as:

$$\zeta = Z(x)\cos\left[\omega t - \delta_\xi(x)\right] = Z_c(x)\cos\omega t + Z_s(x)\sin\omega t \tag{29}$$

$$u = U(x)\cos\left[\omega t - \delta_u(x)\right] = U_c(x)\cos\omega t + U_s(x)\sin\omega t \tag{30}$$

where

$$Z(x) = \sqrt{Z_c^2(x) + Z_s^2(x)} \tag{31}$$

$$U(x) = \sqrt{U_c^2(x) + U_s^2(x)} \tag{32}$$

are the tidal elevation and current velocity amplitudes and

$$\delta_\zeta(x) = \arctan\left(\frac{Z_s}{Z_c}\right) \tag{33}$$

$$\delta_u(x) = \arctan\left(\frac{U_s}{U_c}\right) \tag{34}$$

their respective phase lags.

The following time independent equations are obtained by the substitution of Equations (20) and (30) into Equations (1) and (2). The two first come from the momentum equation and the two last from the mass conservation:

$$\frac{dZ_c}{dx} = -\frac{1}{g}(rU_c + \omega U_s) \tag{35}$$

$$\frac{dZ_s}{dx} = \frac{1}{g}(rU_s + \omega U_c) \tag{36}$$

$$\frac{d(AU_c)}{dx} = -b\omega Z_s \tag{37}$$

$$\frac{d(AU_s)}{dx} = b\omega Z_c \tag{38}$$

Now, these expressions are integrated from the head to the mouth after a discretisation of the spatial dominion in 'n' cross sections equally spaced a distance Δx and being given its co-ordinate x by $x_i = i \bullet \Delta x$, with i varying from 0 to n, being the total channel length $L = n \bullet \Delta x$.

In this way, Equations (37) and (38) were applied on the even sections, i=0, 2, 4,...,n ,yielding the expressions for U_c and U_s :

$$U_{c(i+2)} = -\frac{b_{(i+1)}\omega Z_{s(i+1)} 2\Delta x}{A_{(i+2)}} + \frac{A_{(i)} U_{c(i)}}{A_{(i+2)}} \tag{39}$$

$$U_{s(i+2)} = \frac{b_{(i+1)}\omega Z_{c(i+1)} 2\Delta x}{A_{(i+2)}} + \frac{A_{(i)} U_{s(i)}}{A_{(i+2)}} \tag{40}$$

On the other hand, Equations (35) and (36) were applied on the odd sections, i=1, 3,...,n-1, yielding the expressions for Z_c and Z_s:

$$Z_{c(i+2)} = -\frac{1}{g}(\omega U_{s(i+1} + r_{i+1} U_{c(i+1})2\Delta x + Z_{c(i)} \tag{41}$$

$$Z_{s(i+2)} = \frac{1}{g}(\omega U_{c(i+1)} + r_{(i+1)} U_{s(i+1)})2\Delta x + Z_{s(i)} \tag{42}$$

The computation starts with $i=0$ in Equations (39) and (40), computing the values $U_c(2)$ and $U_s(2)$, at the third section, after fixing the boundary condition at the head, $U_c(0)=U_s(0)=0$ and the values $Z_c(1)=1$ meter and $Z_s(1)=0$, at the second section. The value $Z_s(1)$ is taken as zero in order to establish a zero value for the tidal elevation phase at the head, actually at the second section. Furthermore, the phases for tidal elevation and current velocity at any other section are referred to the tidal elevation

elevation phase at the head. The $U_c(2)$ and $U_s(2)$ values are used to compute the values $Z_c(3)$ and $Z_s(3)$, taking $i=1$ in Equations (41) and (42). In this way, the pairs of Equations (39), (40) and (41), (42) are alternatively used to compute current velocity and tidal elevation. From these values the amplitude and phase lag are recovered from Equations (31) to (34).

The results of the numerical simulation will be resumed in the parameters R, $\Delta\delta$ and $\Delta\varphi$ defined in Equations (15) to (17). In addition it will be useful a new parameter defined as:

$$E_r = \frac{\frac{1}{n}\sum_{i=1}^{n}\left[U_m(x_i) - U_a(x_i)\right]}{\overline{U_m}} \cdot 100 \qquad (43)$$

which represents the along channel averaged value of the differences in each section between the amplitude of the current velocity computed from the numerical model, U_m, and the solution from Equation (26), U_a will be useful. This quantity is divided by the averaged along channel current velocity and it is expressed in percentage. The E_r parameter is used to estimate the degree of approach of the U_a and U_m. It can be read as a % of relative residue with respect to the averaged along channel current velocity amplitude.

3. Results from numerical experiments

The discussion and comments of this section will be referred to the Figures 3 to 5 where the contour maps of parameters R, $\Delta\delta$, $\Delta\varphi$ and E_r for the different values of n and m and for several length of the channel are shown. All results are characterised for the M_2 tidal wave and for a bottom friction coefficient of $2.5 \cdot 10^{-3}$. A characteristic value of the depth at the head, h_0=2 m, has been fixed for all simulations.

In the case of L=10 m, the E_r values never are greater than 2%, so the approach of the simplified model given Equation (16), U_a, to the full model, U_m, is rather good. This is in agreement with the value of R, very close to 1, the value of $\Delta\delta$ close to 0 and a value of $\Delta\varphi$ close to 90 degrees for the full range of n and m. So, the simplified model of Equation (26) can be applied on convergent channels of L=10 km with a depth at the head of h_0=2m and for arbitrary exponential variations of depth and breadth.

For the case of L=20 km, the E_r values are never greater than 4%. So the approach of U_a to U_m is quite satisfactory. However, significant deviations of $\Delta\delta$ from zero can be found for a wide range of n and m values. This means that although the amplitude of the current velocity is being well estimated from Equation (26) in the whole range of n and m, its along channel phase lag gradient is far from zero, leading to an uncertainty in the prediction of the time of maximum current along the channel. However if a small uncertainty is admitted for the phase lag of the current estimation, for $\Delta\delta$ values less than 10 degrees, the relative error parameter, E_r, is less than 6% for any case. This magnitude of the error can be accepted to perform the prediction of the amplitude and phase lag of the current. The Ten Degrees Line (TDL) can be assimilate as a straight line mapped for different combinations of m and n and for all the considered lengths of the channels.

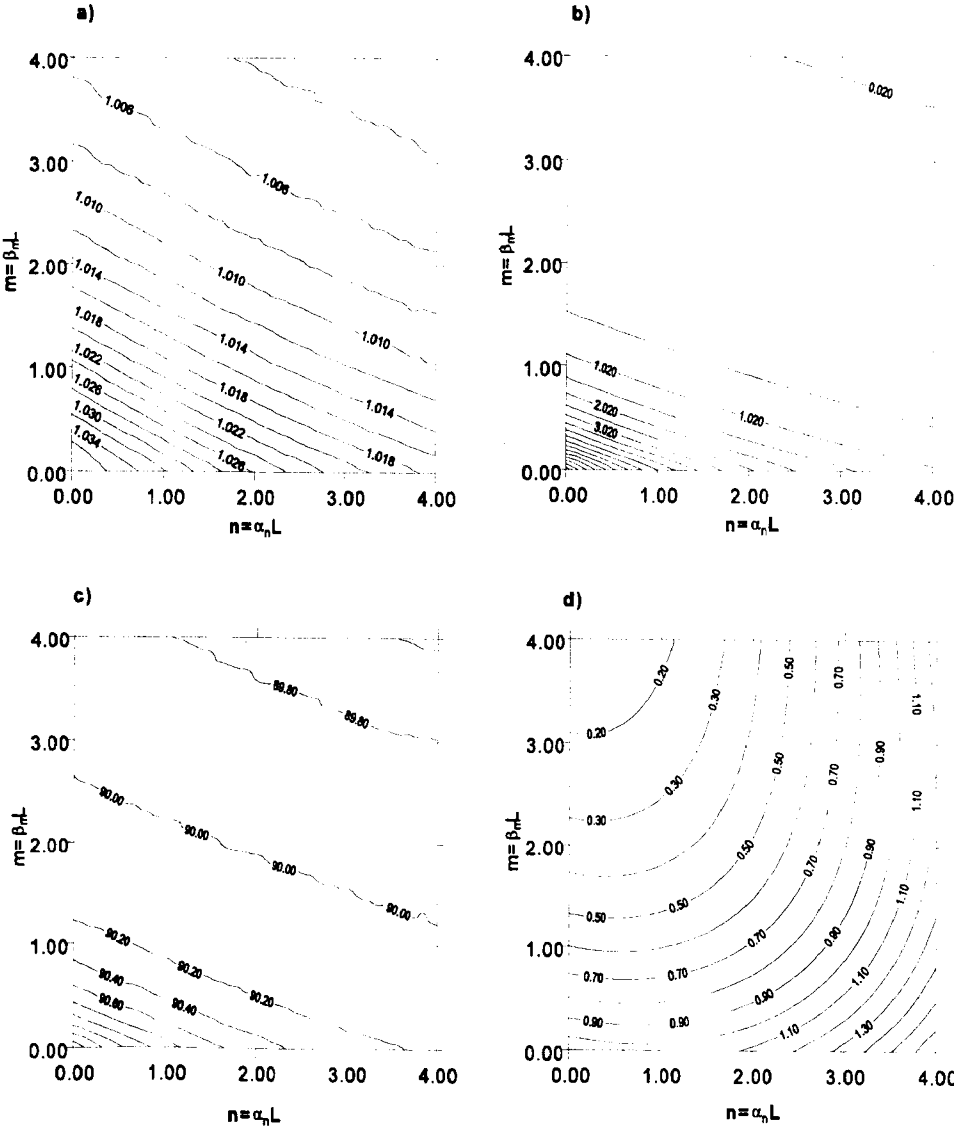

FIG. 3.-Values of the parameters; a) R, b) Δδ, c) Δφ and d) E, for the case L=10 km along different values of n and m. Computations have been done taking the values; $\omega=\omega_{M2}=1.4\ 10^{-4}\ s^{-1}$, $k=2.5\ 10^{-3}$, $h_0=2.0$ m and $Z_0=1$ m.

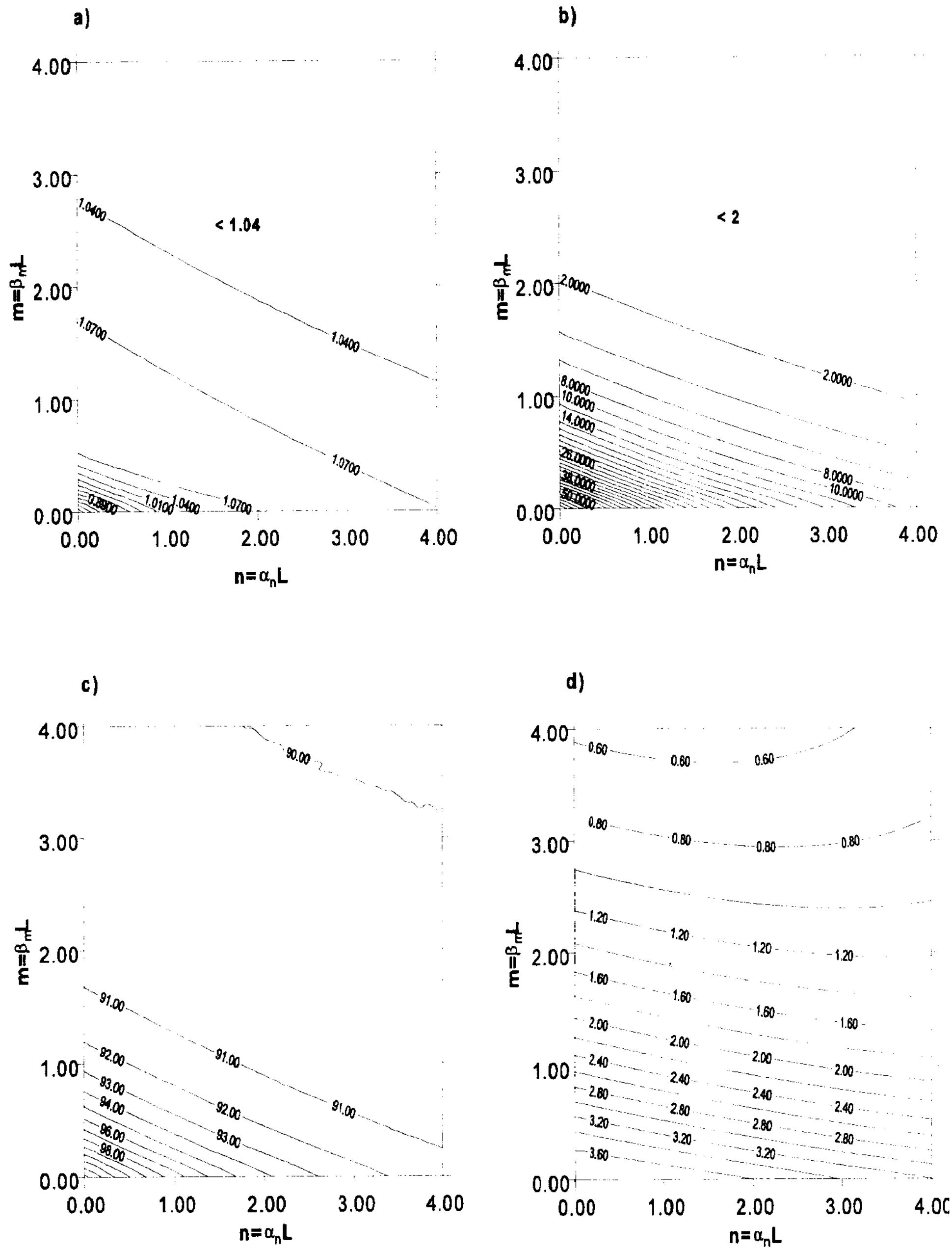

FIG. 4.- Values of the parameters; a) R, b) $\Delta\delta$, c) $\Delta\varphi$ and d) E_r for case L=20 km along different values of n and m. Computations have been done taking the values; $\omega=\omega_{M2}=1.4\ 10^{-4}\ s^{-1}$, $k=2.5\ 10^{-3}$, $h_0=2.0$ m and $Z_0=1$ m.

For the remaining cases, the TDL is located upper, decreasing the validity dominion of Equation (26) (Fig. 6). In this way, this region is reduced in the more limitating case of *L*=50 km to above the line joining the points *n*=0, *m*=3.1 and *n*=4,

m=1.9. It is easy to verify that the validity dominion is greater in channels with greater depth at the head.

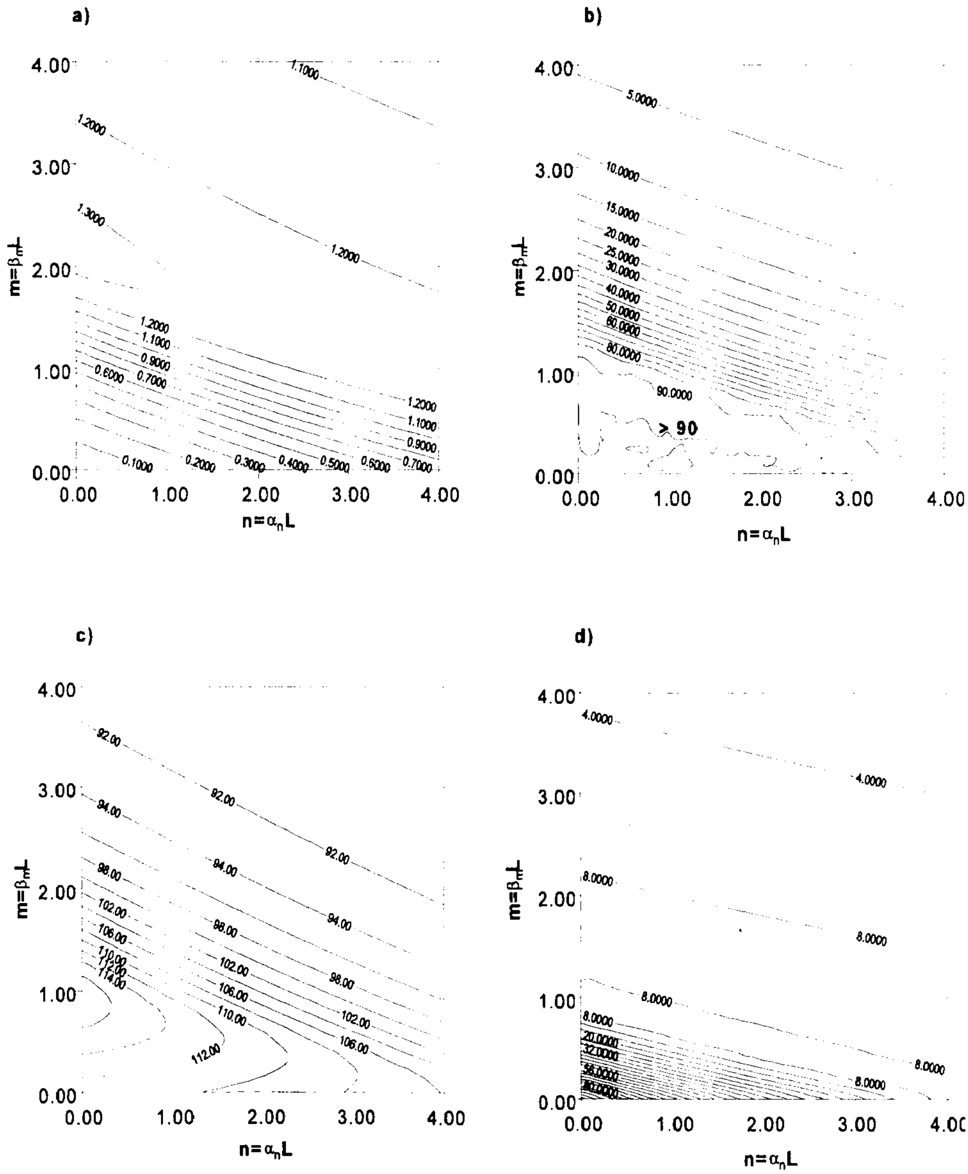

FIG. 5.- Values of the parameters; a) R, b) $\Delta\delta$, c) $\Delta\varphi$ and d) E_r for the case L=50 along different values of n and m. Computations have been done taking the values; $\omega\omega_{M2}$=1.4 10^{-4} s^{-1}, k=2.5 10^{-3}, h_0=2.0 m and Z_0=1 m.

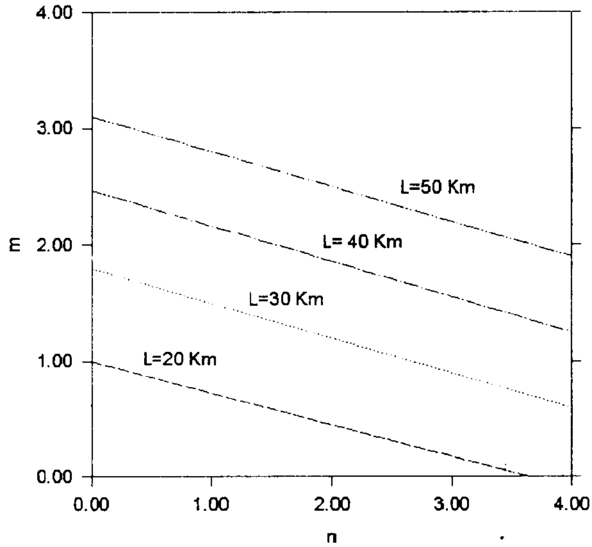

FIG. 6.- TDL plot characterising the iso -Δδ -lines of 10° for different L.

4. Application to the Vigo and Pontevedra Estuaries

In order to analyse the feasibility of the approach given by Equation (26) in a practical context, the method has been applied on two Spanish characteristic short length convergent channels: the Vigo and Pontevedra estuaries. The Vigo and Pontevedra estuaries belong to the Gallega Estuarine System. They are located in the Northwest coast of Spain (Fig. 6). The sections where the tidal elevation and current velocity were estimated are also shown.

The application of the method consists in the computation of the values for the tidal elevation and currents of the M2 tidal wave by the full model and from the simplified model of Equation (26). After this, the computation is on N_2, M_2 and S_2 tidal waves and the comparison with the harmonic constants from current velocity recordings in the locations C1 and C2 (Fig. 2) inside the Vigo Estuarie is made.

a) Comparison with numerical model solution

In Table 1 the length, *L*, the depth at the head, h_0, and the geometrical parameters *n* and *m* characterising the breadth and depth exponential variation of the Vigo and Pontevedra are shown. It can be observed in Figure 6 that the points (*n*,*m*) are located above the respective critical lines corresponding to the two estuaries, Vigo $L \approx 20$ km and Pontevedra $L \approx 10$ km, henceforth a good approximation of U_a to U_n values as well as a close to zero along channel phase lag of the current velocity should be found.

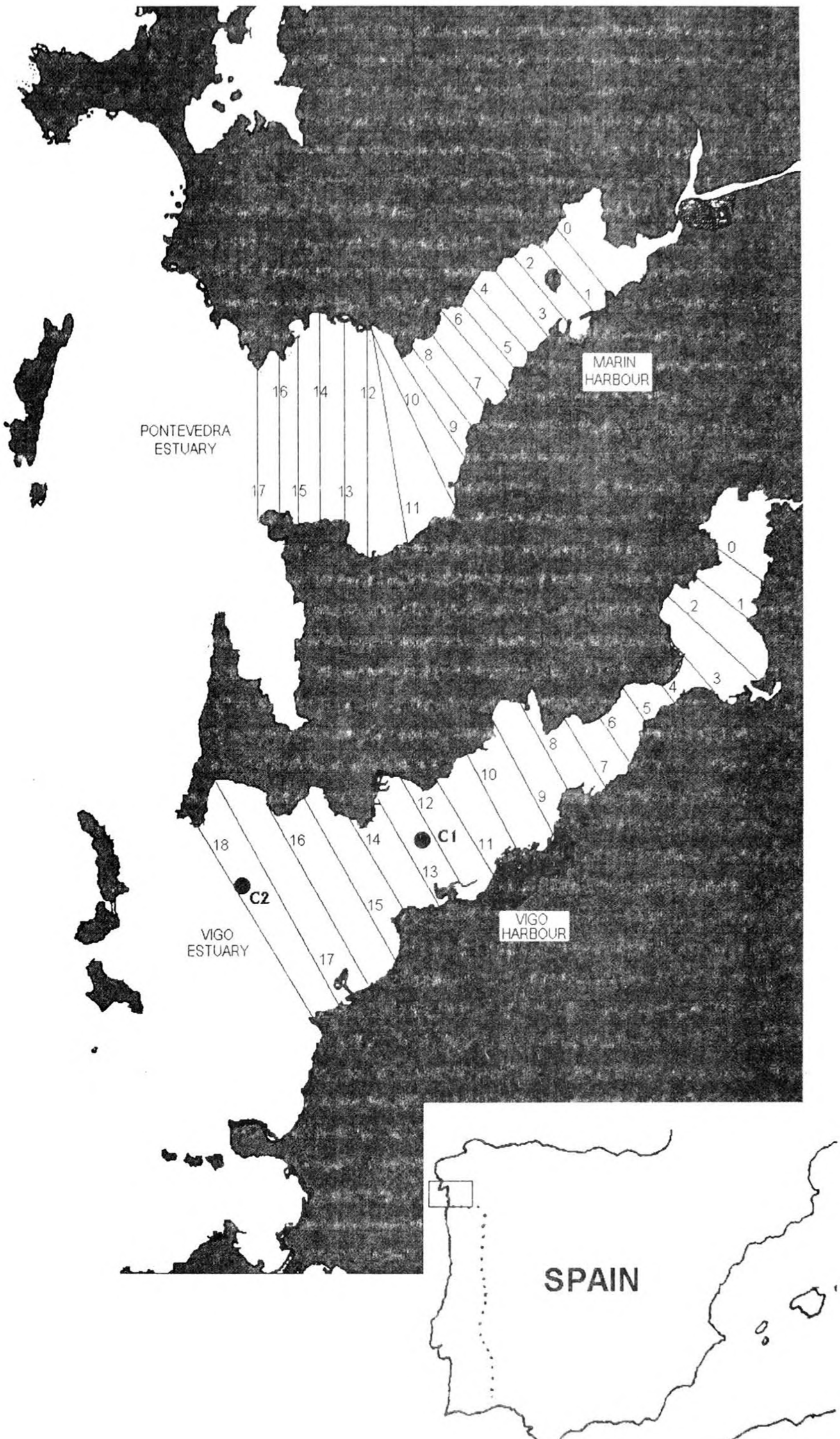

FIG. 7.- Map of the Vigo and Pontevedra Estuaries showing the cross sections used in the spatial discretisation of the estuaries, the placement of current velocity measurements C1 and C2 and the placement of the sea elevation measurements at Marín and Vigo Harbours.

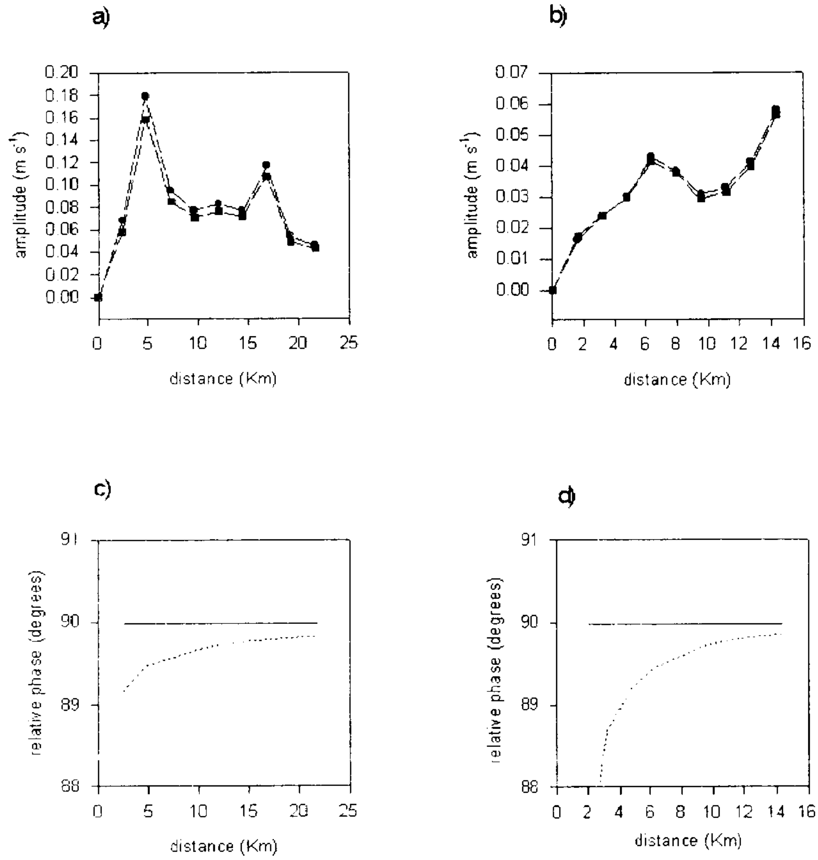

FIG. 8.- Comparison between the amplitudes and phase lags of current velocity of the M2 tidal wave computed by Equation (26) and by the numerical solution described in section 2. In a) for Vigo and b) for Pontevedra solid circles stand for U_m and solid square for U_a. In c) for Vigo and d) for Pontevedra, are presented the relative phase lag of the current velocity with respect to the phase of the elevation at the head, with solid line for Equation (26) and dotted line for the numerical solution.

Table 1 - Values of geometrical parameters for the Vigo and Pontevedra Estuaries

Estuary	**h_o(m)**	**L (Km)**	**α (m^{-1})**	**β (m^{-1})**	**n**	**m**
Vigo	2.0	21.6	8.29E-5	1.27E-4	1.79	2.743
Pontevedra	2.0	14.3	9.54E-5	2.03E-4	1.36	2.905

The phase lag for the current of the simplified and the full model associated to M2 tidal wave is obtained for Vigo and Pontevedra are shown in Figure 9. For both, the computations of U_a and U_m are from the amplitude of the tidal elevation taken from Vigo and Marin Harbours within the Vigo and Pontevedra estuaries respectively and where harmonic constants of tidal elevation were available (Table 2). Considering constant values for them along the estuary in the U_a computation and using them to fix the elevation boundary condition at the head in the U_m computation. The U_a and U_m agree quite well and the along channel current phase lag variation is very close to zero in the two estuaries. So, the presented method to compute tidal current velocities U_a can be successfully applied on those estuaries as it was expected from the numerical experiment results of section 3.

Table 2 - Harmonic constants of the main semidiurnal tidal waves for elevation in Vigo and Marín Harbours. Z means for the amplitude in meters and g_ξ is Greenwich phase lag in degrees

	Vigo Harbour		Marin Harbour	
Component	**Z**	**gξ**	**Z**	**gξ**
N_2	0.219	69.53	0.210	67.00
M_2	1.089	77.94	1.040	82.00
S_2	0.390	102.32	0.380	110.00

b) Comparison with the observed data

This comparison have been carried out by means of the along channel current velocity associated to the M2, N2 and S2 tidal waves. Data were taken from two currentmeters located in the Vigo estuary (Fig. 6) and the harmonic constants have been estimated by a Least Square Harmonic Analysis directly on data (FOREMAN, 1976). In Table 3, the harmonic constants from real data and the ones obtained by Equation (26) are shown together. All quantities, amplitudes and phase lags from real data and model, quite agree, showing that the proposed approach is quite well to modelling the tidal currents in the along axis of the channels that present certain geometrical characteristics.

Table 3 - Amplitudes (in cm s^{-1}) and Greenwich phase lags (degrees) for the main semidiurnal tidal waves in the along channel current velocity at two placements (C1 and C2) within the Vigo Estuary. U_{ob} and g_{ob} stand for amplitudes and Greenwich phases lags obtained by least squares harmonic analysis on recorded data. U_a and g_a stand for the values computed from Equation (26).

Component	U_{ob}	U_a	g_{ob}	g_a	U_{ob}	U_a	g_{ob}	g_a
N_2	2.3	1.4	5.9	-20.5	0.7	0.8	25.8	-20.5
M_2	11.3	7.2	14.3	-12.6	4.3	4.3	34.2	-12.6
S_2	4.0	2.7	39.0	12.3	1.6	1.6	58.6	12.3

5. Conclusions

From the present study it could be detached that in short length elongated convergent channels a very feasible approach of tidal current velocity in the along channel direction can be obtained through the very simple formula represented by Equation (26). The only necessary information are the harmonic constants of the tidal elevation in any placement inside the channel and a resolved enough nautical chart of the channel. In addition, a practical way to inquire when a channel allows the application of Equation (26) is proposed by the use of graphical representations for the 10 degree lines of maximum errors, or TDL (Fig. 6). From the *m* and *n* values characterising the breadth and depth variations of a given channel and provided the point (n,m) locates above the TDL corresponding to the length of the channel L, the authors can expect that the estimation of the tidal current using Equation (26) will yield a successful approach.

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