Article

The Geodetic Properties of the Equidistance Line

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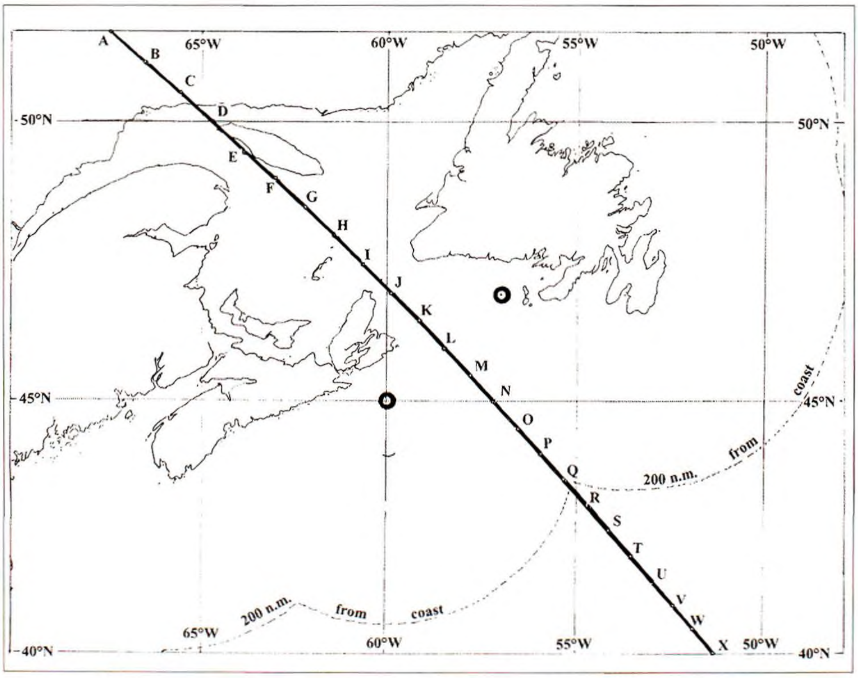
The equidistance line has held pride of place in terms of maritime boundary delimitations – if not for the actual boundary, then for a starting point for discussions. Yet, many people do not understand the mathematical complexities of such a line when it is placed along the surface of the earth. The paper compares the strict equidistance line to the geodesic, the normal section line, the rhumb line and the great circle. The paper also evaluates the equidistance line against straight lines on various map projections.

Introduction

The 1958 Geneva Convention on the Continental Shelf put more weight on the use of the equidistance line method of determining a maritime boundary beyond the territorial sea than on other methods. The equidistance method continues to hold pride of place in many judicial decisions and in State Practice as well as Article 15 of the United Nations Convention on the Law of the Sea (UNCLOS) for delimitation between overlapping Territorial Seas. But do the



Map showing the geographic magnitude of the sample equidistance line used in this paper. The line is equidistant from the two points marked by bold circles legal advisors, programme administrators, even the technical experts really understand the geodetic properties of the equidistance line?

There are few, if any, published papers investigating the geodetic properties of an equidistance line in a quantified manner. This paper confirms the author's intuitive knowledge, but also highlights a few surprises.

The world is not a flat plane: Christopher Columbus (circa 1492) was convinced of that, and Ferdinand Magellan led the flotilla that proved that one could circumnavigate the globe (1518-1521). Navigators, whether using chronometers (developed by John Harrison, 1735-1765) or using the angular distance between the moon and either the sun or stars to determine time, and hence longitude, presumed that the earth was a sphere (Sobel, 1995). During the 1800s, geodesists determined that the earth was actually flattened at the poles; thus, proving a theory developed by Sir Isaac Newton. From a layperson's point of view, if a ball of bread dough were spun rapidly for a long enough time, it would flatten out into a pizza shell.

Geodesists often consider that the earth is an ellipsoid (a solid created by revolving an ellipse about its minor axis). In fact, there are varying concentrations of rocks of differing densities throughout not only the solid crust of the earth, known as the mantle, but also the molten rock within the core, which is generally denser than the mantle. In addition, the mantle tends to have a deep root where there is high topography and is thin in the ocean depths. That means that the sea-level surface of the earth is more like a nubbly pear. The vertical separation of the geoid (the real mean sea level surface) and the mathematical ellipsoid ranges from -40 to +40 metres. For many calculations, these perturbations can be neglected.

In plane, Euclidean, geometry, the strict equidistance line between two points is a straight line at right angles to the line joining the two points, by the rules of congruent triangles. Thus, the equidistance line, being straight, is the shortest distance between two points on that line. In spherical geometry, the equidistance line can be proved to be a great circle, again by the rules of congruent spherical triangles. The great circle is known to be the line of shortest distance between two points on a sphere. But on an ellipsoid where the radii of curvature of the sides of these same triangles are different, the angles at the vertices may not equal the corresponding angles of the other triangle although the three sides correspond. Thus, there is not the automatic link between an equidistance line and a geodesic (the shortest line between two points on an ellipsoid).

Test Data

The typical maritime boundary delimitation specifies a line of a specified geometric character between a series of turning points. Sometimes, delimitation lines are supposed to represent an equidistance line between these turning points. Therefore, it is necessary to compare an equidistance line against the various geometric lines. To create a test case, the author computed a series of points that were exactly equidistant from 2 basepoints (one at 45°N, 60°W, and the other 47°N, 57°W). The ellipsoid used was the Geodetic Reference System 1980 ellipsoid (GRS 80) used in the computations of the North American Datum 1983 and the World Geodetic System 1984. The semi-major (equatorial) diameter (equatorial radius) is 6,378,137 metres and the semi-minor diameter (distance from centre of the earth to either Pole) 6,356,752.314 m. The intermediate points along the equidistance line are at intervals of a half degree of latitude from 51° 30'N to 40° 00'N and are designated 'A' through 'X', inclusive. The end points ('A' and 'X') are over 900 km (almost 500 n.m.) from the two basepoints. This

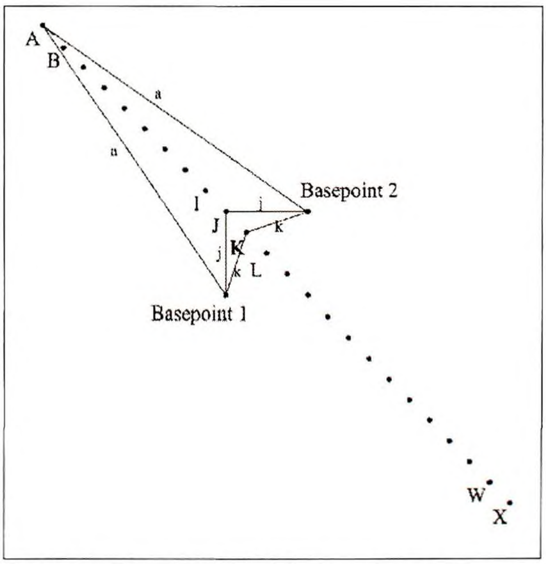


Figure 1: Schematic layout of the two basepoints and the 24 equidistance points. Basepoints 1 and 2 are 174 n.m. apart; 'A' and 'X' are about 950 n.m. apart

length might be the extreme situation for an equidistance line throughout an extended continental shelf under Article 76 of the UNCLOS. The total length of the test line from one end to the other (i.e., from 'A' to 'X') is in the order of 950 n.m. It is intentional that the line is oriented at roughly 45° (actually 135° / 315°) and also that the line is centred at approximately 45°N. These conditions should maximise the rate of change of the mathematical functions involved.

Description of Various Geometric Lines

As well as the equidistance line on the surface of the earth, there are other lines that have recognised geometric properties that are commonly used by geodesists. A short description of each follows.

Great Circle

A few maritime boundary delimitations are defined by series of great circles between the turning points.[1] Great circles assume a spherical earth. They are also the perimeter of slices through the centre of the sphere. The great circle has the geometric property of being the shortest distance along the surface of the sphere between the end-points. If one assumes the earth to be a sphere, the equator and all meridians of longitude would be great circles. A parallel of latitude, other than the equator, is a small circle, since its radius is smaller than the radius of the sphere. For the computation of the equidistance line on the sphere, the radius can be any measurement; however, distances listed in Table 1 and Appendix A were computed using geodetic (ellipsoidal) parameters.

Rhumb Line

Many maritime boundary delimitations are defined as series of rhumb lines (or loxodromes).[2] Rhumb lines have their origin in early navigation. Mariners wished to travel in a single direction from one point until they arrived at the next turning point. They would sail that course until they arrived at that point not knowing the distance travelled with as much precision as their course. They did not mind sailing some extra miles than the shortest distance. A rhumb line is defined as the line that cuts each meridian at the same angle. Thus, it spirals towards each pole, but never actually gets there. Gerhardus Mercator developed a map projection in 1569 where the meridians of longitude are perpendicular to the parallels of latitude. The former are uniformly spaced whereas the latter spread out in proper mathematical relation to compensate for the fact that the meridians get closer together as they approach the pole. It took the development of calculus and the integral of the secant of an angle by Edward Wright in 1599 before the map projection was perfected (Sebert, 2001). Thus, a rhumb line on a Mercator projection map is a straight line. Although originally formulated for a spherical earth, the equations have since been modified for an ellipsoidal earth.

Normal Section

To the author's knowledge, the normal section has not been used for maritime boundary delimitations. The normal section is the line traced on the ellipsoid by the plane that contains the two points on the earth's surface and the vertical at one of the points. Because there is a centrifugal force acting on all bodies on the earth's surface as well as the pure gravitational attraction between bodies, the direction of the total force (i.e., the vertical, also known as the normal) acting on a body is not towards the centre of mass of the earth. In fact, the vertical intersects the axis of rotation of the earth on the opposite side of the equator from the point in question. Except in the unique case where the end-points have identical latitudes, or are North-South of each other, the usual normal section from one end towards the other end does not coincide with the normal section in the reverse direction. The normal section line is the projection onto the ellipsoid of the visual line of sight of a distant point from the observation point. The lateral separation of the two normal sections for the test line (A-X) is about 28 metres. Various geodesists over the past 200 years have developed equations to calculate the length between points and the azimuths at each end of the line given the geo-

[1] Sample boundaries are: 1970 Italy/Yugoslavia, 1975 France/Spain, 1980 Norway/United Kingdom, and 1987 Libya/Malta (ICJ award).

[2] Sample boundaries are: 1966 Denmark/Norway, 1975 France/United Kingdom, 1976 The Gambia/Senegal, 1985 Guinea/Guinea-Bissau (ICJ award), 1988 Dominica/France (Guadeloupe & Martinique), 1989 Guinea-Bissau/Senegal. (Tribunal was of the view that the 1960 boundary line of 240° demarcated by the 1960 agreement for the Territorial Sea & continental shelf was not geodetic, but loxodromic.)

graphic coordinates of the two points (inverse problem), and the geographic coordinates of the end point given the length and the starting azimuth (forward problem). Depending on their complexity, the equations replicate the normal section condition to various accuracies, degrading with length. Theory says that there should be a miniscule difference between the length along the normal section and that along the geodesic, increasing with length of line, with the former being slightly longer. As a test, the author used the data set in this paper, used Clarke's 'best' formula (Clarke, 1880) for the normal section lengths and Sodano's formulae for the geodesic lengths, and computed the differences between the two. The differences built up to 5 cm at 900 km (as was expected), then started to decrease so that the difference was –0.5 m at 1782 km (950 n.m.), i.e., the normal section length being shorter than the geodesic, which was not the expected result. Other formulae, other geometric conditions, and different computers might give different comparisons.

Geodesic

In geodesy, the term 'geodesic' is understood to mean the shortest line on a surface between two points on that surface. The geodesic between two points on a plane is the straight line segment between those two points. The geodesic between two points on a sphere is the shorter arc of a great circle joining the two points. The geodesic on an ellipsoid is, in general, more complex than a second-degree curve (National Geodetic Survey, 1986). It is also reversible – going from A to B is the same line as the line from B to A. The geodesic is based on the international interpretation of 'straight':

Straight line: Mathematically the line of shortest distance between two points in a specified space or on a specified surface (Int. Hydrographic Org., 1993).

Any geodesic has the mathematical property of:

$N \bullet \sin\alpha \bullet \cos\varphi$ = a constant,

Where: N = Radius of curvature in the Prime Vertical; i.e., at right angles to the meridian (a slowly varying number with respect to latitude)

α = instantaneous azimuth

φ = instantaneous latitude.

The accepted best equations for computations using the geodesic are those developed by E.M. Sodano.[3] The equations are reportedly accurate for lines that are up to half way around the earth. The computations are much more complex than those for the normal section, taking noticeably longer to compute, even with modern computers.

The geodesic has been used in several maritime boundary delimitations.[4]

Strict Equidistance Line

The strict equidistance line is the locus of points such that each point has exactly equal geodesic lengths to the two basepoints. This is because the geodesic is the line with the shortest possible length over the surface of the earth. As mentioned earlier, the equidistance line is not exactly a geodesic line itself because computations are being performed on an ellipsoid and not on a sphere or flat plane.

Results

The intercepts of the strict equidistance line, the geodesic, the normal section lines both from the northwest ('A') to the southeast ('X') end and vice versa, the rhumb line (or loxodrome) and the great circle as they cross various latitudes (i.e., latitude fixed, longitude varied) are listed in Appendix A. In summary, the greatest separation between the strict equidistance line and all the others, taken individually is summarised in the Table 1.

The mid point between Basepoints 1 and 2 was 46° 00' 35.74488"N, 58° 31' 37.31242"W. The azimuths from the midpoint to the two basepoints were computed to be 46° 15' 01.07990" and 226° 15' 01.08060" - a miniscule difference from being exactly 180° opposite each other. Using plane Euclidean geometry, the perpendicular would have an azimuth of 316° (or 136°) 15' 01.08025". The actual azimuth to the points on the strict equidistance line varied from –0.01013" to +0.00513" from this theoretical value, except at point 'L' which is only 1.5 km away. These small azimuth differences amount to less than 0.045 metres of offset from the strict equidistance line. The offset at 'L' is less than 0.001 metre.

[3] E.M. Sodano, "A Rigorous Non-Iterative Procedure for very rapid Inverse Solution of very long Geodesics", Bulletin Geodesique, Vol. 48, 1958.

[4] Sample boundaries are: 1970 Iran/Qatar, 1972 Mexico/USA, 1974 Canada/Greenland, 1978 Italy/Spain, 1984 Canada/USA (Gulf of Maine, ICJ decision), 1992 Canada/France (St-Pierre and Miquelon).

Greatest separation of the following lines from the strict equidistance line:	
Geometric line	Greatest separation between the line defined in column 1 and the strict equidistance line.
Rhumb Line	45,017m SW
Great Circle	145 m SW
Normal Section, X to A	15.6 m SW
Normal Section, A to X	13.4 m NE
Geodesic	0.033 m NE

Table 1

Interim Conclusion

The conclusion to draw from this data, particularly from its full version in Appendix A, is that the geodesic line is so close to the strict equidistance line as to be able to say that it can be considered the equidistance line for ALL practical purposes. As a check, the geodesic constant (equation given above) for the points on both the strict equidistance line and on the geodesic line was computed. Although not dead exact, the value was far closer to being constant with the geodesic line than with the strict equidistance line.

To put some perspective on these results, modern satellite positioning in its most refined operating methodology can only define a static position in terms of latitude and longitude to an accuracy of about 0.1 m. At sea, using real-time positioning and differential corrections from a monitor station on shore, satellite positioning is only good to 2 metres.

Map Projections

Maritime boundary lawyers and technical advisers are always wanting to see what the line 'looks like' so that they can make assessments of the practicality of that particular type of line. This means that the line has to be portrayed on maps. But maps can be of many different projections.

Here, it is perhaps best to stop and realise that a map is 'a conventional representation, usually on a plane surface and at an established scale, of the physical features (natural, artificial, or both) of a part or the whole of the Earth's surface. Features are identified by means of signs and symbols, and geographical orientation is indicated.' A map projection is 'a function relating coordinates of points on a curved surface and a coordinate system on a plane.' A map projection cannot preserve all geometric relationships on the curved surface, but it can preserve one or more of them. A conformal map projection preserves angles; an equal-area projection preserves areas; an azimuthal map projection preserves azimuths from a point; and an equidistant map projection preserves distances from a particular point or line. Map projections often map the ellipsoid onto a plane by using a developable surface as intermediary. In a cylindrical map projection the ellipsoid is first mapped onto a cylinder; in a conical map projection the ellipsoid is first mapped onto a cone. Projections directly onto a plane are not designated as such, but are classified according to the location of the centre of projection, location of the plane, etc. (National Geodetic Survey, 1986)[5].

Nautical charts are usually Mercator projection, but atlases can use many varieties - some very sophisticated - and selected to portray the world with minimal distortion in some aspect; be it lengths (scale), direction, area. For example, the National Geographic Atlas (Revised Sixth Edition) issued in 1992 used 16 different projections, ranging from the 1569 Mercator projection to the Robinson projection devised in 1963.

Cylindrical

The cylindrical projection has been used on some very old British Admiralty nautical charts of small areas done at large scale. The construction technique is simple and leads to easy construction of the survey manuscript in the field. One of the two standard parallels (commonly near the mid-latitude) is shown as a straight east-west line, and other parallels of latitude are spaced at distances north or south of that scaling latitude based on a uniform scale. Longitude intervals are derived from the fact that one minute of longitude = one minute of latitude x cosine (scaling latitude). The meridians of longitude are laid down at right angles to the parallels. Thus, the map sheet is uniformly spaced north-south and east-west. The map sheet is only conformal at the mid-latitude where both the east-west and north-south scales are the same. A map projection is said to be conformal when the instantaneous scale in any direction is the same, and the instantaneous directions at a location are true. By

[5] National Geodetic Survey. 1986. Geodetic Glossary. U.S. Dept. of Commerce, Rockville, MD.

that, West is exactly 90° counter-clockwise from North, etc. In the British Admiralty application, one minute of latitude was assumed to be a constant value. This meant that the earth was being assumed to be spherical. In the data presented in this paper, the north-south distances between parallels are based on the GRS-80 ellipsoid used by the WGS-84 coordinate system. For the test, the standard parallels were 42°N and 42°S, so that one minute of longitude equaled 1380.846 m.

Mercator

Mercator is probably the most common map projection used. Certainly it is for nautical charts and often for other applications - even when other projections would be more appropriate. Much has already been discussed about this map projection under the topic of rhumb lines, above.

Mathematically, the north-south distance is the integration of the incremental distances from the equator multiplied by the instantaneous scale. The distance east or west of the Central Meridian (of the map sheet) is the longitude difference converted into metres at the scaling latitude and multiplied by the scale.

For the record, the standard parallel used in the calculations was 47° 20'N, although any distances quoted with respect to this projection have been corrected for the instantaneous scale at that latitude.

Transverse Mercator

Transverse Mercator is, as the name implies, the Mercator projection turned at right angles. Instead of the equator being the starting point, a meridian of longitude is used. Distances east or west of the central meridian are called Eastings, and distances from the equator are called Northings. The central meridian is a straight line, but parallels are curved (concave on the side of the nearer Pole). Other meridians are ever so slightly concave on the side towards the central meridian and would converge at the Pole. To keep the projection from getting outlandish scale problems, the maximum longitude difference from the central meridian is kept to a small value (typically 3° or 1.5°) and a scale factor is applied at central meridian so that the scale within the 6° or 3° zone is very close to the nominal value. The topographic maps of Canada, and of many other nations, use this projection. For this paper, the standard 6° Universal Transverse Mercator projection with a Central Meridian at 57°W was used, even when some positions are more 10° away from the central meridian.

Lambert Conformal Conic

A conic projection in its simplest form can be visualised by placing a cone on a globe with the apex of the cone above the North or South Pole and the cone tangent to the globe at a desired latitude. Then, consider that there is a light source at the center of the earth projecting the image of the globe onto the inside of the cone. Next, remove the cone, slit the side of it and roll out the cone as a flat sheet of paper. The map projection can be improved slightly by considering the cone as slicing the globe at two latitudes instead of being tangent at only one latitude. Parallels of latitude are concentric circles and meridians of longitude are equally spaced straight lines radiating from the centre of these circles.

The Lambert conformai conic projection is a further development of this map projection that is sometimes used for topographic maps or for state-plane coordinate systems - particularly if the coordinated region is narrow North-South and broader East-West. The use of this map projection has not devolved into universal sets of standard parallels, as has the UTM system's selection of map projection constants. In deciding what standard parallels to use, it is usual to determine the minimum and maximum latitudes within the mapping zone, calculate one-sixth the angular distance between those extremes, add that value to the minimum and subtract it from the maximum and then round off the two values. In the examples in this study, two sets of standard parallels were selected; one following this rule; namely, 42°N and 50°N, and the other being a totally inappropriate set; namely, 10°N and 20°N.

Polyconic

As the name implies, a polyconic projection can be visualized as a series of cones, all tangent to their particular parallel of latitude. The Central Meridian is a straight line; all other meridians are concave on the side towards the central meridian. All parallels of latitude, except the Equator, are arcs of circles. A whole globe can be mapped on this projection but it is normally only used in large-scale (small area) applications. One of the advantages of the projection is the fact that it can be computed by hand - as Canadian hydrographers did for years in the preparation of field sheets (the name used for large-scale plans of all the soundings and shoreline detail compiled during a survey). For this study, a Central Meridian of 55°W has been used - purposely not picking one symmetrical to the data set.

Polar Stereographic

Stereographic projection can be visualized as a flat sheet of paper placed tangent to a globe at a location and having a light source at the point diametrically opposite to the point of tangency. A stereographic projection map is conformal, in that the instantaneous scale is the same in all directions. A hemisphere is sometimes shown in atlases using a stereographic projection map - greatly exaggerating areas near the edges. Nautical charts of polar regions often use polar stereographic projection (point of tangency at the Pole) instead of Mercator projection because the Mercator projection is badly distorted in those areas.

Gnomonic

Gnomonic projection can be visualized as a flat sheet of paper placed tangent to a globe at a location and having a light source at the center of the earth. It is impossible to show a whole hemisphere on a gnomonic projection map. Since gnomonic maps are often prepared only at small scales, the earth is assumed to be spherical rather than ellipsoidal. The advantage of the gnomonic projection map is the fact that great circle routes are straight lines. This was important during the Second World War when the Allies tried to locate enemy ships by plotting the radio direction bearings on the same radio message from separated stations. Having a map projection that showed great circle routes (the probable route of the radio signal) as straight lines was important when the targeted ships were in the middle of the ocean. In researching for this paper, the author could only find the formulae for conversion of latitudes and longitudes to map sheet orthogonal (X, Y) coordinates using the spherical earth assumption. For this paper, a point of tangency at 42°N, 55°W was used. The radius of the assumed spherical earth was:

Radius = $\sqrt{(R \bullet N)}$ evaluated at 42°N
= 6,275,898.428 m.

Where: R = Radius of curvature in the Meridian, and
N = Radius of curvature in the Prime Vertical.

Two Studies

There are two studies that can be done using the strict equidistance data set from the first part of the paper. We can look at how well the map projection line compares to the strict equidistance line, and how well a manually constructed equidistance line (using plane geometry techniques on a map sheet) compares with the strict equidistance line.

Comparison of Map Projection Straight Line with Strict Equidistance Line

First, let us look at how well the map projection shows the strict equidistance line as a straight line. This is similar to plotting the two end points of an equidistance line (950 n.m. apart) on the map sheet and drawing the straight line between them and comparing that line to rigorously plotting all the intermediate points along the strict equidistance line. In this exercise, the map sheet orthogonal coordinates (the X, Y coordinates) were computed for the points on the strict equidistance line (points 'A' to 'X') for a map sheet of a certain scale - usually about 1:100,000 to a precision of 0.0001 millimetres (1 cm on the earth's surface). Using the map sheet coordinates of 'A' and 'X' to develop the equation of the straight line on the map projection, the perpendicular distance off the line of the remaining equidistance points ('B' through 'W') was calculated, and converted that back to distances on the earth's surface. The full data set is given in Appendix B, but is summarised in Table 2.

Even with the earth being assumed as a sphere, the gnomonic projection (which portrays great circles as straight lines) shows the strict equidistance line as a straight line significantly better than do the other projections. When one considers these numbers as a function of the dimensions of a 1:1,000,000 map sheet, the line is 1.8 metres long and the line deviates from being straight by just 0.15 mm. This is because, as was discovered above, the strict equidistance line and the geodesic are within 33 millimeters of each other over a length of 950 n.m., and the geodesic is the ellipsoid's version of the sphere's great circle. The Lambert conformal conic (with appropriately selected standard parallels) ranks second, and Transverse Mercator ranks third. For several of the projections, the distances off the line are approximate since the projection is non-conformal. Where possible, the instantaneous scale at the equidistance point locations has been considered, not just the nominal scale. What is evident

Table gives the maximum distance of the strict equidistance line off the straight line on the map projection between the known end points of the equidistance line.

Map Projection	Maximum distance of strict equidistance line off straight line on map projection between ends 950 n.m. apart
Cylindrical Standard Parallels = 42°N, 42°S	70.6 km
Mercator	45.0 km
Lambert Conformal Conic, Std. Parallels 10°N, 20°N	28.9 km
Polar Stereographic	17.1 km
Polyconic, Central Meridian 55°W	3.0 km
Transverse Mercator, Central Meridian = 57°W	1.2 km
Lambert Conformal Conic, Std. Parallels 42°N, 50°N	0.6 km
Gnomonic, Point of tangency = 42°N, 55°W	0.14 km

Table 2

from viewing the whole data sets in Appendix B is that these separations of the strict equidistance line from the straight line on the map projection are very much a function of the parameters used in the map projection. Often the equidistance line is a simple parabola from one end to the other, but some of the lines are 'S' shaped curves.

Comparison of the Right Bisector on the Map Projection with the Strict Equidistance Line

If, for example, one had the basepoints of the two States on a map, one could construct the right bisectors that make up the equidistance line between the appropriate basepoints. Technical experts will often do this as expediency when someone wishes to know, in a broad sense, the location of the equidistance line. However, when the line has to be computed precisely, the technical expert should use proper geodetic formulae to determine the equidistance line. The question being addressed here is how well this manual construction using plane Euclidean geometry (or analytical geometry) can replicate the desired line.

Knowing the geographic positions of the basepoints used to calculate the strict equidistance line, their map sheet orthogonal coordinates (X, Y) were determined and the algebraic equation of the right bisector of the line between the two basepoints. Using that equation, the perpendicular distance off the line of all the known points on the strict equidistance line ('A' through 'X') was calculated. The results are listed in Appendix C and summarised in Table 3.

The Lambert Conformal Conic, if using standard parallels appropriate for the area in question, produces the best results. Transverse Mercator ranks second.

Table gives the maximum distance of the strict equidistance line off the graphical right bisector on the map projection between the basepoints that defined the strict equidistance line.

Map Projection	Maximum distance of strict equidistance line off the graphical right bisector on the map projection between the basepoints that defined the strict equidistance line
Cylindrical Standard Parallels = 42°N, 42°S	145 km
Mercator	49.5 km
Lambert Conformal Conic, Std. Parallels 10°N, 20°N	32.7 km
Polar Stereographic	17.8 km
Polyconic, Central Meridian 55°W	5.6 km
Gnomonic, Point of tangency = 42°N, 55°W	4.1 km
Transverse Mercator, Central Meridian = 57°W	2.4 km
Lambert Conformal Conic, Std. Parallels 42°N, 50°N	1.6 km

Table 3

Point	Latitude	Strict Equidistance	Geodesic	Normal Section A to X	Normal Section X to A	Rhumb Line	Great Circle
A	51° 30'N	67° 26' 36.6269"W	given	given	given	given	given
B	51° 00'N	66° 29' 26.8935"W	66° 29' 26.89281"W 0.008 m NE	66° 29' 26.76193"W 1.6 m NE	66° 29' 27.1774"W 3.5 m SW	66° 40' 00.813"W 8,852 m SW	66° 29' 29.0844"W 27 m SW
C	50° 30'N	65° 34' 20.3056"W	65° 34' 20.30466"W 0.012 m NE	65° 34' 20.0529"W 3.3 m NE	65° 34' 20.8163"W 6.6 m SW	65° 53' 55.126"W 16,580 m SW	65° 34' 24.3238"W 52 m SW
D	50° 00'N	64° 41' 08.0788"W	64° 41' 08.07751"W 0.017 m NE	64° 41' 07.7153"W 4.8 m NE	64° 41' 08.7674"W 9.1 m SW	65° 08' 18.722"W 23,255 m SW	64° 41' 13.6020"W 73 m SW
E	49° 30'N	63° 49' 42.3584"W	63° 49' 42.35688"W 0.021 m NE	63° 49' 41.8949"W 6.3 m NE	63° 49' 43.1828"W 11.1 m SW	64° 23' 10.787"W 28,939 m SW	63° 49' 49.1003"W 91 m SW
F	49° 00'N	62° 59' 56.0877"W	62° 59' 56.08596"W 0.024 m NE	62° 59' 55.5354"W 7.6 m NE	62° 59' 57.0110"W 12.7 m SW	63° 38' 30.536"W 33,686 m SW	63° 00' 03.7923"W 106 m SW
G	48° 30'N	62° 11' 42.8984"W	62° 11' 42.89649"W 0.027 m NE	62° 11' 42.2685"W 8.9 m NE	62° 11' 43.8886"W 14.0 m SW	62° 54' 17.213"W 37,548 m SW	62° 11' 51.3358"W 119 m SW
H	48° 00'N	61° 24' 57.0201"W	61° 24' 57.01810"W 0.029 m NE	61° 24' 56.3245"W 10.0 m NE	61° 24' 58.0503"W 14.8 m SW	62° 10' 30.086"W 40,570 m SW	61° 25' 05.9832"W 129 m SW
I	47° 30'N	60° 39' 33.2048"W	60° 39' 33.20270"W 0.031 m NE	60° 39' 32.4550"W 11.0 m NE	60° 39' 34.2511"W 15.4 m SW	61° 27' 08.451"W 42,791 m SW	60° 39' 42.5059"W 136 m SW
J	47° 00'N	59° 55' 26.6629"W	59° 55' 26.66072"W 0.033 m NE	59° 55' 25.8712"W 11.8 m NE	59° 55' 27.7045"W 15.6 m SW	60° 44' 11.627"W 44,250 m SW	59° 55' 36.1316"W 142 m SW
K	46° 30'N	59° 12' 33.00928"W	59° 12' 33.00712"W 0.033 m NE	59° 12' 32.1880"W 12.5 m NE	59° 12' 34.0286"W 15.5 m SW	60° 01' 38.954"W 44,982 m SW	59° 12' 42.4904"W 144 m SW
L	46° 00'N	58° 30' 48.21746"W	58° 30' 48.21531"W 0.033 m NE	58° 30' 47.3790"W 13.0 m NE	58° 30' 49.1997"W 15.2 m SW	59° 19' 29.798"W 45,017 m SW	58° 30' 57.5690"W 145 m SW
M	45° 30'N	57° 50' 08.5798"W	57° 50' 08.57774"W 0.032 m NE	57° 50' 07.7371"W 13.3 m NE	57° 50' 09.5118"W 14.7 m SW	58° 37' 43.541"W 44,385 m SW	57° 50' 17.6716"W 143 m SW
N	45° 00'N	57° 10' 30.6737"W	57° 10' 30.67174"W 0.031 m NE	57° 10' 29.8394"W 13.4 m NE	57° 10' 31.5449"W 14.0 m SW	57° 56' 19.589"W 43,113 m SW	57° 10' 39.3861"W 140 m SW
O	44° 30'N	56° 31' 51.3320"W	56° 31' 51.33012"W 0.031 m NE	56° 31' 50.5191"W 13.2 m NE	56° 31' 52.1334"W 13.0 m SW	57° 15' 17.364"W 41,226 m SW	56° 31' 59.5547"W 134 m SW
P	44° 00'N	55° 54' 07.6172"W	55° 54' 07.61550"W 0.028 m NE	55° 54' 06.8390"W 12.9 m NE	55° 54' 08.3414"W 12.0 m SW	56° 34' 36.308"W 38,747 m SW	55° 54' 15.2483"W 126 m SW
Q	43° 30'N	55° 17' 16.79938"W	55° 17' 16.79778"W 0.027 m NE	55° 17' 16.0695"W 12.2 m NE	55° 17' 17.4406"W 10.8 m SW	55° 54' 15.879"W 35,699 m SW	55° 17' 23.7445"W 117 m SW
R	43° 00'N	54° 41' 16.3360"W	54° 41' 16.33456"W 0.025 m NE	54° 41' 15.6680"W 11.4 m NE	54° 41' 16.8897"W 9.4 m SW	55° 14' 15.553"W 32,102 m SW	54°41' 22.5077"W 105 m SW
S	42° 30'N	54° 06' 03.8549"W	54° 06' 03.85370"W 0.021 m NE	54° 06' 03.2625"W 10.2 m NE	54° 06' 04.3181"W 8.0 m SW	54° 34' 34.820"W 27,975 m SW	54° 06' 09.1717"W 92 m SW
T	42° 00'N	53° 31' 37.1390"W	53° 31' 37.13800"W 0.018 m NE	53° 31' 36.6362"W 8.8 m NE	53° 31' 37.50955"W 6.5 m SW	53° 55' 13.188"W 23,337 m SW	53° 31' 41.5251"W 77 m SW
U	41° 30'N	52° 57' 54.11238"W	52° 57' 54.11161"W 0.014 m NE	52° 57' 53.7136"W 7.1 m NE	52° 57' 54.38934"W 4.9 m SW	53° 16' 10.177"W 18,204 m SW	52° 57' 57.4972"W 60 m SW
V	41° 00'N	52° 24' 52.82834"W	52° 24' 52.82785"W 0.009 m NE	52° 24' 52.5476"W 5.0 m NE	52° 24' 53.08887"W 4.7 m SW	52° 37' 25.322"W 12,593 m SW	52° 24' 55.1455"W 42 m SW
W	40° 30'N	51° 52' 31.4588"W	51° 52' 31.45848"W 0.006 m NE	51° 52' 31.3109"W 2.7 m NE	51° 52' 31.54954"W 1.6 m SW	51° 58' 58.170"W 6,521 m SW	51° 52' 32.6464"W 22 m SW
X	40° 00'N	51° 20' 48.2839"W	given	given	given	given	given

Appendix A

Point	Cylindrical Standard Parallels= 42°N, 42°S	Mercator	Transverse Mercator, Central Meridian = 57°W	Lambert Conformal Conic, Std. Parallels 42°N, 50°N	Lambert Conformal Conic, Std. Parallels 10°N, 20°N	Polyconic Central Meridian 55°W	Polar Stereo-graphic	Gnomonic, Point of tangency = 42°N, 55°W
A	given	given	given	given	given	given	given	given
B	14.5 km	8.9 km	-0.40 km	0.27 km	5.8 km	-0.80 km	2.9 km	0.03 km
C	27.0 km	16.5 km	-0.72 km	0.45 km	10.9 km	-1.46 km	5.6 km	0.05 km
D	37.6 km	23.3 km	-0.94 km	0.56 km	15.2 km	-1.98 km	8.0 km	0.07 km
E	46.6 km	28.9 km	-1.09 km	0.61 km	18.9 km	-2.38 km	10.1 km	0.09 km
F	54.0 km	33.7 km	-1.18 km	0.61 km	21.9 km	-2.66 km	11.9 km	0.11 km
G	60.0 km	37.5 km	-1.22 km	0.58 km	24.4 km	-2.86 km	13.4 km	0.12 km
H	64.5 km	40.6 km	-1.21 km	0.51 km	26.3 km	-2.96 km	14.6 km	0.13 km
I	67.7 km	42.8 km	-1.16 km	0.41 km	27.6 km	-2.98 km	15.6 km	0.14 km
J	69.8 km	44.2 km	-1.09 km	0.30 km	28.5 km	-2.94 km	16.4 km	0.14 km
K	70.6 km	45.0 km	-0.98 km	0.18 km	28.9 km	-2.85 km	16.9 km	0.14 km
L	70.4 km	45.0 km	-0.86 km	0.06 km	28.8 km	-2.70 km	17.1 km	0.14 km
M	69.1 km	44.4 km	-0.73 km	-0.07 km	28.4 km	-2.52 km	17.1 km	0.14 km
N	66.9 km	43.1 km	-0.60 km	-0.18 km	27.5 km	-2.30 km	16.8 km	0.14 km
O	63.7 km	41.2 km	-0.46 km	-0.29 km	26.2 km	-2.06 km	16.3 km	0.13 km
P	59.6 km	38.7 km	-0.33 km	-0.38 km	24.6 km	-1.80 km	15.5 km	0.12 km
Q	54.7 km	35.7 km	-0.20 km	-0.44 km	22.6 km	-1.53 km	14.5 km	0.12 km
R	49.0 km	32.1 km	-0.10 km	-0.49 km	20.2 km	-1.26 km	13.2 km	0.10 km
S	42.5 km	28.0 km	-0.01 km	-0.50 km	17.6 km	-1.00 km	11.6 km	0.09 km
T	35.4 km	23.3 km	0.06 km	-0.48 km	14.6 km	-0.75 km	9.8 km	0.08 km
U	27.5 km	18.2 km	0.10 km	-0.42 km	11.4 km	-0.52 km	7.8 km	0.06 km
V	18.9 km	12.6 km	0.10 km	-0.33 km	7.9 km	-0.31 km	5.4 km	0.04 km
W	9.8 km	6.5 km	0.07 km	-0.19 km	4.1 km	-0.14 km	2.9 km	0.02 km
X	given	given	given	given	given	given	given	given
	1)		2)			3)		4)

1) distances approximate since non-conformal projection
2) distances not corrected for scale factor
3) distances approximate since non-conformal projection
4) distances not corrected for scale factor, also non-conformal projection

Appendix B

Conclusions

Each study produced its own conclusion. There are also some conclusions drawn from an overall evaluation.

First, once the turning points of an equidistance line have been determined, then the geodesic line is so close to the equidistance line between those turning points that the difference is insignificant.

Second, the equidistance line between two basepoints can be computed on an ellipsoid by finding the mid-point along the geodesic between them, and then computing the geodesic that has an initial azimuth at the mid-point exactly 90° to the instantaneous azimuth of the line joining the two basepoints at the mid-point.

Third, the gnomonic map projection, even if assuming a spherical earth, shows the equidistance line between two points that are known to be equidistant from the same basepoints as a straight line better than any other projection tested. This is because a gnomonic projection is designed to show all great circles as straight lines.

Fourth, in the one example tested (and may not be true for all occasions), the Lambert conformal conic projection was better than the other map projections for use with Euclidean plane geometry techniques to construct the equidistance line on a map sheet. However, for any reasonable amount of precision, particularly with long lines involved, it is recommended that points be computed mathematically along the equidistance line to alleviate problems with map projections.

Fifth, the normal map projections used for nautical charts – Mercator and polar stereographic – are not well suited for displaying the equidistance line accurately. Given other considerations, such as proportionality of areas, consideration might be given to plotting trial lines on equal-area type map projections.

That could be the subject for further study since not one of those map projections was investigated in this study since they are not commonly used for nautical charts or topographic maps. Frankly, the author would need to search out the mathematical formulae for those projections.

Appendix A (Page 32)

The location of points from 'A' to 'X' on lines of various geometric properties. The points on the strict equidistance line are exactly equidistant from 45°N, 60°W and 47°N, 57°W.

Appendix B (Page 33)

The data given is the perpendicular distance of strict equidistance points 'B' through 'W' off the straight line on the map projection between strict equidistance points 'A' and 'X'.

Appendix C (Page 34)

The data given is the distance of strict equidistance points off the graphical right bisector on each map projection between the basepoints that defined the strict equidistance line.

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Point	Cylindrical Standard Parallels= 42°N, 42°S	Mercator	Transverse Mercator, Central Meridian = 57°W	Lambert Conformal Conic, Std. Parallels 42°N, 50°N	Lambert Conformal Conic, Std. Parallels 10°N, 20°N	Polyconic CM 55°W	Polar Stereo-graphic	Gnomonic, Point of tangency = 42°N, 55°W
A	-145 km	49.5 km	2.41 km	-1.57 km	-32.7 km	5.58 km	-17.8 km	3.81 km
B	-123 km	40.4 km	1.85 km	-1.16 km	-26.6 km	4.50 km	-14.9 km	3.44 km
C	-103 km	32.4 km	1.40 km	-0.83 km	-21.3 km	3.57 km	-12.2 km	3.08 km
D	-85 km	25.5 km	1.03 km	-0.58 km	-16.7 km	2.78 km	-9.8 km	2.72 km
E	-70 km	19.5 km	0.74 km	-0.39 km	-12.7 km	2.12 km	-7.7 km	2.36 km
F	-55 km	14.5 km	0.51 km	-0.25 km	-19.4 km	1.57 km	-5.9 km	2.01 km
G	-43 km	10.4 km	0.34 km	-0.15 km	-6.7 km	1.12 km	-4.3 km	1.66 km
H	-32 km	7.1 km	0.21 km	-0.08 km	-4.6 km	0.77 km	-3.1 km	1.31 km
I	-22 km	4.5 km	0.12 km	-0.04 km	-2.9 km	0.49 km	-2.1 km	0.97 km
J	-14 km	2.8 km	0.07 km	-0.02 km	-1.8 km	0.28 km	-1.3 km	0.63 km
K	-8 km	1.8 km	0.04 km	-0.01 km	-1.1 km	0.13 km	-0.8 km	0.29 km
L	-1 km	1.4 km	0.03 km	0.00 km	-0.9 km	0.03 km	-0.6 km	-0.05 km
M	4 km	1.8 km	0.03 km	0.01 km	-1.1 km	-0.03 km	-0.6 km	-0.39 km
N	8 km	2.7 km	0.04 km	0.02 km	-1.7 km	-0.05 km	-0.8 km	-0.73 km
O	10 km	4.3 km	0.05 km	0.04 km	-2.7 km	-0.05 km	-1.4 km	-1.06 km
P	12 km	6.5 km	0.05 km	0.08 km	-4.1 km	-0.03 km	-2.1 km	-1.40 km
Q	13 km	9.2 km	0.05 km	0.14 km	-5.8 km	0.00 km	-3.2 km	-1.73 km
R	13 km	12.5 km	0.03 km	0.22 km	-7.9 km	0.04 km	-4.4 km	-2.06 km
S	12 km	16.3 km	-0.00 km	0.34 km	-10.2 km	0.07 km	-6.0 km	-2.40 km
T	11 km	20.6 km	-0.06 km	0.48 km	-12.9 km	0.09 km	-7.8 km	-2.73 km
U	8 km	25.4 km	-0.14 km	0.66 km	-15.9 km	0.09 km	-9.8 km	-3.06 km
V	5 km	30.7 km	-0.26 km	0.88 km	-19.1 km	0.07 km	-12.2 km	-3.40 km
W	2 km	36.4 km	-0.41 km	1.14 km	-22.6 km	0.02 km	-14.6 km	-3.73 km
X	-3 km	42.6 km	-0.61 km	1.44 km	-26.4 km	-0.07 km	-17.6 km	-4.06 km
	1)		2)			3)		4)

1) distances approximate since non-conformal projection
2) distances not corrected for scale factor
3) distances approxi-mate since non-conformal projection
4) distances not corrected for scale factor, also non-conformal projection

Appendix C

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Biography

David H. Gray has a B.A. Sc. and a M.A.Sc. in geodetic surveying from the University of Toronto. David is a Professional Engineer in the Province of Ontario (P.Eng.) and a Canada Lands Surveyor (CLS).
He was a member of the Canadian Team for the Canada/France maritime boundary arbitration and was the technical expert assigned to assist the Tribunal in the Nova Scotia – Newfoundland offshore resources boundary case. Three Caribbean countries and one African country have had the benefit of his technical advice on their maritime boundaries and limits. He provides technical advice on maritime boundaries and limits to Dept. of Foreign Affairs (Canada).
He has been a technical expert in over 20 fisheries violation cases where has defended and defined technically Canada's maritime boundaries and domestic fishing limits in court in both Canada and the United States.

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