Article

A Method for Detecting and Adjusting Systematic Errors of Singlebeam Sounding Data Acquired in a Grid Pattern*

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Hydrographic surveys are usually organised in survey lines with a grid of main scheme lines and a smaller number of reference lines used to check the data. The disagreements between two groups of sounding data at the crossing points or overlap are usually the only information available to evaluate the quality of sounding data.
To improve the accuracy of depth measurements, a method has been developed in which the differences between two groups of sounding data at crossing points of main scheme sounding lines and reference sounding lines have been used to detect or adjust the systematic sounding errors. Based on the structure of systematic error in a grid of sounding lines and the concept of the rank-defect adjustment, the formulae of the method are derived in the paper. The method has been tested using both the simulated and observed data. The results show the method works well and the accuracy and efficiency of sounding can be greatly improved in traditional single-beam hydrographic surveys. The principle and method in this paper may be beneficial to the data processing for other marine surveys with a grid-pattern, such as marine gravitimetry and magnetic survey.

Introduction

Hydrographic surveys are usually organised in survey lines with a grid of main scheme lines and a smaller number of reference lines used to check the data (Ingham A E and V J Abbott, 1992). The differences or disagreements between two groups of sounding data at crossing points of the main scheme sounding line and the reference sounding line are usually the only information available to evaluate the quality of sounding data using some statistical method in most coastal countries (Liu Y C, 2003). In a general way, the sounding data in a whole survey grid will be acceptable if their differences at crossing points are satisfied with some qualifications, otherwise, the sounding data will be unacceptable and a re-sounding will be needed. It is rarely researched that if these differences could be used, the accuracy and efficiency of sounding will be improved. In fact, three questions should be further answered by hydrographic surveyors in the face of the disagreements between two groups of sounding data at crossing points of main scheme lines and reference lines. Question 1 is what are the characteristic of the

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disagreements, random or systematic? Question 2 is how to discover or detect the systematic disagreements or errors effectively in case there exist systematic errors in the sounding lines; Question 3 is how to adjust the systematic disagreements or errors after the systematic errors have been determined.

In most circumstances of hydrographic survey, sounding along a main or reference line is carried out in a short time. The observation conditions along a sounding line may be regarded as invariable because the velocity of the survey boat is relatively constant and most factors and effects on the sounding data, such as those of tide, wave, heave, draft and so on are non-variant. Therefore, it can be believed that there may be a systematic effect or error on the sounding data to each survey line, which is treated as a relatively stable constant. It is obvious that the systematic errors of all sounding lines have been embodied in the differences of sounding data at crossing points between main scheme lines and reference lines.

An adjustment method is developed here by the authors to detect the sounding-line systematic errors and also to correct or adjust the sounding errors of observations. The rank-defect characteristic of the adjustment of sounding-line grid data is discovered which is similar to that of a free leveling net in land surveys. Based on rank-defect adjustment theory, the formulae of the method are derived and algorithms are tested using both simulated and observed data.

Structure of Error and Principle of Method

The Structure of Error

Suppose there is a sounding-line grid including *m* main scheme lines, *n* reference lines and *mxn* (marked as *mn*) crossing points in which each main scheme line intersects with each reference line. Then at the crossing points, the sounding data on the main scheme lines (shown as matrix D), the sounding data on reference lines (shown as matrix $\tilde{D}$) and their differences or disagreements (shown as matrix Δ) can be expressed as follows:

$$D=\begin{bmatrix} D_{1,1} & D_{1,2} & \cdots & D_{1,n} \\ D_{2,1} & D_{2,2} & \cdots & D_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ D_{m,1} & D_{m,2} & \cdots & D_{m,n} \end{bmatrix}_{m\times n} \quad \tilde{D}=\begin{bmatrix} \tilde{D}_{11} & \tilde{D}_{12} & \cdots & \tilde{D}_{1n} \\ \tilde{D}_{21} & \tilde{D}_{22} & \cdots & \tilde{D}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{D}_{m1} & \tilde{D}_{m2} & \cdots & \tilde{D}_{mn} \end{bmatrix}_{m\times n} \quad \Delta=D-\tilde{D}=\begin{bmatrix} \Delta_{11} & \Delta_{12} & \cdots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & \cdots & \Delta_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \Delta_{m1} & \Delta_{m2} & \cdots & \Delta_{mn} \end{bmatrix}_{m\times n} \tag{1}$$

where D_{ij}, $\tilde{D}_{ij}$ are the observed values at the crossing point of *i* th main scheme line and *j* th reference line respectively, and $\Delta_{ij} = D_{ij} - \tilde{D}_{ij}$.

Were there no errors in the observed data, D_{ij} would be equal to $\tilde{D}_{ij}$ and $\Delta=0$. In fact, $\Delta\neq0$, so an error model based on the structure of the sounding-line systematic error is given as follows:

$$\begin{cases} D_{ij} = d_{ij} + a_i + \delta_{ij} \ (main \quad line) \\ \tilde{D}_{ij} = d_{ij} + b_j + \tilde{\delta}_{ij} \ (reference \quad line) \end{cases} \tag{2}$$

Where $i=1...m$, $j=1...n$, d_{ij} is the true depth value at the crossing point of the *i* th main scheme line and the *j* th reference line; a_i is the systematic effect or error on the *i* th main scheme line; and b_j is that on the *j* th reference line. Both δ_{ij} and $\tilde{\delta}_{ij}$ are random errors at the crossing point for main line and reference line, which are supposed to satisfy normal distribution $N(0,\sigma^2)$. Therefore, the difference at the crossing point is written as

$$\Delta_{ij} = D_{ij} - \tilde{D}_{ij} = (a_i - b_j) + (\delta_{ij} - \tilde{\delta}_{ij}) \tag{3}$$

Let $\xi_{ij} = \delta_{ij} - \tilde{\delta}_{ij}$, then we get $\Delta_{ij}=a_i-b_j+\xi_{ij}$, and in matrix notation as follows:

$$\Delta = A - B + \xi \tag{4}$$

where
$$\underset{m\times n}{A}=\begin{bmatrix} a_1 & a_1 & \cdots & a_1 \\ a_2 & a_2 & \cdots & a_2 \\ \vdots & \vdots & \vdots & \vdots \\ a_m & a_m & \cdots & a_m \end{bmatrix} \quad \underset{m\times n}{B}=\begin{bmatrix} b_1 & b_2 & \cdots & b_n \\ b_1 & b_2 & \cdots & b_n \\ \vdots & \vdots & \vdots & \vdots \\ b_1 & b_2 & \cdots & b_n \end{bmatrix} \quad \underset{m\times n}{\xi}=\delta-\tilde{\delta}=\begin{bmatrix} \xi_{1,1} & \xi_{1,2} & \cdots & \xi_{1,n} \\ \xi_{2,1} & \xi_{2,2} & \cdots & \xi_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ \xi_{m,1} & \xi_{m,2} & \cdots & \xi_{m,n} \end{bmatrix}$$

and
$$\underset{m\times n}{\delta}=\begin{bmatrix} \delta_{1,1} & \delta_{1,2} & \cdots & \delta_{1,n} \\ \delta_{2,1} & \delta_{2,2} & \cdots & \delta_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ \delta_{m,1} & \delta_{m,2} & \cdots & \delta_{m,n} \end{bmatrix} \quad \underset{m\times n}{\tilde{\delta}}=\begin{bmatrix} \tilde{\delta}_{1,1} & \tilde{\delta}_{1,2} & \cdots & \tilde{\delta}_{1,n} \\ \tilde{\delta}_{2,1} & \tilde{\delta}_{2,2} & \cdots & \tilde{\delta}_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\delta}_{m,1} & \tilde{\delta}_{m,2} & \cdots & \tilde{\delta}_{m,n} \end{bmatrix}$$

There are $m+n$ parameters in Eq. (4). The effects of sounding-line systematic errors on the observed data can be removed if parameters a_i and b_j have been estimated.

The Derivation of Adjustment Solution

Let L_{ij} substitute for $-\Delta_{ij}$, and v_{ij} for $-\xi_{ij}$, the following adjustment model may be obtained as follows (the detailed derivation is enclosed in Appendix A):

$$\begin{cases} V = C\hat{X} - L \\ G^T P_X \hat{X} = 0 \\ V^T V = \min \end{cases} \tag{5}$$

where $C_{mn\times(m+n)}=\begin{bmatrix} E_m & -e_m\eta_1^T \\ E_m & -e_m\eta_2^T \\ \vdots & \vdots \\ E_m & -e_m\eta_n^T \end{bmatrix}$, E_m is a $m\times m$ unit matrix, V and L are $mn\times 1$ vectors,

$$e_m^T=(1 \quad 1 \quad \cdots \quad 1)_{1\times m},$$
$$\eta_1^T=(1,0,\cdots,0)_{1\times n},\ \eta_2^T=(0,1,0,\cdots,0)_{1\times n},\ \eta_j^T=(0,0,\cdots,\overset{j}{1},0,\cdots,0)_{1\times n},(j=1,2,\cdots,n),$$
$$\hat{X}_a^T=(\hat{a}_1,\hat{a}_2,\cdots\hat{a}_m)_{1\times m},\ \hat{X}_b^T=(\hat{b}_1,\hat{b}_2,\cdots\hat{b}_n)_{1\times n},$$
$$\hat{X}^T=(\hat{X}_a^T,\hat{X}_b^T),$$
$$G^T=(1,1,\cdots,1)_{1\times(m+n)}=e_{m+n}^T=(e_m^T,e_n^T),$$
$$P_X=diag(p_{a_1},\cdots,p_{a_i},\cdots,p_{a_m},p_{b_1},\cdots,p_{b_j},\cdots,p_{b_n}).$$

P_X is the datum matrix, p_{a_i} the datum factor of parameter a_i, and p_{b_j} the datum factor of parameter b_j. p_{a_i} and p_{b_j} should be 0 or 1, in which p_{a_i} (or p_{b_j}) = 1 means parameter a_i (or b_j) is constrained in the adjustment model of (5); p_{a_i} (or p_{b_j}) = 0 means parameter a_i (or b_j) is not constrained in the adjustment model of (5).

(5) is the defect-rank adjustment model, their solution and precision information can be obtained as follows:

$$\begin{cases} \hat{X} = (N + P_X GG^T P_X)^{-1} C^T L \\ Q_X = (N + P_X GG^T P_X)^{-1} N (N + P_X GG^T P_X)^{-1} \\ \hat{\sigma}_\xi^2 = V^T V / [(m-1)(n-1)] \end{cases} \quad (6)$$

where $N = C^T C = \begin{bmatrix} nE_m & -e_m e_n^T \\ -e_n e_m^T & mE_n \end{bmatrix}$, E_n is a $n \times n$ unit matrix, $e_n^T = (1 \quad 1 \quad \cdots \quad 1)_{1 \times n}$,

Q_x is the parameter coefficient weight matrix, $\hat{\sigma}_\xi$ is the mean standard deviation of random errors.
The parameter values from (6) should be tested to determine whether they are significant or not, the significant parameters will be regarded as systematic errors and used to correct the sounding data along corresponding survey lines. The T-test approach is adopted in this paper, in which the statistic variable is written as:

$$t = \frac{\hat{x} - 0}{\sigma_{\hat{x}}} = \frac{\hat{x}}{\sigma_\xi \sqrt{q_{\hat{x}}}} \quad (7)$$

Where $\hat{x}$ is a parameter value (i.e., the element of $\hat{X}$), $\sigma_{\hat{x}}$ the deviation of $\hat{x}$ and α the significance level (here, we take α=0.05), $\hat{x}$ will be regarded as the estimate of a systematic error, if $|t| > t_{\alpha/2,(m-1)(n-1)}$.

The Determination of the Value of p_{a_i} and p_{b_j}

(detailed derivation is enclosed in *Appendix B*)

In P_X, p_{a_i} the datum factor of parameter a_i, and p_{b_j} the datum factor of parameter b_j. p_{a_i} and p_{b_j} will be 0 or 1. In view of adjustment, parameter a_i (or b_j) should be constrained in (5), i.e., p_{a_i} (or p_{b_j}) = 1, when a_i (or b_j) = 0; parameter a_i (or b_j) should not be constrained in (5), i.e., p_{a_i} (or p_{b_j}) = 0, when a_i (or b_j) ≠0.
Now we can further acquire another explanation about the selection of p_{a_i} (or p_{b_j}) according to the relationship between the parameters.
If there is a systematic error a_i added on the i^{th} main line, the changes of the observed vector L in (5) is ΔL.

$$\Delta L^T = a_i(\mu_i^T, \mu_i^T, \cdots, \underset{i}{\mu_i^T})_{1\times mn}$$

where $\mu_i^T = (0, \cdots, 0, 1, 0, \cdots, 0)_{1\times m}$.

When $\sum_{i=1}^{m} p_{a_i} = M$ and $\sum_{j=1}^{n} p_{b_j} = N$ in the vector of $(p_{a_1}, \cdots, p_{a_i}, \cdots, p_{a_m}, p_{b_1}, \cdots, p_{b_j}, \cdots, p_{b_n})_{1\times(m+n)}$,

the change $\Delta\hat{X}_{P_X}$ of the parameter vector X due to a_i is obtained from (6) as follows (the detailed derivation is enclosed in *Appendix B*)

$$\Delta\hat{X}_{P_X} = a_i \begin{bmatrix} \mu_i - p_{a_i} e_m / (M+N) \\ -p_{a_i} e_n / (M+N) \end{bmatrix} \quad (8)$$

If there is a systematic error b_j added on the j^{th} reference line, $\Delta\hat{X}_{P_X}$ due to b_j could be obtained from (6) as follows

$$\Delta\hat{X}_{P_X} = b_j \begin{bmatrix} -p_{b_j} e_m / (M+N) \\ \eta_j - p_{b_j} e_n / (M+N) \end{bmatrix} \quad (9)$$

It can be seen from (8) that p_{a_i} should be 0 when a_i is a systematic error on i^{th} main line because if

$p_{a_i}=1$, a_i will affect the estimates of the other parameters. p_{a_i} should be 1 when a_i is zero.

From (9), when b_j is a systematic error on j^{th} main line, p_{b_j} should be 0 to avoid the effects of b_j on the other estimates. Otherwise, p_{b_j} should be 1 when b_j is zero.
Further, we continue to discuss the case that there are systematic errors on all the lines.
Let the vector $(a_1, a_2, \cdots, a_m)$ be the systematic errors added on the main lines, $(b_1, b_2, \cdots, b_n)$ on the reference lines. From (8) and (9), the changes of the parameters could be further obtained as follows:

$$\Delta\hat{X}_{P_X} = \begin{bmatrix} \Delta\hat{a}_1 \\ \vdots \\ \Delta\hat{a}_m \\ \Delta\hat{b}_1 \\ \vdots \\ \Delta\hat{b}_n \end{bmatrix}_{P_X} = \begin{bmatrix} a_1-\tau \\ \vdots \\ a_m-\tau \\ b_1-\tau \\ \vdots \\ b_n-\tau \end{bmatrix} \quad (10)$$

where $\tau = (\sum_{i=1}^{m} p_{a_i} a_i + \sum_{j=1}^{n} p_{b_j} b_j)/(M+N)$

In (10), while $\tau \approx 0$, $\Delta\hat{a}_i$ ($\Delta\hat{b}_j$) is able to estimate a_i (b_j). Otherwise $\Delta\hat{a}_i$ ($\Delta\hat{b}_j$) is able to partly estimate a_i (b_j). While τ is much larger than a_i (b_j), $\Delta\hat{a}_i$ ($\Delta\hat{b}_j$) cannot estimate a_i (b_j) but mainly estimate τ. Therefore, τ should be very small or even zero to make $\Delta\hat{a}_i$ $\Delta\hat{b}_j$) estimate a_i (b_j). In order to minimise τ, the datum factors of the non-zero systematic errors should be 0; the datum factors of the zero systematic errors should be 1.

The Selection of Datum Matrix P_{rx}

In order to minimize τ in (10), the datum factors of the non-zero systematic errors should be 0; the datum factors of the zero systematic errors should be 1. However, it is difficult to assure whether there exist a systematic error on a line or not before adjustment calculation, so all the lines can be regarded as the same, i.e. let $P_X = P_{0X}$ (P_{0X} is defined as an unit matrix) for the first adjustment. Then, the value of p_{a_i} or p_{b_j} can be further determined according to the absolute t-test values of $\hat{X}$. The detailed steps of selection are given as follows:

Step 1, In (6), let $P_X = P_{0X}$ (P_{0X} is an unit matrix) and we can obtain the sequence of its corresponding estimates: $\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_m, \hat{b}_1, \hat{b}_2, \cdots, \hat{b}_n$ and the absolute value sequence of t-test by (7):
$|t_{a_1}|, |t_{a_2}|, \cdots, |t_{a_m}|, |t_{b_1}|, |t_{b_2}|, \cdots, |t_{b_n}|$:

Step 2, Let $|t_{\max}|$ be the maximum of the sequence $|t_{a_1}|, |t_{a_2}|, \cdots, |t_{a_m}|, |t_{b_1}|, |t_{b_2}|, \cdots, |t_{b_n}|$, If $|t_{\max}|$ is less than $t_{\alpha/2,(m-1)(n-1)}$, the selection will be ended. If $|t_{\max}|$ is more than $t_{\alpha/2,(m-1)(n-1)}$, the estimate $\hat{a}_I$ (or $\hat{b}_J$) ($1 \le I \le m$, $1 \le J \le n$) corresponding to $|t_{\max}|$ will be considered as a systematic error and its datum factor p_{a_i} (or p_{b_j}) in P_{0X} should have been 0, and then P_{0X} can be modified and changed into P_{1X}.

Step 3, Re-calculate the estimates of $\hat{X}$ with P_{1X} by (6), and then all the estimates whose datum factors are 1 in P_{1X} are tested by the t-test method. Another sequence of the t-test values is obtained: $|t_{a_{I_1}}|, \cdots, |t_{a_{I_{m-1}}}|, |t_{b_{J_1}}|, \cdots, |t_{b_{J_{n-1}}}|$, where $I_1, \cdots, I_{m-1}, J_1, \cdots, J_{n-1}$ $(1 \le I_1, \cdots, I_{m-1} \le m, 1 \le J_1, \cdots, J_{n-1} \le n)$ denote the order of the datum factors that are 1 in P_{1X}. And then re-do the second step, and P_{1X} can be modified and changed into P_{2X}.

Step 4, After doing the second and third steps several times,the sequence of the datum matrix P_{1x}, P_{2x}...P_{rx} can be obtained. If $|t_{max}|$ of the absolute t-test values of $\hat{X}$ in which the corresponding datum factor is 1 in P_{rx} is less than $t_{\alpha/2,(m-1)(n-1)}$, the corresponding estimates will be regarded as random errors and P_{rx} can be regarded as a rational datum for (5). The selection of datum matrix is ended.
The calculations of $\hat{X}$ under P_{rx} will be the last adjustment results from (6) for a sounding line grid and will answer Question 1 and Question 2.

The Correction of Systematic Errors

Now we discuss how to answer Question 3, i.e., how to adjust the systematic disagreements or errors while the systematic errors are determined.
It is obvious that the systematic effects on the observed data should be removed in case the estimates such as $\hat{a}_i$ and $\hat{b}_j$ determined by (6) is significant after the t-test. The method to correct and adjust errors is given as follows.
Let H_i stands for all the observed data on i^{th} main line (of course including D_{ij}), H_j all data on j^{th} reference line (of course including $\tilde{D}_{ij}$). All of the observed data in a grid of sounding lines can be corrected:

$$\begin{cases} H_i' = H_i + (p_{a_i} - 1)\hat{a}_i \\ H_j' = H_j + (p_{b_j} - 1)\hat{b}_j \end{cases} \tag{11}$$

where P_{a_i} and P_{b_j} are the elements of the selected datum matrix P_{rx}.
After the systematic correction using (11), the disagreements between two groups of sounding data at crossing points of main scheme lines and reference lines will only be due to random errors which can be further used to assess the precision of sounding data.

Tests

Tests Using the Simulated Data

Six simulations have been designed to verify that the model (5) can be used to choose the rational datum and also to detect further the systematic errors exactly.

Test 1: designed to test whether the estimates of *X* determined by (6) will be zeros or not when there are no systematic errors.
Simulation conditions: there are 20 main sounding lines and 5 reference lines, in which the standard deviation of sounding is 0.3 metre and $t_{0.025,(20-1)(5-1)}$ =1.99.The differences at their intersections are subject to $N(0,2\text{x}0.3^2)$ and shown in Table 1.

	Reference line 1	**2**	**3**	**4**	**5**
main line 1	-0.29	-0.38	0.49	0.46	-0.27
2	0.06	-0.18	0.35	0.35	-0.58
3	-0.21	-0.13	0.41	-0.02	-0.05
4	-0.52	0.11	-0.19	0.34	0.26
5	-0.29	0.76	-0.09	0.21	-0.59
6	-0.36	-0.15	-0.40	0.38	0.52
7	0.58	0.19	-0.55	-0.34	0.11
8	-0.15	-0.27	0.48	-0.21	0.14
9	0.35	-0.28	0.49	-0.50	-0.05
10	0.46	-0.06	-0.58	0.13	0.06

11	0.26	-0.03	0.11	-0.22	-0.12
12	-0.28	-0.54	0.29	-0.03	0.57
13	-0.24	0.20	-0.18	0.46	-0.24
14	-0.03	0.54	0.00	-0.63	0.12
15	0.24	-0.04	0.16	-0.16	-0.20
16	-0.15	0.53	-0.01	0.42	-0.79
17	0.29	-0.78	0.26	0.24	-0.01
18	0.08	0.62	-0.43	-0.71	0.44
19	0.39	-0.32	-0.80	0.34	0.38
20	-0.19	0.21	0.19	-0.51	0.30

Table 1: The simulated difference data at the crossing points (unit: m)

After using P_{0x}, the estimates and the t-test values of X determined by (6) are shown in Table 2.

No.	Systematic true values		Estimates (m) Under P_{0x}		t-test values	
	a	b	$\hat{a}$	$\hat{b}$	t_a	t_b
1	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00		0.00		0.00	
7	0.00		0.00		0.00	
8	0.00		0.00		0.00	
9	0.00		0.00		0.00	
10	0.00		0.00		0.00	
11	0.00		0.00		0.00	
12	0.00		0.00		0.00	
13	0.00		0.00		0.00	
14	0.00		0.00		0.00	
15	0.00		0.00		0.00	
16	0.00		0.00		0.00	
17	0.00		0.00		0.00	
18	0.00		0.00		0.00	
19	0.00		0.00		0.00	
20	0.00		0.00		0.00	
Note: the t-test critical value ($t_{0.025,(20-1)(5-1)}$) is 1.99						

Table 2: Estimates & t-test values of X under P_{0x}

In Table 2, the estimates are zeros under P_{0x} and even under other datum, which shows the estimates of parameters determined by (6) are consistent with their true values.

Test 2: designed to test whether the estimates of *X* determined by (6) will be exact or not when there is a systematic error only on one line.
In this test, 1.0m as a constant is added to the 3rd main line and 2nd reference line respectively to simulate systematic error. The estimates and t-test values of X are shown in Table 3 and Table 4.

No.	Systematic true values		Estimates (m) Under P_{ox}		t-test values		P_{rx}		Estimates (m) Under P_{rx}		t-test	
	a	b	$\hat{a}$	$\hat{b}$	t_a	t_b	p_a	p_b	$\hat{a}$	$\hat{b}$	t_a	t_b
1	0.00	0.00	-0.04	-0.04	-0.21	-0.44	1	1	0.00	0.00	0.00	0.00
2	0.00	0.00	-0.04	-0.04	-0.22	-0.44	1	1	0.00	0.00	0.00	0.00
3	**1.00**	0.00	**0.96**	-0.04	**5.25**	-0.44	0	1	**1.00**	0.00	**5.25**	0.00
4	0.00	0.00	-0.04	-0.04	-0.22	-0.44	1	1	0.00	0.00	0.00	0.00
5	0.00	0.00	-0.04	-0.04	-0.22	-0.44	1	1	0.00	0.00	0.00	0.00
6	0.00		-0.04		-0.23		1		0.00		0.00	
7	0.00		-0.04		-0.23		1		0.00		0.00	
8	0.00		-0.04		-0.23		1		0.00		0.00	
9	0.00		-0.04		-0.21		1		0.00		0.00	
10	0.00		-0.04		-0.21		1		0.00		0.00	
11	0.00		-0.04		-0.22		1		0.00		0.00	
12	0.00		-0.04		-0.21		1		0.00		0.00	
13	0.00		-0.04		-0.22		1		0.00		0.00	
14	0.00		-0.04		-0.22		1		0.00		0.00	
15	0.00		-0.04		-0.22		1		0.00		0.00	
16	0.00		-0.04		-0.22		1		0.00		0.00	
17	0.00		-0.04		-0.22		1		0.00		0.00	
18	0.00		-0.04		-0.22		1		0.00		0.00	
19	0.00		-0.04		-0.23		1		0.00		0.00	
20	0.00		-0.04		-0.22		1		0.00		0.00	

Note: the t-test critical value is 1.99

Table 3: Estimates & t-test values of X when there is a systematic error on 3rd main line

No.	Systematic true values		Estimates (m) Under P_{ox}		t-test values		P_{rx}		Estimates (m) Under P_{rx}		t-test	
	a	b	$\hat{a}$	$\hat{b}$	t_a	t_b	p_a	p_b	$\hat{a}$	$\hat{b}$	t_a	t_b
1	0.00	0.00	-0.04	-0.04	-0.22	-0.44	1	1	0.00	0.00	0.00	0.00
2	0.00	**1.00**	-0.04	**0.96**	-0.22	10.64	1	0	0.00	**1.00**	0.00	**10.64**
3	0.00	0.00	-0.04	-0.04	-0.22	-0.44	1	1	0.00	0.00	0.00	0.00
4	0.00	0.00	-0.04	-0.04	-0.22	-0.44	1	1	0.00	0.00	0.00	0.00
5	0.00	0.00	-0.04	-0.04	-0.22	-0.44	1	1	0.00	0.00	0.00	0.00
6	0.00		-0.04		-0.22		1		0.00		0.00	
7	0.00		-0.04		-0.22		1		0.00		0.00	
8	0.00		-0.04		-0.22		1		0.00		0.00	
9	0.00		-0.04		-0.22		1		0.00		0.00	
10	0.00		-0.04		-0.22		1		0.00		0.00	
11	0.00		-0.04		-0.22		1		0.00		0.00	
12	0.00		-0.04		-0.22		1		0.00		0.00	
13	0.00		-0.04		-0.22		1		0.00		0.00	
14	0.00		-0.04		-0.22		1		0.00		0.00	

15	0.00		-0.04		-0.22		1		0.00		0.00	
16	0.00		-0.04		-0.22		1		0.00		0.00	
17	0.00		-0.04		-0.22		1		0.00		0.00	
18	0.00		-0.04		-0.22		1		0.00		0.00	
19	0.00		-0.04		-0.22		1		0.00		0.00	
20	0.00		-0.04		-0.22		1		0.00		0.00	
Note: the t-test critical value is 1.99												

Table 4: Estimates & t-test values of X when there is a systematic error on 2nd reference line

In Table 3 and Table 4, the estimates of *X* under P_{0x} are very near to true values. There is twenty-fifth of the systematic error (i.e. 1/(20+5)=-0.04) to affect other parametres. Because $\hat{a}_3$ in Table 3 and $\hat{b}_2$ in Table 4 are significant under P_{0x} by t-test, let $p_{a_3}=0$ to get P_{1x} in Table 3, $p_{b_2}=0$ to get $P_{rx}=P_{1x}$ in Table 4.

Here, the estimates of *X* can still be obtained exactly after the selection of rational P_{rx} according to the t-test values under P_{0x} although it is unknown first that there is 1.0m systematic error on 3rd main line (or 2nd reference line) before calculation.

Test 3: designed to test whether the estimates of *X* will be exactly determined by (6) or not when there are systematic errors on several main sounding lines.

No.	**Systematic true values**		**Estimates (m) Under P_{0x}**		**t-test values**		**P_{rx}**		**Estimates (m) Under P_{rx}**		**t-test**	
	a	*b*	$\hat{a}$	$\hat{b}$	t_a	t_b	p_a	p_b	$\hat{a}$	$\hat{b}$	t_a	t_b
1	0.00	0.00	-0.16	-0.16	-0.86	-1.77	1	1	-0.01	-0.01	-0.05	-0.11
2	0.00	0.00	-0.16	-0.16	-0.88	-1.77	1	1	-0.01	-0.01	-0.05	-0.11
3	0.00	0.00	-0.16	-0.16	-0.88	-1.77	1	1	-0.01	-0.01	-0.05	-0.11
4	**0.20**	0.00	**0.04**	-0.16	**0.22**	-1.77	1	1	**0.19**	-0.01	**1.05**	-0.11
5	0.00	0.00	-0.16	-0.16	-0.88	-1.77	1	1	-0.01	-0.01	-0.05	-0.11
6	0.00		-0.16		-0.89		1		-0.01		-0.05	
7	**0.50**		**0.34**		**1.85**		0		**0.49**		**2.56**	
8	0.00		-0.16		-0.89		1		-0.01		-0.05	
9	0.00		-0.16		-0.86		1		-0.01		-0.05	
10	0.00		-0.16		-0.86		1		-0.01		-0.05	
11	**0.80**		**0.64**		**3.50**		0		**0.79**		**4.41**	
12	0.00		-0.16		-0.86		1		-0.01		-0.05	
13	0.00		-0.16		-0.88		1		-0.01		-0.05	
14	0.00		-0.16		-0.88		1		-0.01		-0.05	
15	**1.00**		**0.84**		**4.59**		0		**0.99**		**5.19**	
16	0.00		-0.16		-0.88		1		-0.01		-0.05	
17	0.00		-0.16		-0.88		1		-0.01		-0.05	
18	**1.50**		**1.34**		**7.33**		0		**1.49**		**7.81**	
19	0.00		-0.16		-0.89		1		-0.01		-0.05	
20	0.00		-0.16		-0.88		1		-0.01		-0.05	
Note: the t-test critical value is 1.99												

Table 5: Estimates & t-test values of X while 4th, 7th, 11th, 15th and 18th main lines with systematic errors respectively

Now 0.2, 0.5, 0.8, 1.0 and 1.5 metres of systematic errors are added to the simulated sounding data along the 4th, 7th, 11th, 15th and 18th main line respectively, the estimates and t-test values of X under P_{0X} rational P_{rX} are shown in Table 5.

In Table 5, there is -0.16 (i.e. $\tau = -(0.2+0.5+0.8+1.0+1.5)/(20+5)=-0.16$ by (10)) to affect all the estimates. The estimates of the parameters are near to the systematic true errors. According to the t-test values under P_{0X}, the selection of P_{rX} have been done as follows:

First, $\hat{a}_{18}$ is significant and p_{a18} should be 0 to get P_{1X}, and then $\hat{a}_{15}$ is detected and P_{a15} should be 0 to get P_{2X}, and then $\hat{a}_{11}$, $\hat{a}_{7}$ are tested by t-test orderly, and P_{a11}, P_{a7} should be 0 to get P_{3X} and P_{4X}, and then $P_{rX} = P_{4X}$. The estimates under P_{rX} are very near to the true values. This shows that the selection of P_{rX} is able to minimise the effects of τ on the estimates.

Test 4: designed to test whether the estimates of *X* will be exactly determined by (6) or not when there are systematic errors on several reference sounding lines.

In this test, -0.2, 0.5, 0.4, 0.3 and 0.1 metres are added to the simulated soundings of the 1st, 2nd, 3rd, 4th and 5th reference line respectively, the estimates and t-test values of X under P_{0X} rational P_{rX} are shown in Table 6.

In Table 6, there is -0.04, i.e. $\tau = -(-0.2+0.5+0.4+0.3+0.1)/(20+5)=-0.04$ by (10), to affect all the estimates. The estimates of the parameters are near to the systematic true errors. According to the t-test

No.	Systematic true values		Estimates (m) Under P_{0X}		t-test values		P_{rX}		Estimates (m) Under P_{rX}		t-test	
	a	b	$\hat{a}$	$\hat{b}$	t_a	t_b	p_a	p_b	$\hat{a}$	$\hat{b}$	t_a	t_b
1	0.00	**-0.20**	-0.04	**-0.24**	-0.23	**-2.70**	1	**0**	0.00	**-0.20**	-0.03	**-2.18**
2	0.00	**0.50**	-0.04	**0.46**	-0.24	**5.05**	1	**0**	0.00	**0.50**	-0.03	**5.28**
3	0.00	**0.40**	-0.04	**0.36**	-0.24	**3.94**	1	**0**	0.00	**0.40**	-0.03	**4.21**
4	0.00	**0.30**	-0.04	**0.26**	-0.24	**2.84**	1	**0**	0.00	**0.30**	-0.03	**3.15**
5	0.00	**0.10**	-0.04	**0.06**	-0.24	**0.62**	1	1	0.00	0.10	-0.03	1.07
6	0.00		-0.04		-0.25		1		0.00		-0.03	
7	0.00		-0.04		-0.25		1		0.00		-0.03	
8	0.00		-0.04		-0.25		1		0.00		-0.03	
9	0.00		-0.04		-0.23		1		0.00		-0.03	
10	0.00		-0.04		-0.23		1		0.00		-0.03	
11	0.00		-0.04		-0.24		1		0.00		-0.05	
12	0.00		-0.04		-0.23		1		0.00		-0.05	
13	0.00		-0.04		-0.24		1		0.00		-0.05	
14	0.00		-0.04		-0.24		1		0.00		-0.03	
15	0.00		-0.04		-0.24		1		0.00		-0.05	
16	0.00		-0.04		-0.24		1		0.00		-0.05	
17	0.00		-0.04		-0.24		1		0.00		-0.03	
18	0.00		-0.04		-0.24		1		0.00		-0.03	
19	0.00		-0.04		-0.25		1		0.00		-0.03	
20	0.00		-0.04		-0.24		1		0.00		-0.03	

Note: the t-test critical value is 1.99

Table 6: Estimates & t-test values of X while 1st, 2nd, 3rd, 4th and 5th reference lines with systematic errors

values under P_{0x}, first, $\hat{b}_2$ is significant by t-test and p_{b_2} should be 0 to get P_{1x}; and then $\hat{b}_3$ under P_{1x} is significant and p_{b_3} should be 0 to get P_{2x}; and then $\hat{b}_4$ under P_{2x} is significant and p_{b_4} should be 0 to get P_{3x}; and then $\hat{b}_1$ under P_{3x} is significant and p_{b_1} should be 0 to get P_{4x}; It can be demonstrated that all the t-test absolute values under P_{4x} except t_{b2}, t_{b3}, t_{b4} and t_{b1} are less than 1.99, and then P_{rx}= P_{4x}. The estimates under P_{rx} are the true values (if neglecting rounding difference).

Test 5: designed to test whether the estimates of *X* will be exactly determined by (6) or not when there are systematic errors on many sounding lines.

Now 0.2, 0.5, 0.8, 1.0,1.5, -0.1, 0.5, 0.6, 1.0 and 0.2 metres of simulated systematic errors are added to the 4th,7th,11th,15th,18th main line and all the 5 reference lines respectively, the estimates and t-test values of *X* under P_{0x} rational P_{rx} are shown in Table 7.

No.	Systematic true values		Estimates (m) Under POX		t-test values		PrX		Estimates (m) Under PrX		t-test	
	a	b	$\hat{a}$	$\hat{b}$	t_a	t_b	p_a	p_b	$\hat{a}$	$\hat{b}$	t_a	t_b
1	0.00	**-0.10**	-0.25	**-0.35**	-1.35	-3.86	1	1	-0.00	**-0.11**	-0.02	**-1.17**
2	0.00	**0.50**	-0.25	**0.25**	-1.36	2.79	1	**0**	-0.01	**0.49**	-0.03	**5.15**
3	0.00	**0.60**	-0.25	**0.35**	-1.36	3.90	1	**0**	-0.01	**0.59**	-0.03	**6.19**
4	**0.20**	**1.00**	**-0.05**	**0.75**	**-0.26**	8.33	1	**0**	**0.19**	**0.99**	**1.07**	**10.36**
5	0.00	**0.20**	-0.25	-0.05	-1.36	-0.53	1	**0**	-0.01	**0.19**	-0.03	**2.02**
6	0.00		-0.25		-1.37		1		-0.01		-0.04	
7	**0.50**		**0.25**		**1.37**		**0**		**0.49**		**2.55**	
8	0.00		-0.25		-1.37		1		-0.01		-0.04	
9	0.00		-0.25		-1.35		1		-0.00		-0.02	
10	0.00		-0.25		-1.35		1		-0.00		-0.02	
11	**0.80**		**0.55**		**3.02**		**0**		**0.79**		**4.12**	
12	0.00		-0.25		-1.35		1		-0.00		-0.02	
13	0.00		-0.25		-1.36		1		-0.01		-0.03	
14	0.00		-0.25		-1.36		1		-0.01		-0.03	
15	**1.00**		**0.75**		**4.11**		**0**		**0.99**		**5.16**	
16	0.00		-0.25		-1.36		1		-0.01		-0.03	
17	0.00		-0.25		-1.36		1		-0.01		-0.03	
18	**1.50**		**1.25**		**6.85**		**0**		**1.49**		**7.76**	
19	0.00		-0.25		-1.37		1		-0.01		-0.04	
20	0.00		-0.25		-1.36		1		-0.01		-0.03	
Note: the t-test critical value is 1.99												

Table 7: Estimates & t-test values of X while 4th,7th,11th, 15th and 18th main lines and all the five reference lines with systematic error

In Table 7, -0.25m (i.e.τ = -0.25) is shared to all the estimates from the systematic error under P_{0x}, and t_{a7} and t_{a4} are less than the t-test critical value, this shows the effect of τ has made the systematic errors a_7 and a_4 unable to be found under P_{0x}, and the estimates of the systematic errors a_4 and b_1 have been strongly affected by τ. The selection of P_{rx} is that, first according to the t-test value under P_{0x}, and then p_{b_4} should be 0 to get P_{1x}, and then $p_{a_{18}}$ should be 0 to get P_{2x} according to the t-test value under P_{1x}, and then p_{b_3} should be 0 to get P_{3x} according to the t-test value under P_{2x}, and then $p_{a_{15}}$ should be 0 to get P_{4x} according to the t-test value under P_{3x}, and then p_{b_2} should be 0 to get P_{5x} according to the t-test value under P_{4x}, and then $p_{a_{11}}$ should be 0 to get P_{6x} according to the t-test value under P_{5x}, and then

p_{a_7} should be 0 to get P_{7x} according to the t-test value under P_{6x}, and then p_{b_5} should be 0 should be 0 to get P_{8x} according to the t-test value under P_{7x}, lastly $P_{rx} = P_{8x}$. It is clear that the selection of P_{rx} is able to minimise the effect of τ on the estimates and the estimates under P_{rx} are very near to the true values. It is noticeable that the systematic errors a_4 and b_1 could not be found under P_{rx} according to the model (5). This shows that there exists the minimal systematic error which could be found by (6).

Test 6: designed to test the minimal systematic error that could be detected by (6) when there is a systematic error only on one line.

It can be seen that P_{ox} (an unit matrix) is adopted as the first adjustment datum when there is no prior information that datum factors should be 0 or 1. Now we discuss the minimal systematic true error which can be detected by (5) under P_{ox}. The estimates and t-test values of *X* are shown in Table 8 when the third main line is added to 0.38m intentionally as a systematic error. The estimates and t-test values of *X* are shown in Table 9 when 2[nd] reference line is added to 0.187m intentionally as systematic error.

In Table 8 and Table 9, when the maximum (t_{a3} or t_{b2}) of the t-test value under P_{ox} is equal to the t-test critical value (1.99), the systematic error on a main line is 0.38m, and the systematic error on a reference line is 0.187m. This shows that the detectability of the model (5) in finding the minimal systematic error on a main line is different from that on a reference line. It also implies that the number of reference lines (or called reference line) should be increased in order to improve the detectability of the model (5) in finding the minimal systematic error on a main line.

No.	Systematic true values		Estimates (m) Under P_{ox}		t-test values	
	a	b	$\hat{a}$	$\hat{b}$	t_a	t_b
1	0.00	0.00	-0.02	-0.02	-0.08	-0.17
2	0.00	0.00	-0.02	-0.02	-0.08	-0.17
3	**0.38**	0.00	**0.36**	-0.02	**1.99**	-0.17
4	0.00	0.00	-0.02	-0.02	-0.08	-0.17
5	0.00	0.00	-0.02	-0.02	-0.08	-0.17
6	0.00		-0.02		-0.08	
7	0.00		-0.02		-0.08	
8	0.00		-0.02		-0.08	
9	0.00		-0.02		-0.08	
10	0.00		-0.02		-0.08	
11	0.00		-0.02		-0.08	
12	0.00		-0.02		-0.08	
13	0.00		-0.02		-0.08	
14	0.00		-0.02		-0.08	
15	0.00		-0.02		-0.08	
16	0.00		-0.02		-0.08	
17	0.00		-0.02		-0.08	
18	0.00		-0.02		-0.08	
19	0.00		-0.02		-0.08	
20	0.00		-0.02		-0.08	
Note: the t-test critical value is 1.99						

Table 8: The minimal systematic true error on a main line corresponding to the t-test critical value under P_{ox}

No.	Systematic true values		Estimates (m) Under P_{ox}		t-test values	
	a	b	$\hat{a}$	$\hat{b}$	t_a	t_b
1	0.00	0.00	-0.01	-0.01	-0.04	-0.08
2	0.00	**0.18**	-0.01	**0.18**	-0.04	**1.99**
3	0.00	0.00	-0.01	-0.01	-0.04	-0.08
4	0.00	0.00	-0.01	-0.01	-0.04	-0.08
5	0.00	0.00	-0.01	-0.01	-0.04	-0.08
6	0.00		-0.01		-0.04	
7	0.00		-0.01		-0.04	
8	0.00		-0.01		-0.04	
9	0.00		-0.01		-0.04	
10	0.00		-0.01		-0.04	
11	0.00		-0.01		-0.04	
12	0.00		-0.01		-0.04	
13	0.00		-0.01		-0.04	
14	0.00		-0.01		-0.04	
15	0.00		-0.01		-0.04	
16	0.00		-0.01		-0.04	
17	0.00		-0.01		-0.04	
18	0.00		-0.01		-0.04	
19	0.00		-0.01		-0.04	
20	0.00		-0.01		-0.04	
Note: the t-test critical value is 1.99						

Table 9: The minimal systematic true error on a reference line corresponding to the t-test critical value under P_{ox}

Tests Using the Observed Data

There are 15 main lines and 12 reference lines in a sounding grid. The sounding data is obtained by an SDH-13D Echo Sounder, made in China (technical character index: working frequency is 208kHz, designed sound velocity 1,460m/s, sounding accuracy ±0.4% depth ±5cm, beam width 8°±1). The range of depth in survey area is from 22 to 28 metres. The differences or disagreements of sounding data at crossing points are shown in Table 10.

	Reference											
main line	**line 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	-0.96	-0.89	-1.00	0.22	0.19	0.15	0.08	0.21	0.21	0.15	0.21	0.27
2	-1.12	-1.02	-0.92	0.20	0.16	0.15	0.22	0.20	0.21	0.11	0.19	0.23
3	-1.43	-0.88	-0.90	0.16	0.26	0.01	0.18	0.15	0.11	0.19	0.24	0.15
4	-1.10	-0.98	-1.13	0.16	0.21	0.13	0.21	0.17	0.14	0.17	0.17	0.20
5	-1.05	-0.95	-1.31	0.20	0.11	0.18	0.13	0.10	0.23	0.17	0.24	0.20
6	-1.13	-0.93	-0.82	0.11	-0.02	0.24	0.17	0.21	0.20	0.02	0.15	0.15
7	-1.35	-1.03	-0.96	0.29	0.31	0.12	0.08	0.20	0.20	0.26	0.17	0.45
8	-1.33	-0.83	-0.97	0.29	0.24	0.11	0.06	0.25	0.13	0.21	0.28	0.28
9	-1.03	-1.02	-0.93	0.18	0.10	0.18	0.15	0.10	0.06	0.05	0.27	0.13
10	-0.88	-1.01	-0.88	0.24	0.10	0.04	0.20	0.18	0.18	0.25	0.23	0.03
11	-0.93	-0.83	-0.78	0.16	0.18	0.14	0.28	-0.09	0.14	0.11	0.23	0.22
12	-1.01	-0.95	-0.87	0.20	0.14	0.05	0.09	0.10	0.20	0.08	0.15	0.17
13	-1.34	-1.01	-1.11	0.16	0.28	0.13	0.17	0.20	0.15	0.21	0.16	0.15
14	-1.27	-0.91	-0.91	0.24	0.19	0.24	0.22	0.11	0.16	0.12	0.23	0.16
15	-0.82	-0.88	-1.05	0.26	0.21	0.20	0.30	0.07	0.14	0.16	0.10	0.26

Table 10: The differences of sounding data at the crossing points (unit: m)

No.	**Estimates Under P_{0x}**		**t-test values Under P_{0x}**		**P_{rx}**		**Estimates Under P_{rx}**		**t-test values**	
	a	b	t_a	t_b	p_a	p_b	a	b	t_a	t_b
1	-0.03	**1.06**	-1.07	**44.91**	1	**0**	0.03	**1.12**	1.00	**45.84**
2	-0.05	**0.89**	-1.80	**37.50**	1	**0**	0.01	**0.94**	0.27	**38.65**
3	-0.08	**0.91**	-2.96	**38.68**	1	**0**	-0.02	**0.97**	-0.90	**39.80**
4	-0.07	**-0.26**	-2.61	-10.97	1	**0**	-0.01	**-0.20**	-0.56	**-8.40**
5	-0.08	**-0.23**	-2.93	-9.81	1	**0**	-0.02	**-0.18**	-0.87	**-7.28**
6	-0.07	**-0.19**	-2.61	-8.15	1	**0**	-0.01	**-0.14**	-0.56	**-5.67**
7	-0.04	**-0.22**	-1.39	-9.47	1	**0**	0.02	**-0.17**	0.68	**-6.95**
8	-0.04	**-0.20**	-1.45	-8.40	1	**0**	0.02	**-0.14**	0.62	**-5.91**
9	-0.08	**-0.22**	-2.96	-9.25	1	**0**	-0.02	**-0.16**	-0.90	**-6.73**
10	-0.04	**-0.21**	-1.57	-8.68	1	**0**	0.01	**-0.15**	0.49	**-6.19**
11	-0.03	**-0.26**	-1.10	-10.82	1	**0**	0.03	**-0.20**	0.97	**-8.27**
12	-0.07	**-0.26**	-2.61	-10.91	1	**0**	-0.01	**-0.20**	-0.56	**-8.35**
13	-0.09		-3.24		1		-0.03		-1.19	
14	-0.05		-1.89		1		0.00		0.17	
15	-0.02		-0.73		1		0.04		1.35	
Note: the t-test critical value is 1.98										

Table 11: The estimates & t-test values of parameters

According to the Chinese Specifications for Hydrographic Survey (China, 1998), the limit of difference at the crossing points is 0.6m with the confidence level 95 per cent while the depth range is from 20 to 30m and sounding data is acceptable while the percentage of beyond the limit at all the crossing points is under 15 per cent.

In Table 10, there are 45 differences or disagreements at the crossing points beyond the limit. The percentage of differences beyond the limit is 45 per cent. In a traditional way, the observed data will be rejected as unacceptable data contaminated by blunders and re-sounding should be needed. Now (6) is used to detect and determine the systematic errors of the sounding data in Table 10 so as to improve their accuracy and efficiency of sounding. The estimates and t-test values of X are obtained under P_{0x} and P_{rx} and shown in Table 11.

In Table 11, the estimates $\hat{b}_1$, $\hat{b}_2$ and $\hat{b}_3$ are larger than that of other parameters under P_{0x}. It may be deduced that there are systematic errors on reference line 1, 2 and 3. After examining the data processing carefully, it is found that the water level corrections have not been done to these lines due to carelessness. The true water level corrections should have been 1.20, 1.15 and 1.15m respectively. It shows that the model (6) is able to find systematic errors though the sources of error are often unknown in practical surveys.

According to the t-test values under P_{0x} in Table 11, the rational P_{rx} is selected as in Table 11. The order to select P_{rx} is $p_{b_1}=0$, $p_{b_3}=0$, $p_{b_2}=0$, $p_{b_{12}}=0$, $p_{b_{11}}=0$, $p_{b_4}=0$, $p_{b_5}=0$, $p_{b_7}=0$ $p_{b_9}=0$, $p_{b_{10}}=0$, $p_{b_8}=0$, and $p_{b_6}=0$, and then $P_{rx}=P_{12x}$. It can be seen that the differences between the estimates: $\hat{b}_1$, $\hat{b}_2$ and $\hat{b}_3$ under P_{rx} and their water level corrections are very small which may be due to other factors.

Now we can carry out the corrections using (11). The corrected differences are shown in Table 12, in which the maximum is 0.30m, the minimum is -0.34m. This shows that the quality of corrected sounding data has been improved and that corrected sounding data can be accepted and re-sounding will not be needed.

The method in this paper can be used effectively to detect and estimate systematic errors when there are systematic errors in a sounding grid. It should be determined by hydrographic surveyors according to the criteria whether systematic errors will be corrected and adjusted or not and whether re-sounding will be needed or not.

	Reference line 1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
main line 1	0.16	0.05	-0.03	0.02	0.01	0.01	-0.09	0.07	0.05	-0.00	0.01	0.07
2	-0.00	-0.08	0.05	-0.00	-0.02	0.01	0.05	0.06	0.05	-0.04	-0.01	0.03
3	-0.31	0.06	0.07	-0.04	0.08	-0.13	0.01	0.01	-0.05	0.04	0.04	-0.05
4	0.02	-0.04	-0.16	-0.04	0.03	-0.01	0.04	0.03	-0.02	0.02	-0.03	-0.00
5	0.07	-0.01	**-0.34**	-0.00	-0.07	0.04	-0.04	-0.04	0.07	0.02	0.04	-0.00
6	-0.01	0.01	0.15	-0.09	-0.20	0.10	0.00	0.07	0.04	-0.13	-0.05	-0.05
7	-0.23	-0.09	0.01	0.09	0.13	-0.02	-0.09	0.06	0.04	0.11	-0.03	0.25
8	-0.21	0.11	-0.00	0.09	0.06	-0.03	-0.11	0.11	-0.03	0.06	0.08	0.08
9	0.09	-0.08	0.04	-0.02	-0.08	0.04	-0.02	-0.04	-0.10	-0.10	0.07	-0.07
10	0.24	-0.07	0.09	0.04	-0.08	-0.10	0.03	0.04	0.02	0.10	0.03	-0.17
11	0.19	0.11	0.19	-0.04	0.00	0.00	0.11	-0.23	-0.02	-0.04	0.03	0.02
12	0.11	-0.01	0.10	-0.00	-0.04	-0.09	-0.08	-0.04	0.04	-0.07	-0.05	-0.03
13	-0.22	-0.07	-0.14	-0.04	0.10	-0.01	0.00	0.06	-0.01	0.06	-0.04	-0.05
14	-0.15	0.03	0.06	0.04	0.01	0.10	0.05	-0.03	-0.00	-0.03	0.03	-0.04
15	**0.30**	0.06	-0.08	0.06	0.03	0.06	0.13	-0.07	-0.02	0.01	-0.10	0.06

Table 12: The corrected difference at the crossing points (unit: m)

Conclusion

Through theoretical derivation, the principle and algorithms of the adjustment for the sounding-line grid are presented in the paper. The tests and analyses with simulated and observed data have verified that the method can work well to detect and estimate systematic errors and also to improve the quality of data in a sounding grid. But it should be pointed out that the adjustment and correction should be carried out after determining the sources of the systematic errors so as to guarantee the corrections more reasonable and reliable. Furthermore, the principle and method developed in this paper may be beneficial to the data processing for other marine survey, such as marine gravimetry and magnetic survey.

Appendix A: The derivation of adjustment model and its formulas

Let L_{ij} be instead of $-\Delta_{ij}$ and v_{ij} instead of $-\xi_{ij}$, (4) in the section of error structure could be expressed as follows:

$$V_{m\times n} = \hat{A}_{m\times n} - \hat{B}_{m\times n} - L_{m\times n} \qquad \text{(A-1)}$$

where $V_{m\times n} = \begin{bmatrix} v_{1,1} & v_{1,2} & \cdots & v_{1,n} \\ v_{2,1} & v_{2,2} & \cdots & v_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ v_{m,1} & v_{m,2} & \cdots & v_{m,n} \end{bmatrix}$, $\hat{A}_{m\times n} = \begin{bmatrix} \hat{a}_1 & \hat{a}_1 & \cdots & \hat{a}_1 \\ \hat{a}_2 & \hat{a}_2 & \cdots & \hat{a}_2 \\ \vdots & \vdots & \vdots & \vdots \\ \hat{a}_m & \hat{a}_m & \cdots & \hat{a}_m \end{bmatrix}$

$$\hat{B}_{m\times n} = \begin{bmatrix} \hat{b}_1 & \hat{b}_2 & \cdots & \hat{b}_n \\ \hat{b}_1 & \hat{b}_2 & \cdots & \hat{b}_n \\ \vdots & \vdots & \vdots & \vdots \\ \hat{b}_1 & \hat{b}_2 & \cdots & \hat{b}_n \end{bmatrix}, L_{m\times n} = \begin{bmatrix} L_{1,1} & L_{1,2} & \cdots & L_{1,n} \\ L_{2,1} & L_{2,2} & \cdots & L_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ L_{m,1} & L_{m,2} & \cdots & L_{m,n} \end{bmatrix}.$$

$V_{m\times n}$ can be changed into a $mn\times 1$ vector, and expressed as:

$V^T=[V_{1,1},V_{2,1},\dots,V_{m,1},V_{1,2},V_{2,2},\dots,V_{m,2},V_{3,1},\dots,v_{m-1,n},V_{m,n}]_{1xm}$

Similarly, $L_{m\times n}$ can be expressed as:

$L^T=[L_{1,1},L_{2,1},\dots,L_{m,1},L_{1,2},L_{2,2},\dots,L_{m,2},L_{3,1},\dots,L_{m-1,n},L_{m,n}]^{1xmn}$

Further, $\hat{A}_{m\times n} - \hat{B}_{m\times n}$ can be expressed as $C_{mn\times(m+n)}\hat{X}_{(m+n)\times 1}$

where $C_{mn\times(m+n)} = \begin{bmatrix} E_m & -e_m\eta_1^T \\ E_m & -e_m\eta_2^T \\ \vdots & \vdots \\ E_m & -e_m\eta_n^T \end{bmatrix}$, *$E_m$ is a mxm unit matrix, $e_m^T=(1\ 1\ \dots\ 1)_{1\times m}$,*

$\eta_1^T = (1,0,\cdots,0)_{1\times n}$, $\eta_2^T = (0,1,0,\cdots,0)_{1\times n}$, $\eta_j^T = (0,0,\cdots,\overset{j}{1},0,\cdots,0)_{1\times n}$, ($j$ =1,2,...,n),

$\hat{X}^T = (\hat{X}_a^T, \hat{X}_b^T)$, $\hat{X}_a^T = (\hat{a}_1,\hat{a}_2,\cdots \hat{a}_m)_{1\times m}$, $\hat{X}_b^T = (\hat{b}_1,\hat{b}_2,\cdots \hat{b}_n)_{1\times n}$.

So, (A-1) can be changed as follows:

$$V_{mn\times 1} = C_{mn\times(m+n)}\hat{X}_{(m+n)\times 1} - L_{mn\times 1} \qquad \text{(A-2)}$$

According to the least square criterion V^TV=min (Wells D E and Krakiwsky E J, 1973), the normal equation could be obtained from (A-2) as follows:

$$N\hat{X} = C^T L \tag{A-3}$$

where $N = C^T C = \begin{bmatrix} nE_m & -e_m e_n^T \\ -e_n e_m^T & mE_n \end{bmatrix}$, $e_n^T = (1 \quad 1 \quad \cdots \quad 1)_{1\times n} = \sum_{j=1}^{n} \eta_j^T$.

It can be proved that the rank of matrix N (or C) is $m+n-1$ and its rank-defect number is 1. This shows that it lacks of initial data using (A-1) to determine parameters $\hat{X}$.

There are many solutions to (A-3) from the theory of rank-defect adjustment (Huang W B, 1992). Therefore, the following constraint condition is used here:

$$G^T P_X \hat{X} = 0 \tag{A-4}$$

where G is a matrix which should be satisfied with NG=0, and here G^T= $(1,1,\cdots,1)_{1\times(m+n)}=e^T_{m+n}=(e^T_m, e^T_n)$, and its rank is 1. P_X is a diagonal matrix as well as a datum matrix for (A-3).

Then let $P_X = diag(p_{a_1}, \cdots, p_{a_i}, \cdots, p_{a_m}, p_{b_1}, \cdots, p_{b_j}, \cdots, p_{b_n})$, p_{a_i} is the datum factor of a_i, and p_{b_j} is the datum factor of b_j, and they should be 0 or 1, in which p_{a_i} (or p_{b_j}) = 1 means parameter a_i (or b_j) is constrained in the adjustment; p_{a_i} (or p_{b_j}) = 0 means parameter a_i (or b_j) is not constrained in the adjustment.

So the following adjustment model could be obtained

$$\begin{cases} V = C\hat{X} - L \\ G^T P_X \hat{X} = 0 \\ V^T V = \min \end{cases} \tag{A-5}$$

(A-5) is the rank-defect adjustment model for a sounding-line grid in hydrographic survey. Using (A-3), (A-5) can be further expressed as:

$$\begin{cases} N\hat{X} = C^T L & (i) \\ G^T P_X \hat{X} = 0 & (ii) \end{cases} \tag{A-6}$$

In (A-6), (ii) multiplies $P_X G$ leftward and adds to (i), the following equation can be obtained:

$$(N + P_X G G^T P_X)\hat{X} = C^T L \tag{A-7}$$

In (A-7), $N+P_XGG^TP_X$ is of full rank because G holds NG=0. The result formulas of (A-5) are given as follows:

$$\begin{cases} \hat{X} = (N + P_X G G^T P_X)^{-1} C^T L \\ Q_X = (N + P_X G G^T P_X)^{-1} N (N + P_X G G^T P_X)^{-1} \\ \hat{\sigma}_\xi^2 = V^T V / [(m-1)(n-1)] \end{cases} \tag{A-8}$$

Appendix B: the determination of the values of p_{a_i} and p_{b_j} in (A-4)

Here, we discuss how to determine the value of p_{a_i} and p_{b_j}. Suppose that there is a systematic error a_i added on the i^{th} main line, the change of the observed matrix L in (A-1) is ΔL, then

$$\Delta L_{m\times n} = \begin{bmatrix} 0_{1,1} & 0_{1,1} & \cdots & 0_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{i-1,1} & 0_{i-1,2} & \cdots & 0_{i-1,n} \\ a_i & a_i & \cdots & a_i \\ 0_{i+1,1} & 0_{i+1,2} & \cdots & 0_{i+1,n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{m,1} & 0_{m,1} & \cdots & 0_{m,n} \end{bmatrix} = a_i \begin{bmatrix} 0_{1,1} & 0_{1,1} & \cdots & 0_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{i-1,1} & 0_{i-1,2} & \cdots & 0_{i-1,n} \\ 1 & 1 & \cdots & 1 \\ 0_{i+1,1} & 0_{i+1,2} & \cdots & 0_{i+1,n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{m,1} & 0_{m,1} & \cdots & 0_{m,n} \end{bmatrix} = a_i [\mu_i \quad \mu_i \quad \cdots \quad \mu_i] \tag{B-1}$$

where $\mu_i^T = (0,\cdots,0,\overset{i}{1},0,\cdots,0)_{1\times m}$ ..

ΔL in (B-1) can be changed into a vector notation related to (A-2):

$$\Delta L^T = a_i(\mu_i^T, \mu_i^T, \cdots, \mu_i^T)_{1\times mn} \tag{B-2}$$

From (A-8), The changes of the estimates caused by ΔL due to a_i is

$$\Delta\hat{X}_{P_X} = (N + P_X GG^T P_X)^{-1} C^T \Delta L \tag{B-3}$$

while $P_X = P_{0X}$ (P_{0X} is here defined as a unit matrix), $\Delta\hat{X}_{P_X}$ due to a_i will be

$$\Delta\hat{X}_{P_{0X}} = (N + P_{0X} GG^T P_{0X})^{-1} C^T \Delta L \tag{B-4}$$

Because

$$N + P_{0X} GG^T P_{0X} = \begin{bmatrix} nE_m & -e_m e_n^T \\ e_n e_m^T & mE_n \end{bmatrix} + e_{m+n} e_{m+n}^T = \begin{bmatrix} nE_m + e_m e_m^T & 0_{m\times n} \\ 0_{n\times m} & mE_n + e_n e_n^T \end{bmatrix}$$

where $0_{m\times n}$ is a $m\times n$ zero matrix, $0_{n\times m}$ a nxm zero matrix.
And then,

$$\left[N + P_{0X} GG^T P_{0X}\right]^{-1} = \begin{bmatrix} E_m / n - e_m e_m^T /(m+n)n & 0_{m\times n} \\ 0_{n\times m} & E_n / m - e_n e_n^T /(m+n)m \end{bmatrix} \tag{B-5}$$

Using (B-4) and (B-5), $\Delta\hat{X}_{P_{0X}}$ due to a_i can be obtained as follows

$$\Delta\hat{X}_{P_{0X}} = \begin{bmatrix} \Delta\hat{a}_1 \\ \vdots \\ \Delta\hat{a}_{i-1} \\ \Delta\hat{a}_i \\ \Delta\hat{a}_{i+1} \\ \vdots \\ \Delta\hat{a}_m \\ \Delta\hat{b}_1 \\ \vdots \\ \Delta\hat{b}_n \end{bmatrix}_{P_{0X}} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ a_i \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \begin{bmatrix} a_i/(m+n) \\ \vdots \\ a_i/(m+n) \\ a_i/(m+n) \\ a_i/(m+n) \\ \vdots \\ a_i/(m+n) \\ a_i/(m+n) \\ \vdots \\ a_i/(m+n) \end{bmatrix} = a_i \begin{bmatrix} \mu_i \\ 0_{n\times 1} \end{bmatrix} - a_i/(m+n) \begin{bmatrix} e_m \\ e_n \end{bmatrix} = a_i \begin{bmatrix} \mu_i - e_m/(m+n) \\ -e_n/(m+n) \end{bmatrix} \tag{B-6}$$

Similarly, if there is a systematic error b_j added on the j^{th} reference line, $\Delta\hat{X}_{P_{0X}}$ due to b_j can be obtained as follows:

$$\Delta\hat{X}_{P_{0X}} = \begin{bmatrix} \Delta\hat{a}_1 \\ \vdots \\ \Delta\hat{a}_m \\ \Delta\hat{b}_1 \\ \vdots \\ \Delta\hat{b}_{j-1} \\ \Delta\hat{b}_j \\ \Delta\hat{b}_{j+1} \\ \vdots \\ \Delta\hat{b}_n \end{bmatrix}_{P_{0X}} = b_j \begin{bmatrix} -e_m/(m+n) \\ \eta_j - e_n/(m+n) \end{bmatrix} \tag{B-7}$$

While P_X in (B-3) is arbitrary, $\Delta\hat{X}_{P_X}$ can be obtained from $\Delta\hat{X}_{P_{0X}}$ as follows:

$$\Delta\hat{X}_{P_X} = (E_{m+n} - G(G^T P_X G)^{-1} G P_X)\Delta\hat{X}_{P_{0X}} \tag{B-8}$$

Where E_{m+n} is a $(m+n)\times(m+n)$ unit matrix,

here we give the proof of (B-8).
Firstly, we prove

$$N(N+P_X GG^T P_X)^{-1} = E_{m+n} - P_X G(G^T P_X G)^{-1} G^T \quad \text{(B-9)}$$

In (B-3), $N+P_X GG^T P_X$ satisfies $(N+P_X GG^T P_X)^{-1}(N+P_X GG^T P_X)=E_{m+n}$, and then

$$(N+P_X GG^T P_X)^{-1} N = E_{m+n} - (N+P_X GG^T P_X)^{-1} P_X GG^T P_X \quad \text{(B-10)}$$

When (B-10) multiplies G leftward, we have the following equation in view of $NG=0$

$$(N+P_X GG^T P_X)^{-1} P_X GG^T P_X G = G$$

and then

$$(N+P_X GG^T P_X)^{-1} P_X G = G(G^T P_X G)^{-1}$$

So, (B-10) will be

$$(N+P_X GG^T P_X)^{-1} N = E_{m+n} - G(G^T P_X G)^{-1} G^T P_X$$

and then

$$N(N+P_X GG^T P_X)^{-1} = E_{m+n} - P_X G(G^T P_X G)^{-1} G^T$$

Secondly, we prove

$$\Delta\hat{X}_{P_X} = (E_{m+n} - G(G^T P_X G)^{-1} G^T P_X)\Delta\hat{X}_{P_{0X}}$$

From (B-4), we have

$$C^T \Delta L = (N + P_{0X} GG^T P_{0X})\Delta\hat{X}_{P_{0X}}$$

So, (B-3) can be rewritten as

$$\begin{aligned}\Delta X_{P_X} &= (N+P_X GG^T P_X)^{-1}(N+P_{0X} GG^T P_{0X})\Delta X_{P_{0X}} \\ &= (N+P_X GG^T P_X)^{-1} N\Delta X_{P_{0X}} + (N+P_X GG^T P_X)^{-1} P_{0X} GG^T P_{0X}\Delta X_{P_{0X}}\end{aligned}$$

Considering (B-9) and $G^T P_{0X}\Delta\hat{X}_{P_{0X}} = 0$, we have

$$\Delta\hat{X}_{P_X} = (E_{m+n} - G(G^T P_X G)^{-1} G^T P_X)\Delta\hat{X}_{P_{0X}}$$

The proof of (B-8) is completed.

When $\sum_{i=1}^{m} p_{a_i} = M$ and $\sum_{j=1}^{n} p_{b_j} = N$ in the vector of $(p_{a_1}, \Lambda, p_{a_i}, \cdots, p_{a_m}, p_{b_1}, \cdots, p_{b_j}, \cdots, p_{b_n})_{1\times(m+n)}$,

$$(G^T P_X G)^{-1} = (e_{m+n}^T P_X e_{m+n})^{-1} = (\sum_{i=1}^{m} p_{a_i} + \sum_{j=1}^{n} p_{b_j})^{-1} = (M+N)^{-1}.$$

Then, (B-10) can be rewritten as

$$\Delta\hat{X}_{P_X} = \Delta\hat{X}_{P_{0X}} - GG^T P_X \Delta\hat{X}_{P_{0X}} / (M+N) \quad \text{(B-11)}$$

From (B-11) and (B-6), $\Delta\hat{X}_{P_X}$ due to ai can be obtained as follows

$$\Delta\hat{X}_{P_X} = \begin{bmatrix} \Delta\hat{a}_1 \\ \vdots \\ \Delta\hat{a}_{i-1} \\ \Delta\hat{a}_i \\ \Delta\hat{a}_{i+1} \\ \vdots \\ \Delta\hat{a}_m \\ \Delta\hat{b}_1 \\ \vdots \\ \Delta\hat{b}_n \end{bmatrix}_{P_X} = a_i \begin{bmatrix} \mu_i - e_m/(m+n) \\ -e_n/(m+n) \end{bmatrix} - e_{m+n}e_{m+n}^T P_X a_i \begin{bmatrix} \mu_i - e_m/(m+n) \\ -e_n/(m+n) \end{bmatrix} /(M+N)$$

$$= a_i \begin{bmatrix} \mu_i - e_m/(m+n) \\ -e_n/(m+n) \end{bmatrix} - a_i \begin{bmatrix} p_{a_1} & \cdots & p_{a_m} & p_{b_1} & \cdots & p_{b_n} \\ p_{a_1} & \cdots & p_{a_m} & p_{b_1} & \cdots & p_{b_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{a_1} & \cdots & p_{a_m} & p_{b_1} & \cdots & p_{b_n} \end{bmatrix}_{(m+n)\times(m+n)} \left[\begin{bmatrix} \mu_i \\ 0_{n\times1} \end{bmatrix} - \begin{bmatrix} e_m \\ e_n \end{bmatrix} /(m+n) \right] /(M+N) \quad \text{(B-12)}$$

$$= a_i \begin{bmatrix} \mu_i - e_m/(m+n) \\ -e_n/(m+n) \end{bmatrix} - a_i \left[p_{a_i} \begin{bmatrix} e_m \\ e_n \end{bmatrix} - \left(\sum_{i=1}^{m} p_{a_i} + \sum_{j=1}^{n} p_{b_j}\right) \begin{bmatrix} e_m \\ e_n \end{bmatrix} /(m+n) \right] /(M+N)$$

$$= a_i \begin{bmatrix} \mu_i - e_m/(m+n) \\ -e_n/(m+n) \end{bmatrix} - a_i \left(p_{a_i} - \frac{M+N}{m+n}\right) \begin{bmatrix} e_m \\ e_n \end{bmatrix} /(M+N)$$

$$= a_i \begin{bmatrix} \mu_i - p_{a_i} e_m/(M+N) \\ -p_{a_i} e_n/(M+N) \end{bmatrix}$$

From (B-11) and (B-7), $\Delta\hat{X}_{P_X}$ due to b_j can be obtained as follows

$$\Delta\hat{X}_{P_X} = \begin{bmatrix} \Delta\hat{a}_1 \\ \vdots \\ \Delta\hat{a}_m \\ \Delta\hat{b}_1 \\ \vdots \\ \Delta\hat{b}_{j-1} \\ \Delta\hat{b}_j \\ \Delta\hat{b}_{j+1} \\ \vdots \\ \Delta\hat{b}_n \end{bmatrix}_{P_X} = b_j \begin{bmatrix} -p_{b_j} e_m/(M+N) \\ \eta_j - p_{b_j} e_n/(M+N) \end{bmatrix} \quad \text{(B-13)}$$

From (B-12), p_{a_i} should be 0 when a_i is a systematic error on i^{th} main line, because if p_{a_i} =1, a_i will affect the estimates of the other parameters. p_{a_i} should be 1 if a_i is zero.
From (B-13), if b_j is a systematic error j^{th} main line, p_{b_j} should be 0 to avoid the effects of b_j on the other estimates. Otherwise, p_{b_j} should be 1 if b_j is zero.

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Biography

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