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MEASURING AREAS ON LARGE SCALE NAUTICAL CHARTS ON MERCATOR-SECANT

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Abstract

This short paper presents a highly accurate module of surface deformation which permit to calculate, once and for all, the ellipsoid area corresponding to the area measured on nautical charts on Mercator-Secant.

The accuracy of the measured surface increases together with the increase of the scale because it depends on the graphic error.

PRESENTATION

Nautical charts produced at the *Istituto Idrografico della Marina* are Mercator-Secant.

The secant parallel φ_o is known as middle parallel because, in general, it is selected in the central area of the chart; its value is indicated in the chart title.

The surface deformation module of the Mercator-Secant chart may be expressed as follows:

$$(1) \qquad M = \frac{\Delta X \cdot \Delta Y}{[\Delta\lambda] \cdot N_O^2 \cdot [\sin\phi_B - \sin\phi_A] \cdot S_{*}^{2}} \quad {}^{(2)}$$

where:

$$N_o = a \cdot \left(1 - e^2 \cdot \sin^2\varphi_o\right)^{-1/2}$$

while the parametres of the HAYFORD ellipsoid are:

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[2] The expressions in parentheses are given in radians

$$\begin{cases} a = 6378388 \text{ meter} \\ e^2 = 0{,}00672267 \end{cases}$$

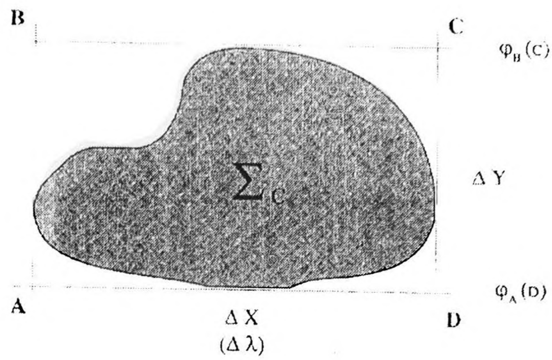


Figure 1

In Figure 1, ΔX and ΔY are measured on the chart as if they were the sides of a rectangle containing the surface to be measured. They should be expressed with the same measure unit as N_o.

[$\Delta\lambda$] is the difference between longitude of radians of the rectangle above, as measured on the chart.

Taking into consideration the following formulae:

$$\phi_A = f(\varphi_A)$$
$$\phi_B = f(\varphi_B)$$

$$(2) \quad \begin{cases} \psi = \tan^{-1} \cdot \left(\tan\varphi \cdot \left(1 - e^2\right)\right) \\ \phi = \psi + \sin^{-1} \cdot \left(\cos\psi \cdot e^2 \cdot \sin\varphi_o\right) \end{cases}$$

S_o = Chart scale

$$S_* = \frac{\cos\varphi_o}{\cos\bar{\phi}} \cdot S_o = \text{scale of middle parallel of surface } \Sigma_C$$

$$\bar{\phi} = \frac{\phi_A + \phi_B}{2}$$

The area Σ_C may be measured on the chart with analytic, graphic or mechanical methods, such as - for instance - the Amsler Polar planimetre.

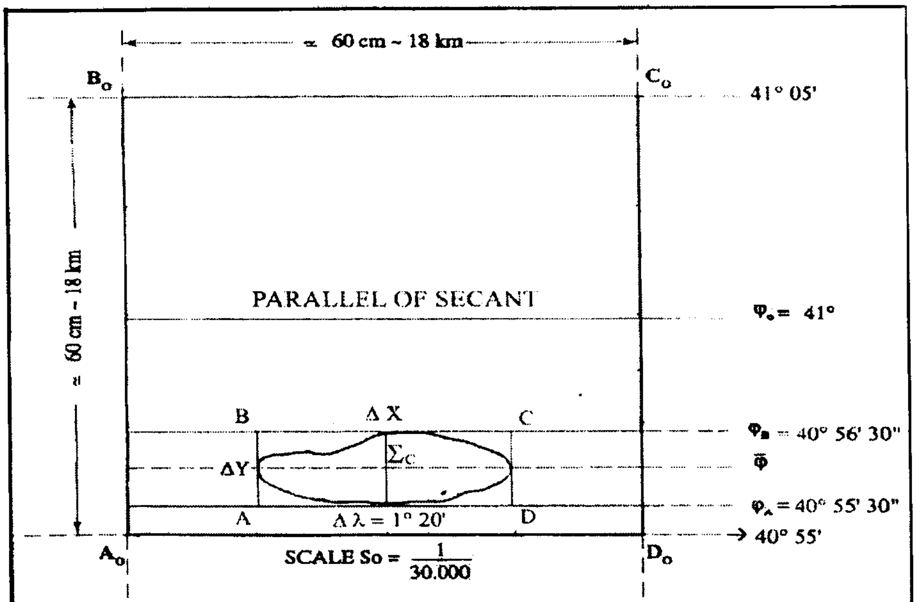
The equation (1), i.e., the module M of surface deformation, is derived from the "normal sphere" of A. VASSALLO, which is a sphere with radius N_o, and thus tangent to the ellipsoid along the whole parallel $\varphi_o = \phi_o$ of the ellipsoid itself.

Such sphere, within the band $\Delta\varphi = 2°$ ($\varphi_o \pm 1°$), represents the conterminous ellipsoid band with geodetic accuracy.

Consequently, after measuring the surface Σ_C on the chart (fig. 1), the corresponding surface on the ellipsoid will be:

$$(3) \quad \Sigma_E = \frac{\Sigma_C}{M}$$

It is again reminded that the accuracy of the measurement of Σ_E increases together with the increase of the scale. Therefore, it depends on the graphic error.



$$N_O = a \cdot (1 - e^2 \sin^2 \varphi_o)^{-\frac{1}{2}}$$

$$M = \frac{\Delta X \cdot \Delta Y}{[\Delta Y]\, N_O^2\, [\sin\phi_B - \sin\phi_A]\, S^2_o} \quad (1)$$

$$\begin{cases} \psi = \tan^{-1}(\tan\varphi\,(1 - e^2)) \\ \phi = \psi + \sin^{-1}(\cos\psi - e^2 \cdot \sin\varphi_o) \end{cases} \quad (2)$$

$$\Sigma = \frac{\Sigma_C}{M} \quad (3)$$

Figure 2

NUMERICAL EXAMPLE

The ellipsoid surface Σ_E is calculated after measuring the surface Σ_C on the chart, in scale below (see Figure 2):

$$S_O = \frac{1}{30000}$$

With an appropriate simulation, we obtain :

$$\Delta X = 6{,}22 \text{ cm.} = 0{,}0622 \text{ m.}$$

$$\Delta Y = 6{,}15 \text{ cm.} = 0{,}0615 \text{ m.}$$

$$AC = \Delta X \cdot \Delta Y = 0{,}0038253 \text{ m}^2$$

Assuming that:

$$\Sigma_C = 0{,}00229518 \text{ m}^2$$

$$[\Delta\lambda] = 0{,}000387850944887 \text{ (in radians)}$$

$$N_o = 6387636{,}09537 \text{ m.}$$

$$\phi_A = 40°{,}925288508$$

$$\phi_B = 40°{,}9418909773$$

$$\bar{\phi} = 40°{,}9335897426$$

$$S_* = 0{,}99899411396 \cdot S_O = \frac{1}{30030{,}2069661}$$

$$M = \frac{\Delta X \cdot \Delta Y}{[\Delta\lambda] \cdot N_o^2 \cdot [\sin\phi_B - \sin\phi_A] \cdot S_*^2} = 0{,}995795086214$$

$$\Sigma_E = \frac{\Sigma_C}{M} = 0{,}00230487178715 \, \text{m}^2$$

The real surface on the ellipsoid will result from the following:

$$\Sigma_{EO} = \frac{\Sigma_E}{S_*^2} = 2078564{,}10258 \, \text{m}^2 = 2{,}07856410258 \, \text{Km}^2$$

Note: In order to facilitate the determination of surfaces on nautical charts, titles should indicate the following elements:

N_o and formulae (1), (2) and (3).

Furthermore, to measure surfaces Σ_C, a small hyperbolic square may be used, attached to the chart.

References

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