Ray-path Stability in Coastal Waters

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The effect of sound speed fluctuations on acoustic travel time and corresponding depth estimates is studied. Dependence of this effect on beam angle and depth is investigated by means of stochastic analysis and numerical simulation. The analytical approach is based on linearisation of the travel time integral in fluctuations about a given sound speed profile. Simulations are performed by repeatedly exercising a ray trace algorithm for a layered medium with random perturbations at the interfaces. Results generally indicate a stable range of launch angles up to a critical angle, above which errors accumulate rapidly. A critical angle close to 45° is found in many cases, but this can vary as a function of the parameters of the model, especially the magnitude of fluctuations at transducer depth, correlation length in the medium and, in the case of planar transducer arrays, the rotation of the head from horizontal. Variance of travel times as a function of depth is found to satisfy a power law in the range from linear to quadratic depending on environmental parameters and system configuration. Survey data, though necessarily never taken under ideal conditions for statistical analysis, shows consistency with theoretical results.

In natural waters acoustic ray paths are affected by spatial and temporal variations in the speed of sound causing them to deviate from the straight line trajectories they would follow in a uniform medium. Corrections are routinely made for these refraction effects by periodically taking measurements of the sound speed profile at standard depths and applying the principles of geometrical acoustics to determine depth and range as a function of travel time and launch angle. However, even if such measurements were perfect they would represent only a snapshot of the sound speed profile at a particular time and position and at discrete depths. Acoustic travel times would still be influenced by random fluctuations of the medium at different times and positions. Random errors in the sound speed measurements would have a similar effect on processed data. It has been observed that travel times display greater variability at higher launch angles and this may be associated with greater sensitivity of the ray path to environmental fluctuations. Thus, the problem of ray-path stability becomes increasingly important as multibeam sonar systems attempt to utilise higher launch angles.

Introduction

In the present paper three approaches will be employed to study the effect of random fluctuations in sound speed. An analytical approach will be based on linearisation of the travel time integral in perturbed quantities. This allows the variance of travel times to be expressed in terms of the covariance of sound speed fluctuations, with implications for dependence on launch angle and depth. The existence of a critical (from nadir) launch angle is noted. Variance below the critical angle is stable but grows rapidly at higher than critical angles. The critical angle is typically but not necessarily in the neighbourhood of 45°. Dependence on depth is found to follow a power law in the range from linear to quadratic depending on environmental conditions. Numerical simulation is a second method of investigation. A layered medium is assumed with sound speeds perturbed randomly at the interfaces. A ray trace algorithm is applied for each realisation to produce output travel times with statistical fluctuations. Since this method does not employ linearisation, the good agreement obtained with analytical results may be taken as justification for that assumption. Finally, of course, it is always of interest to compare theoretical predictions with empirical data. Survey data can never satisfy the ideal of repeated trials under identical conditions that statistical theory requires. However, examination of data can at least support the statement that results from the field are consistent with theory. A previous treatment by Eeg [Ref. 2] also deals with problems of the type considered here.

Travel Time along a Ray

One way acoustic travel-time, T say, along a ray may be expressed by the following integral:

$$T = \int \frac{ds}{c(s)} \tag{1}$$

where c(s) is the sound speed as a function of arc length s along the ray. We assume vertical stratification in the z-direction so that the sound speed is a function of z only. The integral may then be written as follows:

$$T = \int \frac{dz}{c(z)\cos\theta(c(z))} \tag{2}$$

where $\theta(c(z))$ is the ray angle, the angle between the vertical and the local tangent to the ray. The above integral is used to calculate travel-time to a fixed depth, rather than travel-time between fixed points, as in an eigenray problem. The ray angle can be referred back to the launch point by means of Snell's law:

$$\sin\theta(c(z)) = \frac{c(z)}{c_0}\sin\theta_0 \tag{3}$$

where θ_0 is the launch angle from nadir of the ray and c_0 the sound speed at the launch point. At this point we should distinguish between cylindrical transducer arrays and flat face (line) arrays. For cylindrical arrays the launch angle is fixed. However, flat face arrays do not have fixed launch angles. Instead they are steered to a nominal or design launch angle θ_0 using a design sound speed c_0. The launch angle θ_w in the water follows from a refraction calculation:

$$\frac{\sin\theta_w}{c_w} = \frac{\sin\theta_D}{c_D} \tag{4}$$

where θ_w and θ_D are measured from the transducer normal. For a horizontal transducer $\theta_w = \theta_0$. If, however, the transducer is rotated with respect to the horizontal then a transformation must be performed from the transducer-fixed system to the Earth-fixed system. We obtain:

$$\frac{\sin\theta_0}{c_0} = \frac{1}{c_0}\sin\left(\phi + \sin^{-1}\left(\frac{c_0}{c_D}\sin\theta_D\right)\right) \tag{5}$$

where ϕ is the angle of head rotation.

Random Fluctuations

We are interested in studying the effect of random fluctuations of sound speed. We therefore make the replacements:

$$
\begin{aligned}
c(z) &\to c(z) + c'(z) \\
c_0 &\to c_0 + c_0' \\
\theta_0 &\to \theta_0 + \theta_0'
\end{aligned} \tag{6}
$$

where the stochastic process $c'(z)$ and the random variables c_0', θ_0' represent small random fluctuations about mean quantities $c(z)$, c_0, and θ_0. Although only depth dependent fluctuations are considered here, horizontal dependence may also be important when modelling small scale phenomena, such as turbulent microstructure. For a flat array θ_0' follows from c_0' and Eq. (5). For a cylindrical array $\theta_0' = 0$. The quantity c_0' is introduced as a separate variable because it appears explicitly in Snell's law. Consequently integrals below should be interpreted as follows:

$$\int \equiv \lim_{\varepsilon \to 0} \int_{\varepsilon}^{d} \tag{7}$$

where d is the depth reached by the ray.

Perturbation Analysis

Sound speed fluctuations affect the trajectory of the path as well as the travel-time along the path. Since the fluctuations are small the travel-time integral can be linearised in these quantities. This calculation can proceed directly from Eq. (2) but it is of interest to identify path perturbation as a separate effect. To linear order we may write

$$T = T_1 + T_2 \tag{8}$$

where T_1 represents the effect of sound speed fluctuations along the unperturbed path,

$$T_1 = \int \frac{dz}{(c(z) + c'(z)) \cos\theta(c(z))} \tag{9}$$

and T_2 represents the effect of path perturbation,

$$T_2 = \int \frac{dz}{c(z) \cos(\theta(c(z) + c'(z)))} \tag{10}$$

Linearisation

Linearisation in the random quantities is straightforward. With the use of either Eq. (3) or Eq. (5) the following results are obtained for travel-time fluctuations (deviations of T_1 and T_2 from their mean values):

$$\delta T_1 = \int dz \frac{c'(z)}{c_0^{\,2}} \frac{\sin^2\theta_0}{\cos^3\theta} - \int dz \frac{c'(z)}{c(z)^2} \frac{1}{\cos^3\theta} \tag{11}$$

$$\delta T_2 = \int dz \frac{c'(z)}{c_0^{\,2}} \frac{\sin^2\theta_0}{\cos^3\theta} - \int dz \frac{c(z) c_0'}{c_0^{\,3}} \frac{F(\theta_0)}{\cos^3\theta} \tag{12}$$

where

$$F(\theta_0) = \sin^2\theta_0 \tag{13}$$

for a cylindrical array, and

$$F(\theta_0) = \sin^2\theta_0 - \sin\theta_0 \cos\theta_0 \tan(\theta_0 - \phi) \tag{14}$$

for a flat array.

Total Sound Speed Deviation

Equations (11 & 12) have been deliberately written in a form that will emphasise how their leading terms combine additively. It will be shown below that this produces an effect associated with a critical launch angle. The sum of δT_1 and δT_2 gives the total sound speed deviation, which will now be expressed as follows:

$$\delta T = \int_0^d dz\, c'(z)\, K_1(z) + c_0' \int_0^d dz\, K_0(z) \tag{15}$$

where the kernel functions are defined by:

$$K_1(z) = -\frac{1-2[c(z)/c_0]^2 \sin^2\theta_0}{c(z)^2\left[1-(c(z)/c_0)^2 \sin^2\theta_0\right]^{3/2}} \tag{16}$$

$$K_0(z) = -\frac{[c(z)/c_0]^2 F(\theta_0)}{c_0 c(z)\left[1-(c(z)/c_0)^2 \sin^2\theta_0\right]^{3/2}} \tag{17}$$

Critical Launch Angle

The numerator of the expression for $K_1(z)$ deserves special attention, where the factor of 2 should be noted. Since sound speed excursions from any reference profile are small the ratio $c(z)/c_0$ is close to unity. Therefore, for launch angles close to 45° this numerator will be small or zero over the range of integration. The existence of such a critical launch angle will be demonstrated in simulations presented below. From Eqs. (11 & 12) it is seen that both sound speed variability and path perturbation contribute to this effect. It is often assumed, in applications such as travel-time tomography, that path perturbations can be neglected. That is not the case in the present application.

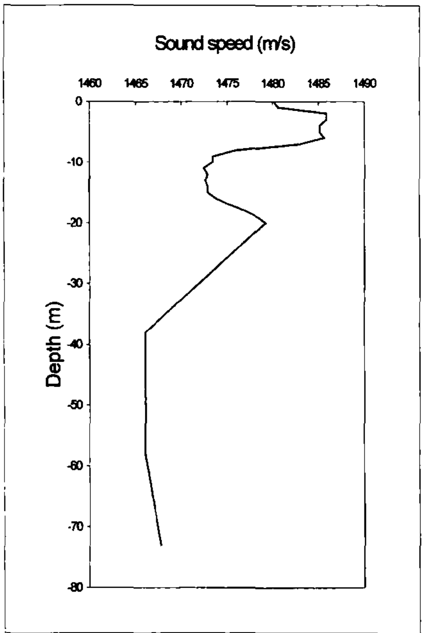

Figure 1: *Sound speed profile used in examples*

Variance of Travel Time Fluctuations

We wish to calculate the variance of travel-time fluctuations. We can express the square of δT as follows:

$$\delta T^2 = \iint dz dz' c'(z)c'(z')K_1(z)K_1(z') + \iint dz dz' \{c_0' c'(z)K_1(z)K_0(z') + c_0' c(z')K_1(z')K_0(z)\} + c_0'^2\left[\int dz K_0(z)\right]^2 \tag{18}$$

The variance of δT follows from the expectation of δT^2. Thus,

$$\mathrm{var}\{\delta T\} = \iint dz dz' \gamma(z,z')K_1(z)K_1(z') + 2\int dz \gamma_0(z)K_1(z)\int dz K_0(z) + \sigma_0^2\left[\int dz K_0(z)\right]^2 \tag{19}$$

where $\gamma(z,z')$ is the autocovariance of $c'(z)$, $\gamma_0(z)$ the covariance of c_0' with $c'(z)$, and σ_0^2 the variance of c_0'.

Uniform Medium

As a special case consider a uniform medium with uncorrelated perturbations. If we assume the latter to be white noise then

$$\gamma(z,z') = \sigma^2\delta(z-z') \tag{20}$$

and K_1 and K_0 are no longer functions of z,

$$K_1 = -\frac{1-2\sin^2\theta_0}{c_0^2\cos^3\theta_0} \tag{21}$$

$$K_0 = -\frac{F(\theta_0)}{c_0^2\cos^3\theta_0} \tag{22}$$

We obtain

$$\mathrm{var}\{\delta T\} = K_1^2\sigma^2 d + K_0^2\sigma_0^2 d^2 \tag{23}$$

where, as a consequence of Eq.(20), the dimensions of σ^2 and σ_0^2 differ by a length coefficient.

Dependence on Depth

We note that the first term on the right hand side above is linear in the depth. This is expected because as the integral of white noise it is a Wiener process Ref. [4, p.502], or Brownian motion, which is known to have this property. The travel time anomaly would also, therefore, be expected to display diffusion-like properties as a function of depth. The second term, which is proportional to the depth squared, is identified with fluctuations at transducer depth. Snell's law as given by Eq. (3) was used to trace rays back to the launch point, thus introducing the quantity c_0. For a flat array this was carried one step further and the ray was referred back to the design sound speed and launch angles of the array. This is especially simple for a horizontal array for which we have:

$$\frac{\sin\theta_0}{c_0} = \frac{\sin\theta_D}{c_D} \tag{24}$$

In this case the quantity c_0 is eliminated in favor of c_D, which, of course, is not a random variable. This has the effect of causing K_0 to vanish in Eq. (23), as can be seen from Eqs.(14 & 23) with $f = 0$. However, it will be seen below that any advantage obtained from this is quickly degraded as the head is rotated.

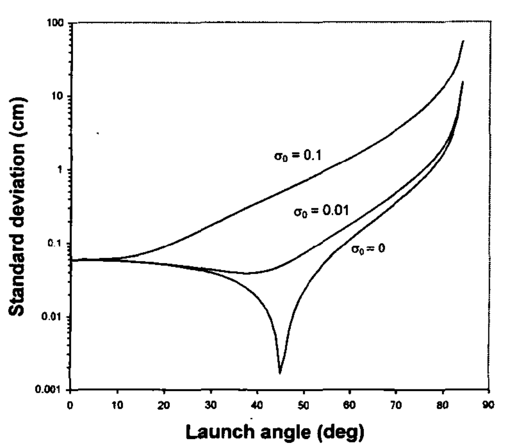

Figure 2: *Standard deviation of depth estimates (cm) vs. launch angle (deg) for cylindrical array*
Bottom to top: Standard deviation of sound speed at transducer 0, 0.01, 0.1 m/s
Standard deviation of sound speed at layer interfaces is 0.1 m/s

Discretisation

For the purpose of performing calculations with actual sound speed profiles it is useful to introduce a discretised version of Eq. (19). Let c_i be sampled (possibly measured) values of sound speed at depths z_i for i = 0, ..., N. The integrals can be approximated by Riemann sums yielding:

$$\mathrm{var}\{\delta T\} = \sum_{i=1}^{N}\sum_{j=1}^{N} \sigma_{ij}^2 K_1(z_i)K_1(z_j)\Delta z_i \Delta z_j + 2\sum_{i=1}^{N} \sigma_{i0}^2 K_1(z_i)\Delta z_i \sum_{i=1}^{N} K_0(z_i)\Delta z_i + \sigma_0^2 \left(\sum_{i=1}^{N} K_0(z_i)\Delta z_i\right)^2 \quad (25)$$

where

$$\begin{aligned} \sigma_{ij}^2 &= \mathrm{cov}(c_i, c_j) \\ \sigma_{i0}^2 &= \mathrm{cov}(c_i, c_0) \\ \sigma_0^2 &= \mathrm{var}(c_0) \\ \Delta z_i &= z_i - z_{i-1} \end{aligned} \quad (26)$$

An alternative approach to obtaining a discretised expression would be to interpolate between sample data points and use a ray-trace algorithm to calculate the travel-time anomaly. In effect the ray-trace becomes a different numerical integration method, but one that does not employ linearisation. The details will not be reported here, but this has been done assuming linear interpolation. Results were virtually identical with the present method, which is strong evidence for the validity of the linearisation assumption.

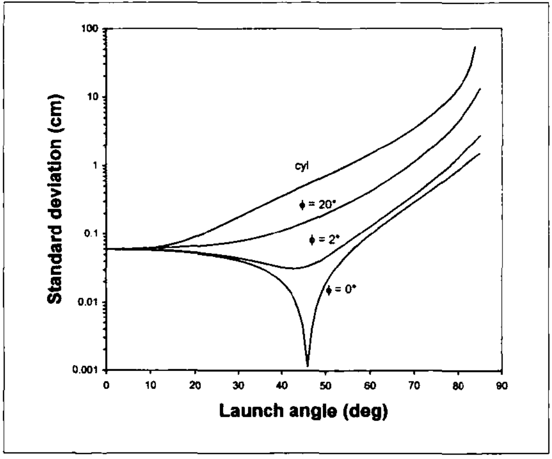

Figure 3: *Standard deviation of depth estimates (cm) vs. launch angle (deg). First three curves bottom to top are for flat array with head rotation from horizontal of f = 0°, 2°, 20°. Top curve is for cylindrical array. Standard deviation of sound speed at transducer depth and at layer interfaces is 0.1 m/s*

Numerical Simulations

A ray-trace algorithm has been used as a tool for performing numerical experiments. The method used assumes a piecewise linear profile connecting data points (c_i, z_i), so that the ray-path segments are circular arcs. The sound speed values are perturbed by uniform random deviates and the program exercised many times to calculate variability. The sound speed profile shown in Figure 1 will be used for examples. This profile was acquired during normal survey activities and was chosen from many available because it displays typical structure. Travel-time has so far been emphasised as the fundamental variable affected by variability of the medium. However, it is the effect of this variable on depth estimates that is of the most practical interest. Therefore, some examples presented will be expressed in terms of the standard deviation of depth estimates, which can be regarded as a proxy for travel-time variability. This is accomplished as follows. The perturbed travel time has been expressed as a function of depth and sound speed, *i.e.*, $T' = f(d, c+c')$. A depth estimate d' obtained from T' would be based on the unperturbed sound speed, and thus d' is a root of $T' = f(d, c+c') = f(d', c)$ for an assumed unperturbed depth d.

Cylindrical Array

Results for a cylindrical array are shown in Figure 2 for the profile perturbed at 1 m intervals by means of a random number generator. The sound speed fluctuations are assumed independent with standard deviations of 0.1 m/s at interfaces below the transducer. For the perturbation at transducer depth three cases are presented, *viz.*, 0, 0.01, and 0.1 m/s. This example shows the importance of fluctuations at transducer depth. For $\sigma_0 = 0$ a critical launch angle at 45° is clearly evident from the deep notch at that value. When the value of σ_0 is increased to 0.01 m/s the notch becomes less prominent and its position shifts slightly to a smaller launch angle. When σ_0 is increased further to 0.1 m/s the notch is no longer present and the critical angle, no longer clearly defined, has shifted to a significantly smaller value. These calculations were also performed analytically using the linearised result expressed by Eq. (25). Agreement with simulations was within ± 4%. Since simulation does not employ linearisation, the good agreement obtained is further evidence of the validity of the linearisation assumption.

Flat Array: Effect of Rotation

It has been shown above that travel time measurements from a flat array in the horizontal position are not affected by sound speed fluctuations at transducer depth, but that this property does not hold if the head is rotated from horizontal. An example is shown in Figure 3 for the case in which independent sound speed fluctuations at 1 m intervals all have a standard deviation of 0.1 m/s. Head rotations of 0°, 2°, and 20° are considered. In the horizontal position results are identical to those obtained for a cylindrical array with $\sigma_0 = 0$, and increasing the head rotation has the same effect as increasing σ_0 for the cylindrical case, an example of which is also displayed in Figure 3 for comparison.

Correlated Fluctuations

Perfectly uncorrelated fluctuations of the medium are an interesting idealised case, which is revealing of phenomena but is questionable on physical grounds. Therefore, we now consider correlated random fluctuations. These will be developed by introducing the concept of a correlation length, L say, which is defined in terms of a correlation coefficient ρ_{ij} as follows:

$$\rho_{ij} \equiv \frac{\mathrm{cov}(c_i', c_j')}{\sqrt{\mathrm{var}(c_i')\,\mathrm{var}(c_j')}} = \exp(-(z_i - z_j)^2 / L^2) \quad (27)$$

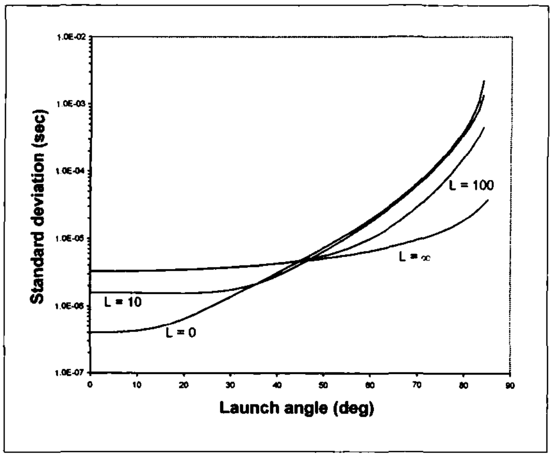

Figure 4: Standard deviation of travel time (sec) vs. launch angle (deg) for a cylindrical array. From top to bottom on the right hand side the correlation length is 0, 10, 100 m, infinity. The standard deviation of sound speed at the transducer and at all layer interfaces is 0.1 m/s

In simulations a set of random variables having this property can be generated by an autoregressive moving average process Ref. [3, p.166]. A set of independent random variables is created by means of a random number generator. A random number γ_1 is generated by forming a weighted sum of variables 1, .., N. A second number γ_2 is generated by forming a weighted sum of variables 2, .., N+1, etc. Random numbers that have overlapping ranges of the independent variables will be correlated. We define a general member of the set as follows.

$$y_i = \sum_{n=1}^{N} \alpha_n rv_{i,n} \quad (28)$$

where random variables $rv_{1,n}$ are independent and $rv_{i+1,n-1}=rv_{i,n}$. It can be shown that the following relation is satisfied:

$$\mathrm{cov}(y_i, y_j) = \sum_n \alpha_{n+j-i}\alpha_n \quad (29)$$

We wish to choose the coefficients a_{ij} so that Eq.(26) is approximately satisfied, *i.e.*,

$$\sum_n \alpha_{n+j-1}\alpha_n \approx \sigma_i \sigma_j e^{-(z_i - z_j)^2/L^2} \quad (30)$$

The left hand side of Eq. (30) has the form of a convolution product, and since the right hand side is a Gaussian function it would seem that the best choice for α_{ij} might be a Gaussian function as well. That is indeed the case. By trial and error it has been found that excellent agreement is found when, for evenly spaced data at intervals of Δ, we choose:

$$\alpha_n = \exp\left[-2(n-N)^2 \Delta^2 / L^2\right] \qquad 1 \le n \le 2N-1 \qquad N\Delta \approx 2L \qquad (31)$$

Effect of Correlation

The results of a simulation using the above method are shown in Figure 4 for a cylindrical array. In this figure are displayed the standard deviations of travel times as a function of launch angle with the correlation length equal to 0 m (independent perturbations), 10 m, 100 m, and infinity (perfect correlation). A trade-off between two competing effects may be observed in these results. For launch angles below critical the effect of correlation is to increase travel time errors. This can be attributed to the fact that the advantage gained by averaging random numbers is reduced when they are correlated. However, a competing effect becomes more prominent for higher launch angles, *viz.*, a lessening of the path perturbation effect due to the presence of correlation. When the launch angle is fixed at the source the effect of correlation tends to cause the medium to behave like a single layer and therefore to reduce perturbation of the path. As evidence of this we can note that for perfect correlation this effect vanishes altogether, as we see from Eq. (12) in which (for a cylindrical array) δT_2 vanishes when we set $c'(z)$ equal to c_0'. The effect is greatest above the critical angle where path perturbation is dominant. For a planar array in the horizontal position the launch angle is not fixed at the source and the path perturbation effect is not found, *i.e.*, the effect of correlation is simply to increase variance over the entire range of beam numbers. However, when the head is rotated from horizontal it behaves like a cylindrical array in this respect.

Dependence on Depth

We have noted that for a white noise process the first term of Eq. (19) is proportional to the depth d, as is known to be the case for a Wiener process. When fluctuations are correlated we no longer have a Wiener process but instead an Ornstein-Uhlenbeck process Ref. [5, p.97], and the variance should increase as a higher power of depth. We can demonstrate this by choosing a particular beam (a launch angle within the stable range was used, *viz.*, 22°) and by simulation calculating the variance as a function of depth as the correlation length is varied. A cylindrical array with $\sigma_0 = 0$ was assumed because in this case only the first term of Eq. (19) is expected to be present. The log-log plot in Figure 5 shows the power laws. For correlation length equal to zero we have a linear law, for very large correlation length the variance is proportional to depth squared. For intermediate correlation lengths the law falls between the first and second powers of the depth. Within the first correlation length the quadratic law is followed because within this range the medium tends to act like a single layer. Thereafter, a transition is made to a linear law as propagation is influenced by somewhat consolidated multiple layers.

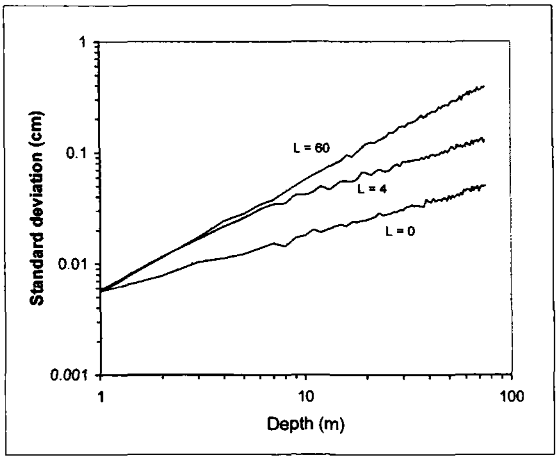

Figure 5: *Log-log plot: standard deviation of depth estimates (cm) vs. depth (m) for cylindrical array with launch angle = 22°. Bottom to top: correlation length L = 0, 4, 60 m. The standard deviation of sound speed fluctuations is 0 m/s at transducer depth and 0.1 m/s at layer interfaces*

Comparison with Survey Data

Ideal data for comparison with theory would be taken by a vessel traversing the same line many times over a perfectly flat horizontal bottom with tides temporarily suspended. Of course, real survey data cannot comply with these conditions. We can however, search for conditions close to ideal and attempt to extract the random component of the data. Analysis has shown a dependence of variance on launch angle and depth. For a given beam the launch angle will vary because of the roll and pitch of the vessel. However we will neglect this effect and consider only the nominal launch angle of the beam. Thus, data for a given beam is treated as if the launch angle were constant. It is *therefore desirable for depth to be approximately constant in the along track direction but not* necessarily in the cross-track direction, for in such a case the depth would still remain approximately constant for a given beam. However, experience has shown that it is never close enough to constant so that its effect can be entirely neglected. We consider the deviation of data from a suitably defined mean state, which must be defined as a function of depth. For this purpose we will use 'loess', a well studied and documented data smoothing algorithm [Ref. 1]. A local regression is performed by fitting a quadratic at each data point according to a weighted least squares principle over a running window of N nearest neighbors, where N must be chosen by the user. Let α be the fraction of the total number of data represented by N. As α becomes larger the fitted curve becomes smoother. The parameter α is too large when the curve is so smooth that it no longer represents the data. On the other hand α is too small when the fitted curve contains spurious structure not present in the data. Our purpose in applying smoothing is to isolate the random component of the data. This can be checked by examining the residuals, the deviations of the data from the smooth curve. Ideally they should be perfectly random. The test for randomness used was a cumulative periodogram with the Kolmogoroff-Smirnov criterion for significance Ref. [3, pp.234-237], with the confidence level chosen to be 95%.

As an example the variability of soundings from a cylindrical array system, the Reson model 8101, is shown in Figure 6. The data are from a survey in Sydney Harbour on 1 January 1999. The system uses 101 beams at 1.5° intervals covering a sector of 150° from port to starboard. There were 204 swaths along the line chosen for this example. The variation in depth in the along track direction was 0.2 m, and in the cross-track direction it was 3 m with the port side beams the deepest. Figure 6 displays the rms of travel-time deviations as a function of beam angle. The value of α determined from the randomness criterion was 0.1. It can be seen that critical angle behavior is present in the results. Fluctuations are stable roughly in the range between ±45° and grow rapidly outside of this range. The graph is asymmetric with larger fluctuations on the port side as would be expected from the greater depths on that side. In general it has been found that as α is increased beyond the point where the randomness criterion is violated graphs such as this one tend to become V-shaped or U-shaped rather than bathtub shaped.

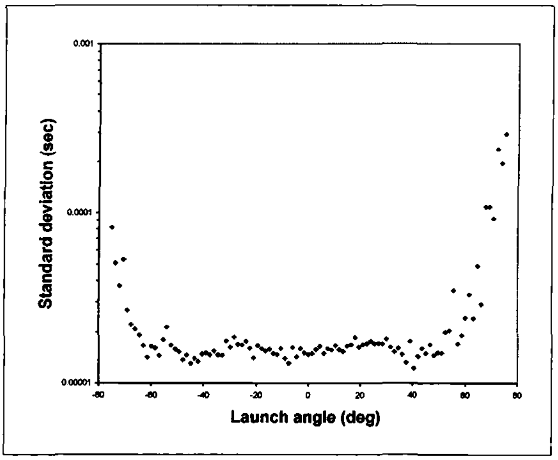

Figure 6: *Standard deviation of travel time (sec) vs. launch angle (deg) for survey data from Sydney Harbour, starboard beams are negative (deg)*

Discussion

Various system properties have been considered that affect travel time variability along a ray. These have included beam angle, depth, variability of sound speed at transducer depth and below, correlation length in the medium, and, in the case of flat arrays, head rotation. The existence of a critical launch angle was identified, which may be sharply defined under certain conditions, but which is probably always present to some extent. Roughly one can say that travel time fluctuations are stable in the range up to the critical angle and increase rapidly above it. Sound speed fluctuations at transducer depth are the most important factor affecting the critical angle. When these are small the critical angle is sharply defined at about

45°, whereas for larger fluctuations the critical angle becomes smaller and less sharply defined. A distinction has been made between the effects of sound speed perturbation and path perturbation. Generally it can be said that above the critical angle the effect of path perturbation is the more prominent one. The distinction between fluctuations at transducer depth and in layers below is also important when one investigates dependence on depth. When fluctuations are negligible at transducer depth and independent below this level, travel time variance is expressed as a stochastic integral that is proportional to depth. When present, however, they contribute a term proportional to the depth squared. The latter term vanishes in the special case of a flat array in the horizontal position. A difference between cylindrical and flat arrays results from different methods of beam steering. Launch angles are fixed for cylindrical arrays but not for flat arrays, which are instead steered to design beam angles that are affected by refraction at the head. As the head is rotated with respect to sound speed layers quadratic dependence on depth no longer vanishes and behavior of flat arrays becomes similar to cylindrical arrays.

The effect of independent fluctuations is an interesting and logical case to consider. Nevertheless, it is not physically plausible to expect fluctuations to be independent no matter how closely placed they may be. Therefore, the effect of correlation has been considered. No dramatic changes are introduced by this assumption. In general, for independent fluctuations dependence on depth is through a linear and a quadratic term. When correlation is present depth dependence is intermediate between these forms. Within the first correlation length travel time variance closely follows the depth squared law, since in this range the medium has a tendency to behave as a single layer. However, as the depth increases further a transition is made to a linear law as the medium starts to have a multi-layered character (though with consolidated layers). Correlation also affects how variance depends on beam angle. The effect is to increase variance below the critical angle and decrease variance above it. The latter may be explained by the tendency of correlation to reduce path perturbation by consolidating layers. It follows that the effect is greatest above the critical angle where path perturbation is dominant.

An ideal comparison of empirical (survey) data with theory would separate launch angle dependence from depth dependence, *i.e.*, multiple trials would be run over various fixed depths. Of course, this is not possible in practice. However, the effect of variable depth can be mitigated in two ways: (1) by searching for data in which the along track depth variation is relatively small, and (2) by de-trending the data. The purpose of the latter is to isolate the random component of the data. In the present study this was accomplished by fitting a smooth curve to the data and testing the residuals for randomness as the criterion for determining the degree of smoothing to be performed. The fine structure of resulting graphs is somewhat sensitive to the degree of smoothing applied. For example, the critical angle is more sharply defined if the smoothing is carefully adjusted. Little is known about the statistics of fluctuations in the water column so no attempt at a quantitative comparison with theory can be made. However, a project supported by NOAA hydrographic vessels is now underway to gather the necessary data.

Acknowledgement

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Biography

Thomas J. Eisler is a physicist with the Coast Survey Development Laboratory of the National Ocean Service. His research interests include radiation, propagation, and scattering of underwater sound. He received the the Ph.D. degree in physics from the University of Michigan in 1962 and has taught at the University of Michigan, M.I.T., and The Catholic University of America.