MEASURING AREAS ON LARGE SCALE NAUTICAL CHARTS ON MERCATOR-SECANT

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Abstract

This short paper presents a highly accurate module of surface deformation which permit to calculate, once and for all, the ellipsoid area corresponding to the area measured on nautical charts on Mercator-Secant.

The accuracy of the measured surface increases together with the increase of the scale because it depends on the graphic error.

PRESENTATION

Nautical charts produced at the Istituto Idrografico della Marina are Mercator-Secant.

The secant parallel $\varphi_0$ is known as middle parallel because, in general, it is selected in the central area of the chart; its value is indicated in the chart title.

The surface deformation module of the Mercator-Secant chart may be expressed as follows:

\[ M = \frac{\Delta X \cdot \Delta Y}{[\Delta \lambda] \cdot N_o^2 \cdot [\sin \phi_B - \sin \phi_A] \cdot S^2} \]  

where:

\[ N_o = a \cdot \left(1 - e^2 \cdot \sin^2 \varphi_0\right)^{-\frac{1}{2}} \]

while the parameters of the HAYFORD ellipsoid are:

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2 The expressions in parentheses are given in radians
\[ \begin{align*}
  a &= 6378388 \text{ meter} \\
  e^2 &= 0.00672267
\end{align*} \]

In Figure 1, ΔX and ΔY are measured on the chart as if they were the sides of a rectangle containing the surface to be measured. They should be expressed with the same measure unit as \( N_0 \).

\([Δλ]\) is the difference between longitude of radians of the rectangle above, as measured on the chart.

Taking into consideration the following formulae:

\[ \begin{align*}
  \phi_A &= f(\phi_a) \\
  \phi_B &= f(\phi_B)
\end{align*} \]

\[ (2) \begin{align*}
  \psi &= \tan^{-1}\left(\tan\phi \cdot (1 - e^2)\right) \\
  \phi &= \psi + \sin^{-1}\left(\cos\psi \cdot e^2 \cdot \sin\phi_0\right)
\end{align*} \]

\[ S_0 = \text{Chart scale} \]

\[ S_x = \frac{\cos\phi_0}{\cos\phi} \cdot S_0 \quad \text{scale of middle parallel of surface } \Sigma_C \]

\[ \phi = \frac{\phi_A + \phi_B}{2} \]

The area \( \Sigma_C \) may be measured on the chart with analytic, graphic or mechanical methods, such as - for instance - the Amsler Polar planimetre.
The equation (1), i.e., the module $M$ of surface deformation, is derived from the "normal sphere" of A. VASSALLO, which is a sphere with radius $N_0$, and thus tangent to the ellipsoid along the whole parallel $\phi_0 = \phi_0$ of the ellipsoid itself.

Such sphere, within the band $\Delta \phi = 2^\circ$ ($\phi_0 \pm 1^\circ$), represents the conterminous ellipsoid band with geodetic accuracy.

Consequently, after measuring the surface $\Sigma_C$ on the chart (fig. 1), the corresponding surface on the ellipsoid will be:

$$\Sigma_E = \frac{\Sigma_C}{M}$$

It is again reminded that the accuracy of the measurement of $\Sigma_E$ increases together with the increase of the scale. Therefore, it depends on the graphic error.

Figure 2
NUMERICAL EXAMPLE

The ellipsoid surface $\Sigma_E$ is calculated after measuring the surface $\Sigma_C$ on the chart, in scale below (see Figure 2):

$$S_0 = \frac{1}{30000}$$

With an appropriate simulation, we obtain:

- $\Delta X = 6.22$ cm. = 0.0622 m.
- $\Delta Y = 6.15$ cm. = 0.0615 m.
- $AC = \Delta X \cdot \Delta Y = 0.0038253$ m$^2$

Assuming that:

- $\Sigma_C = 0.00229518$ m$^2$
- $[\Delta \lambda] = 0.000387850944887$ (in radians)
- $N_c = 6387636.09537$ m.
- $\phi_A = 40^\circ.925288508$
- $\phi_B = 40^\circ.9418909773$
- $\phi = 40^\circ.9335897426$

$$S_* = 0.99899411396 \cdot S_0 = \frac{1}{30030.2069661}$$

$$M = \frac{\Delta X \cdot \Delta Y}{[\Delta \lambda] \cdot N_c^2 \cdot [\sin \phi_B - \sin \phi_A] \cdot S_*^2} = 0.995795086214$$

$$\Sigma_E = \frac{\Sigma_C}{M} = 0.00230487178715 \text{ m}^2$$

The real surface on the ellipsoid will result from the following:

$$\Sigma_{EO} = \frac{\Sigma_E}{S_*^2} = 2078564.10258 \text{ m}^2 = 2.07856410258 \text{ Km}^2$$

Note: In order to facilitate the determination of surfaces on nautical charts, titles should indicate the following elements:
Furthermore, to measure surfaces $\Sigma_c$, a small hyperbolic square may be used, attached to the chart.

References


