# Use of a Multi-Reference GPS Station Network for Precise 3D Positioning in Constricted Waterways

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Numerous coastal and inland marine operations, including navigation in shallow constricted waterways require time-consuming and expensive maintenance that includes frequent precise multi-beam hydrographic surveys and dredging operations. In addition, environmental and safety concerns lead to the establishment of stringent regulations regarding the minimum under keel clearance for commercial shipping operations. The clearance is partly a function of the navigation channel charting accuracy and the ability to determine the instantaneous water level in real time. The use of real-time kinematic (RTK) GPS to provide a three-dimensional accuracy of better than 10 cm has the potential to improve the effectiveness of channel maintenance and commercial navigation. In order for RTK GPS to yield such a high level of accuracy, carrier phase observables must be used. One of the most important limitations is the requirement for short distances between the ship and shore-based fixed reference stations. With the current GPS capability, the distance should be kept to less than 15 to 20 km to assure a continuous service. Establishing reference stations with such a high density is time-consuming, logistically difficult and results in high maintaining cost and operational reliability issues. In this paper a method to substantially reduce the number of reference stations is investigated through field trials conducted along the St. Lawrence Seaway, Canada, in 1998 and 1999. The proximity of the trials to a solar maximum resulted in a very high level of atmospheric activity and provided an opportunity to examine the advantages and limitations of both the conventional and multi-reference station RTK methods under such conditions. The results of the trials show that the new approach results in a substantial improvement of up to 60%.

## Introduction

Numerous coastal and inland waterways operations, including navigation in, and maintenance of, constricted navigation channels requires the highest possible level of positioning accuracy. Given that in many cases, the minimum under-keel clearance is well below the one-metre level, the vertical component is particularly critical. A prime example is the St. Lawrence navigation channel, which includes the Seaway as depicted in Figure 1. From Québec City to the Great Lakes, the riverbed is very shallow and the navigation channel requires dredging along numerous and long stretches. In order for ocean-going vessels to make their way to and from Montréal and the Great Lakes harbours, the channel bed and water level are

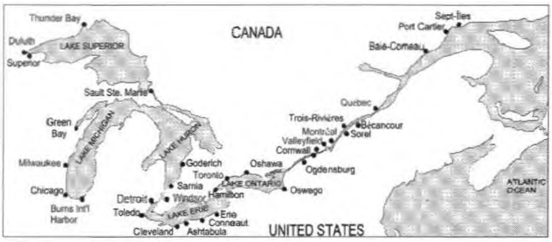

**Figure 1:** *St. Lawrence Seaway (http://seaway.ca)*

closely monitored to assist vessels in adjusting their draft to meet the minimum under-keel clearance of 60 cm set by the Canadian Coast Guard (CCG). The situation is made more complex by the spring ice break-up which can result in boulders of up to one metre in diameter to move into the navigation channel. Immediately after the ice break-up, CCG hydrographic vessels equipped with sweep acoustic systems are used to re-survey the channel in order to detect and remove any boulders and sediment that may have partially filled it during Spring. Given the above operational conditions, a vertical positioning accuracy of one decimetre or better is highly desirable for hydrographic and dredging operations and ship navigation in constricted waterways. A number of other shipborne applications requiring a high degree of accuracy would also benefit from a cm-level positioning system.

The use of high performance real-time kinematic (RTK) Differential GPS (DGPS) methods that require integer carrier phase ambiguity resolution is clearly the method of choice to possibly meet the sub-decimetre accuracy objective stated above. The vertical component is the most critical as depth accuracy will have a significant impact on obstruction detection and dredging cost. In addition, if the bed profile can be accurately and directly established with respect to shore-based reference stations, ships can use the same system to measure their under-keel clearance. Such a capability, integrated with a real-time water level prediction system and an electronic chart, would improve safety of navigation and could eventually lead to a decrease in the minimum clearance requirement, thereby improving the capacity of the Seaway.

In order for RTK DGPS methods to be effective under most situations, including a high level of ionospheric activity, the distance between the ship and the closest reference station must not exceed approximately 15 to 20 km. The high density of reference stations required is generally not acceptable from an operational aspect in view of high deployment and maintenance costs. An alternative is to use a multi-reference station approach to resolve integer ambiguities. An effective method to accomplish this was developed by the University of Calgary during the past few years [Raquet, 1998; Raquet et al., 1998a]. The formulation is straightforward and makes operational implementation in real-time possible. This method is used herein to analyse the density and distribution of reference stations that might be required for sub-decimetre three-dimensional navigation in the St. Lawrence River between Québec City and Montréal. Field tests were carried out in November, 1998, and August, 1999, using GPS measurements collected at up to five permanent radiobeacon stations along the St. Lawrence River and at temporary sites inland. During the November 1998 test, data was collected at five radiobeacon stations and on the hydrographic vessel F.C.G. Smith. During the August 1999 test, three of the five radiobeacon stations were used in addition to four temporary sites.

## Multi-Reference Station Approach

The NetAdjust multi-reference station approach was proposed by Raquet [1998], in order to model errors that affect GPS differential code and carrier-phase kinematic positioning applications [Raquet et al., 1998a; Townsend et al., 1999]. The principle of the method is that as long as the code and carrier-phase observable errors are corrected (or minimised), it is possible to resolve integer ambiguities over longer distances, which increases the achievable accuracy of the user.

The equations used to compute the corrections to the carrier-phase observables are as follows:

$$\delta\hat{l}_r = C_{\delta l_r,\delta l}B^T(BC_{\delta l}B^T)^{-1}(B\overline{\Phi}-\lambda\Delta\nabla N) \quad (1)$$

$$\delta\hat{l} = C_{\delta l}B^T(BC_{\delta l}B^T)^{-1}(B\overline{\Phi}-\lambda\Delta\nabla N) \quad (2)$$

where,

| | |
|---|---|
| $\delta \hat{l}_r$ | are the corrections to carrier-phase observables collected at the rover receiver, in metres, |
| $\delta \hat{l}$ | are the corrections to carrier-phase observables collected at the reference stations, in metres, |
| $\overline{\Phi}$ | is the measurement-minus-range carrier-phase observable ( $\overline{\Phi} = \Phi - \rho$ ), in metres, assuming that the reference station coordinates are known in order to compute the geometric range $\rho$, |
| $\Delta\nabla N$ | are the double difference integer ambiguities between the reference stations (assumed to be known), in cycles, |
| $\lambda$ | is the carrier-phase wavelength, in metres, |
| $B$ | is the double difference matrix ( $B = \partial \Delta\nabla\overline{\Phi} / \partial \overline{\Phi}$ ) (made up of the values +1, -1 and 0), |
| $C_{\delta l}$ | is the covariance matrix of the carrier-phase observables collected at the reference stations, and |
| $C_{\delta l_r, \delta l}$ | is the cross-covariance matrix between the carrier-phase observables collected at the rover receiver and at the reference stations. |

The above equations can be derived using the principle of least-squares prediction (collocation), as shown by Fortes [1998].

Through equations (1) and (2), it can be seen that the double difference ambiguities between the reference stations must be known, along with precise coordinates for the reference stations. The ambiguities can either be solved as integer or real numbers. Integer ambiguities are expected to yield a higher level of performance. The covariance matrices $C_{\delta l}$ and $C_{\delta l_r, \delta l}$ are also required to apply the method (this is actually a requirement of least squares prediction). The procedure used to determine the integer ambiguities and the covariance matrix is described in the next sections.

## November 1998 Test

GPS data was acquired from five CCG DGPS radiobeacon stations, namely Cardinal, Rivière-du-Loup, St-Jean-sur-Richelieu, Trois-Rivières and Lauzon, and by a GPS receiver onboard of the hydrographic surveying vessel F.C.G. Smith, during November 23 to December 4, 1998. Data was collected at a rate of once per second at CCG radiobeacons and on the F.C.G. Smith. All stations were equipped with Ashtech Z-12™ receivers. The ionosphere was found to be very active during the test, with a RMS differential effect of 3 to 4 ppm and a maximum differential effect well in excess of 10 ppm in the position coordinate domain. If the distance between two receivers is 100 km under such conditions, relative position errors of 30 to 40 cm occur even if integer ambiguities are resolved correctly, unless ionospheric-free ambiguities can be used. This will be discussed later.

Two days of data were selected for processing and analysis. The November 25 data was processed to evaluate the performance when the F.C.G. Smith moved through the network area. Unfortunately the St-Jean-sur-Richelieu reference station data was not available on that day. An additional day was processed in which all the reference station data was available, however the F.C.G. Smith was docked (November 27). Figure 2 shows a sketch of the reference network, including the vessel's trajectory during November 25, computed using the University of Calgary's $C^3$NAV™ software [Cannon and Lachapelle, 1997]. Precise coordinates of the reference station network were computed using three days of data (November 25, 27 and 28) and GPSurvey™ [Trimble, 1996]. The relative reference station coordinates were determined with an estimated accuracy of better than a few cm.

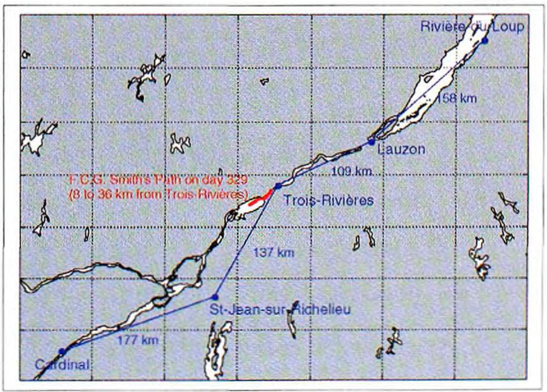

***Figure 2:*** *Reference network and vessel's trajectory on 25 November 1998*

As previously mentioned, it is necessary to know the double difference integer ambiguities between the reference stations in order to apply equations (1) and (2). GPSurvey™ was also used for this purpose. Operationally, this would be done in real-time as described in Sun et al. [1999]. A real-time network ambiguity estimation software, namely NetAR, is being tested by the University of Calgary. All L1 and L2 integer ambiguities between the various reference stations were successfully resolved to integers, which confirmed the high data quality. This does not mean that the residual atmospheric effects after double differencing were small, but that the data was clean with few cycle slips. The Quality Check (QC) program was used to assess the data quality [UNAVCO, 1994], and this showed good results as well.

The covariance matrices $C_{\delta l}$ and $C_{\delta l_r,\delta l}$ have to be known in order to compute the corrections using equations (1) and (2). Each element of these matrices can be calculated based on knowledge of the mathematical functions that map how the correlated errors (atmospheric delays and satellite position errors) behave over the region covered by the network and their dependency on the satellite elevation. In addition, it is necessary to know the variance of the uncorrelated errors (multipath effects and receiver noise) for each station in the network. Thus, elements of the covariance matrices can be properly estimated by combining the correlated and uncorrelated variances by a covariance function, according to the procedure described in detail in Raquet [1998] and Raquet et al. [1998b].

Two mathematical functions for each type of observable (L1 carrier-phase – L1; and Widelane – WL) are used in the approach describe above. The first one maps the zenith variance of the correlated errors over the covered region, and is given by the following equation:

$$\sigma^2_{c_z}(p_m, p_n) = k_1 d + k_2 d^2 \qquad (3)$$

where $\sigma^2_{c_z}(P_m, P_n)$ is the zenith variance of correlated errors between two points $p_m$ and $p_n$ (in cycles$^2$ for the L1 and WL carrier-phase), d is the distance between them (in km) and $k_1$ and $k_2$ are coefficients (see units in Table 1). The second function maps the zenith correlated and uncorrelated errors to a specific satellite elevation, and is given by the equation:

$$\mu(\varepsilon) = \frac{1}{\sin\varepsilon} + k_\mu\left(0.53 - \frac{\varepsilon}{180^\circ}\right)^3 \qquad 4)$$

where $\mu(\varepsilon)$ is a scale factor (unitless) which is multiplied by the zenith errors to obtain the error at a specific satellite elevation $\varepsilon$ (in degrees). $k_\mu$ is a coefficient (unitless).

| | L1 | WL |
|---|---|---|
| $k_1$[a] | 5.90069e-004 | 3.04498e-005 |
| $k_2$[b] | 1.16116e-006 | 2.90428e-008 |
| $k_\mu$[c] | 18.005 | 27.485 |
| $\sigma^2_{u_z}$ (Cardinal)[d] | 4.4273e-05 | 3.0794e-05 |
| $\sigma^2_{u_z}$ (St-Jean-sur-Richelieu)[d] | 4.4273e-05 | 3.0794e-05 |
| $\sigma^2_{u_z}$ (Trois-Rivières)[d] | 4.4273e-05 | 3.0794e-05 |
| $\sigma^2_{u_z}$ (Lauzon)[d] | 4.4273e-05 | 3.0794e-05 |
| $\sigma^2_{u_z}$ (Rivière-du-Loup)[d] | 4.4273e-05 | 3.0794e-05 |

a: cycles$^2$/km; b: cycles$^2$/km$^2$; c: unitless; d: cycles$^2$

***Table 1:*** *k1, k2 and $k_\mu$ coefficients and variances of the uncorrelated errors at the zenith ($\sigma^2_{u_z}$) for each station computed using data collected on November 27, 1998*

Raquet [1998] describes how to compute $k_1$, $k_2$ and $k_\mu$ and the variances of the uncorrelated errors at the zenith ( $\sigma^2_{u_z}$) for each station using field data. This procedure was carried out in this project using the double difference misclosures for all baselines on November 27. The computed values for each observable are shown in Table 1.

The L1 and WL uncorrelated error variances can only be computed using very short baselines, as otherwise the correlated errors dominate the carrier-phase observations. Therefore, the corresponding values included in the table were obtained from Raquet [1998], which is not a rigorous procedure, considering that Raquet computed those values using another network. However, as the baselines present in the network are not very short, it is expected that their influence in the correction computation is negligible.

Figure 3 shows a graphical representation of the functions given by equations (3) and (4) for L1 and WL using the coefficients in Table 1 and those computed using data collected in Norway in September 1997 and 1998, for comparison purposes. It is evident that the correlated errors are much larger in the present case and they degrade faster towards the horizon, mainly due to the fact that the ionosphere was more active.

Once the ambiguities were fixed in the reference network, the carrier phase corrections for each reference network station and for the 'rover' units were derived according to equations (1) and (2) using software NetAdjust, which can also operate in real-time. NetAdjust uses either integer or real number ambiguities as input.

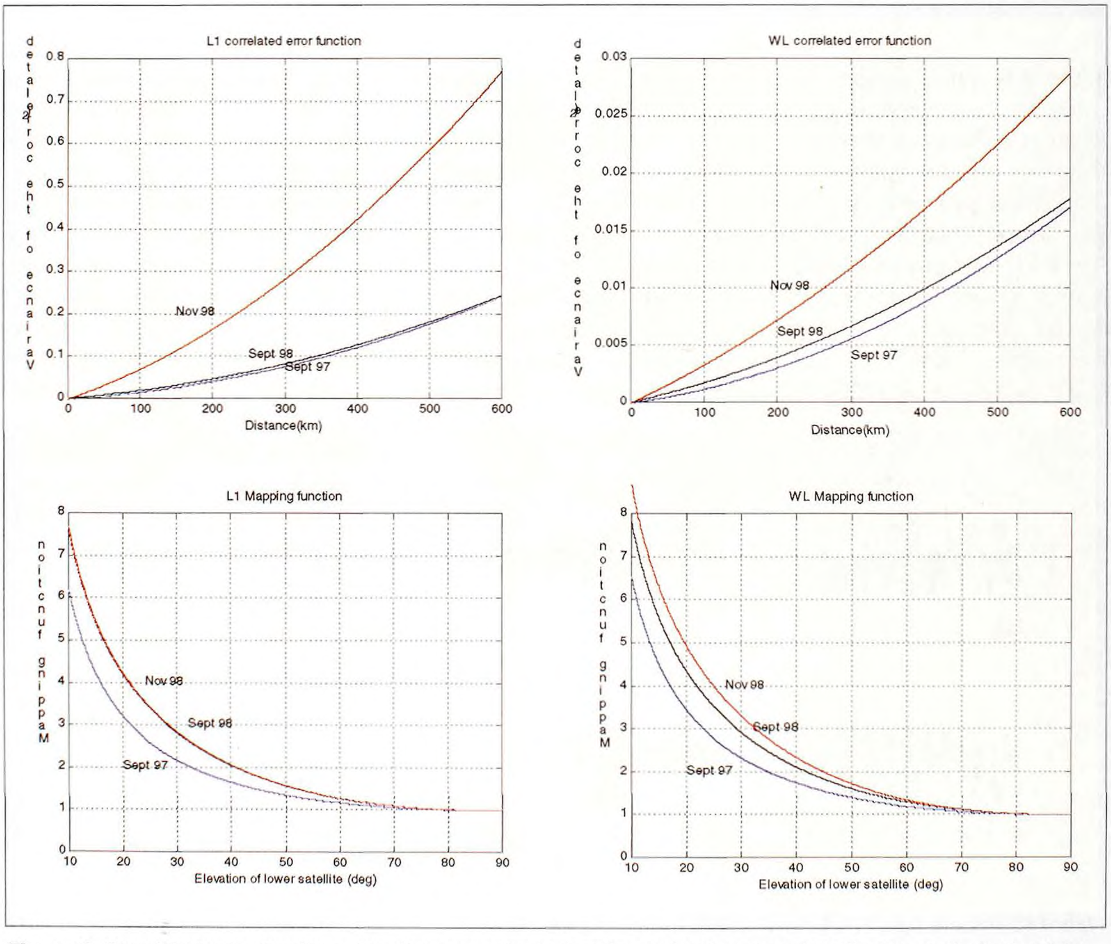

***Figure 3:*** *Correlated error functions (two figures on the top) and mapping functions (two figures on the bottom) for L1 and WL using data from this project (Nov 1998) and data collected in Norway in Sept 1997 and 1998*

## Improvement with the Multi-Reference Station Approach over the Single Reference Station Approach

Results of the comparison between a single station and multi-reference station approaches were computed for the observation, position and ambiguity domains. The objective was to assess how much improvement the multi-reference station approach developed by the University of Calgary gives over the standard OTF method, which uses only one reference station.

| Baseline | L1 (m) | | | WL (m) | | |
|---|---|---|---|---|---|---|
| | Single Ref.St. | Multi Ref.St. | Improv. | Single Ref.St | Multi Ref.St | Improv. |
| Lauzon → Trois-Rivières (109 km) | 0.18 | 0.12 | 30% | 0.19 | 0.15 | 25% |
| St-Jean-sur-Richelieu → Trois-Rivières (137 km) | 0.21 | 0.12 | 39% | 0.22 | 0.15 | 34% |

***Table 2:*** *Single and multi-reference station double difference RMS misclosures and respective improvement for Lauzon to Trois-Rivières (109 km) and St-Jean-sur-Richelieu to Trois-Rivières (137 km) for November 27, 1998*

In the observation domain, the single reference station L1 and WL double difference carrier phase misclosures were compared with those generated after applying the corrections. As it is necessary to know the ambiguities between stations in order to compute the misclosures, only baselines between reference stations were used in this test. Trois-Rivières was assumed to be a 'rover' station with unknown coordinates. Corrections generated using equation (2) were then used to correct the single reference observations of the St-Jean-sur-Richelieu and Lauzon stations, while corrections using equation (1) were computed for the Trois-Rivières 'rover' position. The root mean square (RMS) of the single and multi-reference station double difference misclosures for the Lauzon to Trois-Rivières (109 km) and St-Jean-sur-Richelieu to Trois-Rivières (137 km) baselines, as well as the improvement percentage for November 27 are shown in Table 2. The improvement reaches 39%. This is a very significant but still at a lower level than that obtained in previous tests [e.g. Townsend et al 1999]. This is due to the fact that the absolute values of the single reference station double difference carrier-phase misclosures were much higher in this project (as much as 2 to 3 times) due to a very active ionosphere during the field campaign, as described earlier.

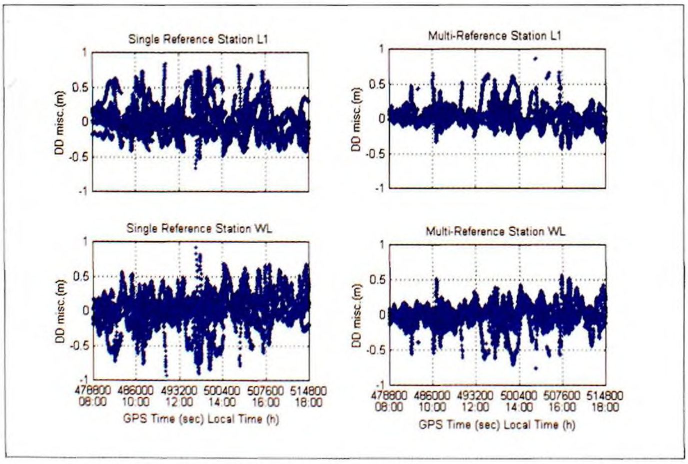

***Figure 4:*** *Single and multi-reference station L1 and WL double difference misclosures for St-Jean-sur-Richelieu to Trois-Rivières baseline (137 km) for November 27, 1998*

Figure 4 shows single and multi-reference station L1 and WL double difference misclosures for the St-Jean-sur-Richelieu to Trois-Rivières baseline. Despite a marked improvement with the multi-reference station approach, some double difference misclosures with absolute values greater than 0.50 m still remain. These values are associated with low elevation satellites whose observation errors were not as well modelled by the method. It was verified that most of the errors

| Coordinates component | L1 (m) | | | WL (m) | | | IF (m) |
|---|---|---|---|---|---|---|---|
| | Single Ref. St. | Multi Ref. St. | Improv. | Single Ref. St. | Multi Ref. St. | Improv. | Multi Ref. St. |
| Latitude | 0.16 | 0.14 | 15% | 0.19 | 0.16 | 16% | 0.02 |
| Longitude | 0.19 | 0.09 | 53% | 0.16 | 0.11 | 35% | 0.01 |
| Height | 0.27 | 0.25 | 8% | 0.36 | 0.30 | 16% | 0.02 |

**Table 3:** *Single and multi-reference station position RMS differences and respective improvement for Trois-Rivières for November 27, 1998*

that were not well corrected corresponded to low satellites not aligned with the network which may suggest that the reference network geometry was not adequate (all reference stations were located more or less along a line that runs in the NE and SW quadrants – see Figure 2).

In the position domain, the known double difference integer ambiguities (resolved previously) were used, since the objective was to verify how much improvement the method brings independent of the ambiguity fixing process. Due to this requirement, the test was carried out again using the Lauzon to Trois-Rivières baseline (109 km), assuming that the Trois-Rivières station was a 'rover' station in the correction's computation. Single and multi-reference station observations were used in FLYKIN™, an OTF software program developed at the University of Calgary [Lu et al., 1994]. In order to use the known ambiguities, the software was modified to read ambiguities from a file instead of trying to resolve them. Coordinates of Trois-Rivières, computed by FLYKIN™ using single and multi-reference station observations for November 27, were compared with the known values. The results are summarised in Table 3. The single reference station L1 and widelane (WL) RMS differences in the vertical component are 27 and 36 cm, respectively, a level of performance that falls below the 10-cm target. The use of the network does improve the agreement to 25 and 30 cm, respectively, but this still falls short of the target. The reason for this is that the network configuration is not sufficiently strong to deal effectively with the extremely high level of ionospheric activity that took place during the test.

In order to confirm that the remaining errors are mostly due to the ionosphere, the results of a third solution are also included in Table 3. This third solution was obtained using an ionospheric-free (IF) linear combination. This linear combination removes the first-order effect of the ionosphere and was calculated with the formula $\phi_1 - (f_2/f_1)\phi_2$, where $\phi_1$ and $\phi_2$ are the L1 and L2 carrier-phase observables in cycles, and $f_1$ and $f_2$ their corresponding frequencies [e.g., Raquet, 1998]. The double difference ambiguities are not integer numbers in the IF case, but they are computed using the known L1 and L2 integer ambiguities by substituting $\phi_1$ and $\phi_2$ for $N_1$ and $N_2$ in the above formula. This can only be done if both the WL and L1 integer ambiguities can be resolved ($N_2$ ambiguities can be derived by extension). L1 integer ambiguities can be relatively difficult to resolve, even if WL integer ambiguities are resolved successfully. The rate of suc-

| Coord Comp. | November 25, 1998 (3,384 epochs in the morning) | | | November 27, 1998 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | All day (33,723 epochs) | | | Morning (17,449 epochs) | | | Afternoon (16,274 epochs) | | |
| | Single Ref St | Multi Ref St | Improv | Single Ref St | Multi Ref St | Improv | Single Ref St | Multi Ref St | Improv | Single Ref St | Multi Ref St | Improv |
| Lat. | 0.64 | 0.65 | -1% | 0.79 | 0.36 | 54% | 0.95 | 0.31 | 68% | 0.56 | 0.41 | 27% |
| Lon. | 0.71 | 0.42 | 41% | 0.79 | 0.39 | 51% | 0.92 | 0.34 | 63% | 0.62 | 0.43 | 30% |
| Height | 1.79 | 0.76 | 58% | 1.57 | 0.95 | 40% | 1.56 | 0.53 | 66% | 1.57 | 1.25 | 21% |

**Table 4:** *F.C.G. Smith single and multi-reference station coordinates RMS differences (in metres) between FLYKIN™ Suite solutions from Lauzon and the respective reference trajectory computed from Trois-Rivières, and respective improvement*

cess in resolving L1 ambiguities once WL ambiguities are resolved will be addressed later, since this becomes important to further improve accuracy performance for applications when the level of ionospheric activity is high. Table 3 shows that the RMS agreement in the vertical component is 2 cm using the multi-reference station approach with IF observables, which is excellent and meet the target 10-cm accuracy stated initially. This high level of accuracy indicates that the multi-reference station approach is very effective to model the troposphere and satellite orbit errors. The differential ionospheric delays were too large and non-linear, given the network configuration, to be properly modelled.

For the ambiguity domain test, the F.C.G. Smith's trajectory was determined from Lauzon by processing both single and multi-reference station WL observations collected on November 25 (ship was moving), and November 27 (ship was docked), using FLYKIN Suite™ [GEOsurv, 1998]. As there were no reference ambiguities to compare the estimated values to, the analysis was only carried out in the position domain. It was then necessary to generate a reference trajectory for the ship by computing it with respect to Trois-Rivières, which was the closest reference station. It had to be ensured that the trajectories computed by FLYKIN Suite™ from Trois-Rivières did not include errors that could deteriorate the results. The adopted solution consisted of calculating each trajectory independently in the forward and reverse time directions. These two solutions were then intercompared. Ship positions that differed more than 5 cm were rejected, in order to generate 'true' reference coordinates with a cm-level precision. The number of accepted epochs for November 25 was only about 10% of the number for November 27. The combination of high ionospheric effects and the fact that the distance between the ship and Trois-Rivières was longer on the first day (varying from 8 to 36 km, instead of only 8 km when the ship was docked) explain this low percentage. Positions corresponding to the accepted epochs were then compared with the ones obtained using the single and multi-reference station observations, with Lauzon as the reference station. When the ship's positions were calculated using Lauzon in the multi-reference station approach, Trois-Rivières was left out, otherwise the results would have been overly optimistic given the proximity of Trois-Rivières from the ship. In order to generate statistics to actually measure the improvement in the ambiguity domain, it was necessary to force FLYKIN Suite™ to re-start the ambiguity search at fixed time intervals, so that enough samples could be generated for each session. Thus every five minutes a new ambiguity set was searched, which generated 120 samples for each ten-hour session. Results of the comparison using WL carrier phase observables are summarised in Table 4. The relatively large RMS differences, which reach 1.79 m, are due to the large differential ionospheric errors over a distance of 109 km and, possibly, to wrongly fixed ambiguity solutions. The effect of the diurnal variation of the ionosphere is obvious. On November 27, the morning results are substantially better than the afternoon results when ionospheric activity reaches a peak. The RMS differences in the vertical components are 0.53 m and 1.25 m using the multi-reference station approach for the morning and afternoon session, respectively. It can be seen that the multi-reference station approach improves the single reference solution up to 68% in some cases. There were more correct FLYKIN™ ambiguities computed using multi-reference station observations than using single reference observations, as expected.

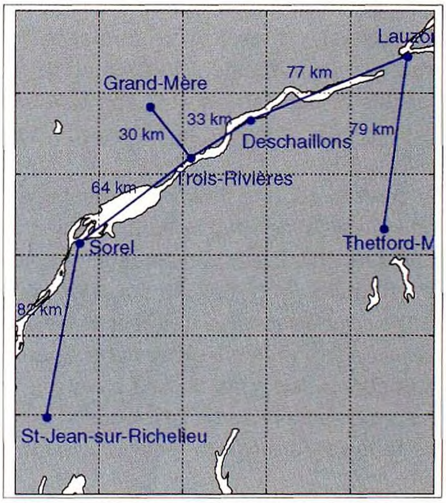

***Figure 5:*** *Reference Network for August 1999 test*

The above results show that, under an active ionosphere, a reference network with inter-station distances of 110-180 km is still too sparse to resolve the integer ambiguities with the availability and reliability required for operational use. The August 1999 test was undertaken to assess performance with a denser reference station network.

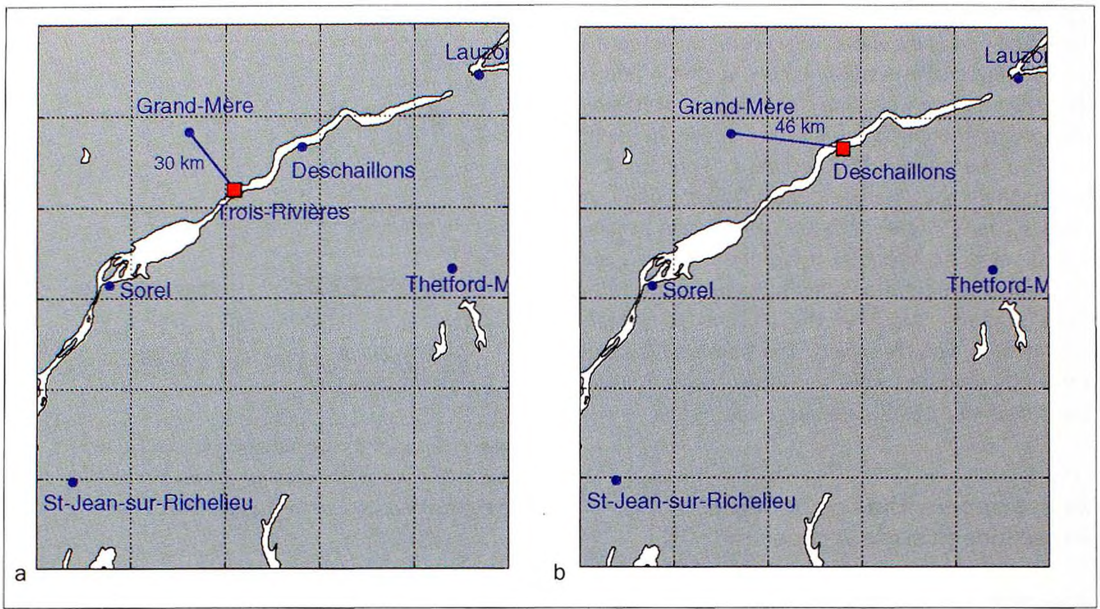

***Figure 6:*** *Network configurations used to test the improvement brought by the multi-reference station approach for August 1999. The 'rover ' stations are represented as a red square and the shown baselines connect them to the closest reference network station*

## August 1999 Test

The August 1999 observation campaign was undertaken in order to assess the multi-reference station approach using a stronger network geometry. From August 2 to 7, four temporary NovAtel MiLLennium™ GPS receivers were used in Grand-Mère, Deschaillons, Thetford-Mines and Sorel, in addition to the CCG radiobeacons located in Lauzon, Trois-Rivières and St-Jean-sur-Richelieu, as shown in Figure 5. No ship was used during this test, as simulating any of the reference stations as a 'rover' was deemed to be adequate. It can also be seen that shorter baselines were involved in this test, as a way to overcome high ionospheric residual effects. Since all observations were made in the static mode, a data rate of 15 s was used for the data reduction. The ionosphere was found to be still relatively active during the test, with a RMS differential effect of 2 to 3 ppm and a maximum differential effect in excess of 7 ppm in the position coordinate domain.

| Baseline | Length (km) | L1 (m) | | | WL (m) | | |
|---|---|---|---|---|---|---|---|
| | | Single Ref. St. | Multi Ref. St. | Improv. | Single Ref. St. | Multi Ref. St. | Improv. |
| GM-TR | 30 | | | | | | |
| August 4 | | 0.04 | 0.02 | 50% | 0.06 | 0.03 | 50% |
| August 5 | | 0.04 | 0.02 | 50% | 0.06 | 0.03 | 50% |
| GM-D | 46 | | | | | | |
| August 4 | | 0.06 | 0.03 | 50% | 0.08 | 0.04 | 50% |
| August 5 | | 0.06 | 0.03 | 50% | 0.08 | 0.04 | 50% |

***Table 5:*** *Single and multi-reference station double difference RMS misclosures and respective improvement for Grand-Mère to Trois-Rivières (30 km) and Grand-Mère to Deschaillons baselines (46 km) for August 4 and 5, 1999*

Two days for which all seven stations were simultaneously tracking satellites were selected for processing and analysis, namely August 4 and 5. The QC program was also used as a pre-processing step and showed a high data quality in general. The Bernese GPS Software, version 4.0, [Rothacher and Mervart, 1996] was used to reduce the GPS data in order to generate precise coordinates for the new reference stations, and to resolve integer ambiguities between the reference stations. Most of the L1 and L2 integer ambiguities were resolved. The same covariance function computed with the data collected during the November 1998 campaign was used.

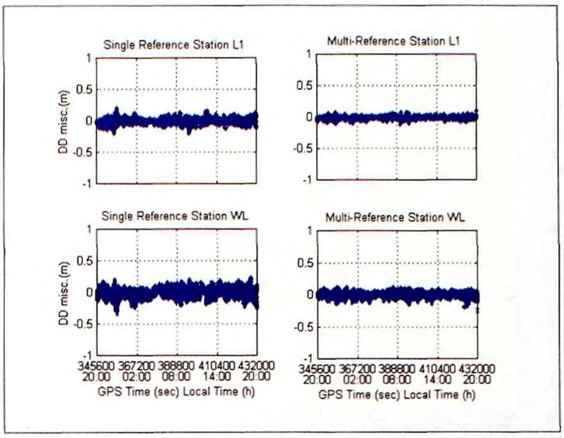

***Figure 7:*** *Single and multi-reference station L1 and WL double difference misclosures for Grand-Mère to Trois-Rivières baseline (30 km) for August 1999*

## Improvement Using the Multi-Reference Station Approach

To assess the improvement using the multiple reference station approach, two network configurations were analysed, as shown in Figure 6. In the first configuration (Figure 6a), Trois-Rivières was treated as the 'rover' receiver and all the remaining stations acted as reference stations to generate network corrections. Since Grand-Mère is the closest station to Trois-Rivières, the 30-km baseline defined by these two stations was processed using single and multi-reference station observations. (Actually the selection of the closest reference station is necessary for the single-reference station approach since, in the multi-reference station approach, the use of any reference station with corrected observations gives the same results). The second configuration (Figure 6b), implemented in order to test the impact of the method when using longer baselines, consisted of choosing Deschaillons as the 'rover', with all the remaining sta-

| Coord. Component | L1 (m) | | | WL (m) | | | IF (m) |
|---|---|---|---|---|---|---|---|
| | Single Ref. St. | Multi Ref. St. | Improv. | Single Ref. St. | Multi Ref. St. | Improv. | Multi Ref. St. |
| **Grand-Mère → Trois-Rivières (30 km), August 4** | | | | | | | |
| Latitude | 0.05 | 0.02 | 53% | 0.07 | 0.03 | 51% | 0.02 |
| Longitude | 0.03 | 0.02 | 53% | 0.04 | 0.02 | 47% | 0.01 |
| Height | 0.08 | 0.05 | 36% | 0.08 | 0.06 | 35% | 0.03 |
| **Grand-Mère → Trois-Rivières (30 km), August 5** | | | | | | | |
| Latitude | 0.05 | 0.02 | 64% | 0.07 | 0.04 | 50% | 0.02 |
| Longitude | 0.03 | 0.01 | 56% | 0.04 | 0.02 | 49% | 0.01 |
| Height | 0.07 | 0.05 | 31% | 0.08 | 0.06 | 31% | 0.04 |
| **Grand-Mère → Deschaillons (46 km), August 4** | | | | | | | |
| Latitude | 0.05 | 0.03 | 33% | 0.06 | 0.04 | 30% | 0.01 |
| Longitude | 0.07 | 0.03 | 56% | 0.08 | 0.04 | 54% | 0.01 |
| Height | 0.08 | 0.07 | 19% | 0.10 | 0.08 | 15% | 0.03 |
| **Grand-Mère → Deschaillons (46 km), August 5** | | | | | | | |
| Latitude | 0.04 | 0.04 | 23% | 0.05 | 0.04 | 20% | 0.02 |
| Longitude | 0.06 | 0.03 | 58% | 0.08 | 0.03 | 63% | 0.01 |
| Height | 0.08 | 0.05 | 43% | 0.12 | 0.07 | 38% | 0.03 |

***Table 6:*** *Single and multi-reference station position RMS differences and respective improvement for August 4 and 5, 1999*

| Baseline | Length | % of corrected fixes of WL ambiguities | | Mean number of epochs to fix WL ambiguities | | % of WL ambiguities reliably converted to L1 | |
|---|---|---|---|---|---|---|---|
| | | Single Ref. St. | Multi Ref. St. | Single Ref. St. | Multi Ref. St. | Single Ref. St. | Multi Ref. St. |
| GM-TR | 30 km | | | | | | |
| August 4 | | 92% | 98% | 9 | 7 | 17% | 28% |
| August 5 | | 93% | 98% | 12 | 8 | 17% | 28% |
| GM-D | 46 km | | | | | | |
| August 4 | | 93% | 94% | 10 | 9 | 21% | 32% |
| August 5 | | 88% | 91% | 17 | 13 | 20% | 25% |

**Table 7:** *Ambiguity domain improvement for Grand-Mère to Trois-Rivières and Grand-Mère to Dechaillons baselines for August 4 and 5, 1999*

tions but Trois-Rivières used to compute network corrections. Grand-Mère was again the closest station to Deschaillons and then the 46-km baseline formed by these two stations was processed using successively the single and multiple reference station approach.

The L1 and WL single and multi-reference station carrier phase double difference misclosures were intercompared in the observation domain using the same procedure as for the November 1998 test. The RMS values of the single and multi-reference station misclosures for Grand-Mère to Trois-Rivières (30 km) and Grand-Mère to Deschaillons (46 km), as well as the improvement percentage are shown in Table 5. The agreement between the two days is excellent. The improvement obtained using the multi-reference station approach is 50%. Increasing the density and improving the layout of the network improved the results significantly, as compared to the November 1998 results. The differential effect of the ionospheric is much reduced, thanks largely to shorter baselines. Figure 7 shows the double difference L1 and WL misclosures using single and multi-reference station observations for Grand-Mère to Trois-Rivières on August 5. The shorter baselines also result in a much better agreement in the position domain, as can be seen by examining Table 6. The multi-reference station approach resulted in an improvement of up to 64%.

| Coord. Component | L1 (m) | | WL (m) | | IF (m) |
|---|---|---|---|---|---|
| | Multi Ref. St. | Improv. | Multi Ref. St. | Improv. | Multi Ref. St. |
| **Grand-Mère → Trois-Rivières (30 km), August 4** | | | | | |
| Latitude | 0.04 | 31% | 0.04 | 42% | 0.02 |
| Longitude | 0.02 | 34% | 0.03 | 35% | 0.02 |
| Height | 0.07 | 15% | 0.06 | 32% | 0.04 |
| **Grand-Mère → Trois-Rivières (30 km), August 5** | | | | | |
| Latitude | 0.03 | 43% | 0.04 | 31% | 0.01 |
| Longitude | 0.02 | 43% | 0.02 | 40% | 0.01 |
| Height | 0.05 | 23% | 0.06 | 20% | 0.04 |
| **Grand-Mère → Deschaillons (46 km), August 4** | | | | | |
| Latitude | 0.03 | 38% | 0.04 | 35% | 0.02 |
| Longitude | 0.03 | 53% | 0.04 | 51% | 0.02 |
| Height | 0.06 | 26% | 0.09 | 18% | 0.04 |
| **Grand-Mère → Deschaillons (46 km), August 5** | | | | | |
| Latitude | 0.03 | 44% | 0.03 | 35% | 0.02 |
| Longitude | 0.02 | 67% | 0.03 | 66% | 0.02 |
| Height | 0.05 | 42% | 0.06 | 41% | 0.03 |

**Table 8:** *Multi-reference station position RMS differences for August 4 and 5, 1999, using real ambiguities in the network*

| Coordinate Component | L1 (m) | | | IF (m) |
|---|---|---|---|---|
| | Single Ref. St. | Multi Ref. St. | Improv. | Multi Ref. St. |
| **INTEGER ambiguities in the network** | | | | |
| Latitude | 0.04 | 0.03 | 36% | 0.02 |
| Longitude | 0.04 | 0.02 | 41% | 0.02 |
| Height | 0.12 | 0.09 | 29% | 0.05 |
| **REAL ambiguities in the network** | | | | |
| Latitude | 0.04 | 0.03 | 28% | 0.03 |
| Longitude | 0.04 | 0.03 | 28% | 0.02 |
| Height | 0.12 | 0.09 | 28% | 0.05 |

***Table 9:*** *Single and multi-reference station position RMS differences and respective improvement for Grand-Mère to Trois-Rivières baseline (30 km) for August 4, 1999, using real ambiguities between reference and rover*

The fixed integer WL mode with the multi-reference station approach, which would be the easiest integer mode to use operationally, yields an RMS agreement better than 10 cm for any coordinate component. Because the differential effects of the atmosphere and orbital errors were modelled effectively with the multi-reference station approach, the use of ionospheric-free observables, while still improving the accuracy by a few cm, is not as critical as in the November 1998 test.

For the ambiguity domain analysis, both baselines were processed using single and multi-reference station observations for the two 24-hour periods with the ambiguity computations re-started every ten minutes in order to generate about 144 samples for each day. The resolved integer ambiguities were then compared with the ones obtained independently with the Bernese software in a batch mode. The results for the WL integer ambiguities, which are summarised in Table 7, show the improvement in terms of percentage of corrected fixes, mean number of epochs to fixed ambiguities and percentage of ambiguities reliably converted to L1. It can be seen that, by using the multi-reference approach, improvements in all three types of comparisons are achieved. The percentage of corrected fixes, even using single reference observations, was above 88%, which can be explained by the use of WL observables over relatively short baselines. The conversion to L1 ambiguities is important if one wants to derive IF solutions, as discussed earlier. Resolution of L1 integer ambiguities is however relatively difficult due to an unfavourable ratio between wavelength and differential errors. This is why the success rate was only 21% or less when using the single reference station approach. The use of the multi-reference station approach improved L1 ambiguity resolution by about 10%.

In order to assess how much accuracy degradation occurs in the position domain when using real number ambiguities, two additional tests were performed using the two August 1999 network configurations described earlier and shown in Figure 7. The two networks were used to generate real ambiguity corrections. The real ambiguities were computed in batch mode. The use of a batch mode should be representative of the real-time case after the network ambiguities have been 'initialised'. In the first test, fixed integer solutions between one of the reference stations, namely Grand-Mère, and the two stations selected as rovers, namely Trois-Rivières and Deschaillons, were then computed using successively the single reference and multi-reference station approach. A correct integer ambiguity file was provide to FLYKIN™ to perform the computations in order to assess the improvement in the position domain, with no external influence from the ambiguity resolution process. The results are summarised in Table 8. The position coordinate RMS agreement is better than 10 cm in all cases. The average improvement over the single reference station mode, whose results are summarised in Table 6, is about 37%. When network integer ambiguities were used (Table 6), the average improvement was about 42%. This shows that the accuracy degradation is minimal when network real ambiguities are used. This provides more operational flexibility.

In the second test, the Grand-Mère to Trois-Rivières baseline was processed using the real ambiguity mode, with both the single and multi-reference station approach. In the latter case, the integer and real ambiguity mode were successively used to derive the network corrections. The results were derived using GPSurvey™ and are summarised in Table 9. Since a continuous sequence of 24 hours of data was processed, the single reference station approach is likely to yield optimistic results. Nevertheless, the multi-reference station approach yields an average improvement of about 28% to 35% over the single reference station approach. The improvement when using integer ambiguities instead of real values in the network is only about 7%. The IF solution yields slightly better results as expected. A comparison of the real ambiguity results with those obtained when using integer ambiguities everywhere (Table 6) shows a degradation of 16%, for the same baseline in the same day.

## Conclusions

The experiments described herein demonstrate the advantages of the NetAdjust multi-reference station approach over the single-reference station approach to resolve integer ambiguities more effectively and to improve accuracy performance to 10 cm, a highly desirable accuracy level for the vertical component for hydrographic operations and navigation in constricted navigation channels. The multi-reference station approach permits a larger spacing between reference stations and thus reduces logistics, operational complexity and cost very significantly. The reference station spacing allowed with the multi-reference station approach is however still dependent on the level of differential GPS errors, the major one being the ionosphere. The tests described herein were conducted under one of the highest levels of ionospheric activity anticipated for Eastern Canada. Under a high level of ionospheric activity, the use of ionospheric-free data may become preferable to single frequency or widelane data, even if the integer nature of the carrier phase ambiguities is lost. In summary, the following was achieved using the multi-reference approach:

- In the observation domain, improvements over the single reference station approach of up to 50%
- In the position domain, improvements of up to 64%
- In the ambiguity domain, up to 31% of L1 ambiguities were successfully resolved versus up to 21% using single stations. This relatively small percentage is explained by the high ionospheric effects during the respective campaign
- The use of floating ambiguities in the reference network was shown to degrade the final multi-reference station approach's results in the position domain by an average of only 5% to 7%. When floating ambiguities were also used to position the rover, an additional average degradation of 8 to 9% occurred, totaling up to a 16% degradation when using floating ambiguities in the reference network and to the rover versus using integer ambiguities everywhere. However, the use of floating ambiguities in the multi-reference approach, which is operationally more robust than that of integer ambiguities, was shown to still deliver a level of accuracy superior to 10 cm

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## References

Cannon, M. E., Lachapelle, G. C3NAV Operating Manual, Combined Code and Carrier for GPS NAVigation, Version 1.3. Department of Geomatics Engineering, The University of Calgary, 1997.

Fortes, L. P. GPS Kinematic Carrier-Phase Positioning Collocation. ENGO629 Minor Seminar, Department of Geomatics Engineering, The University of Calgary, 1998.

GEOsurv. Flykin Suite‰ & Flykin Suite+ Preliminary User's Manual, Rev. 1.8. GEOsurv Inc., Nepean, Ontario, 1998.

Lu, G., Cannon, M. E., Chen, D., Lachapelle, G. FLYKIN‰ Operator's Manual, Version 2.0. Department of Geomatics Engineering, The University of Calgary, 1994.

Raquet, J. Development of a Method for Kinematic GPS Carrier-Phase Ambiguity Resolution Using Multiple Reference Receivers. PhD Thesis, UCGE Report Number 20116, The University of Calgary, 1998. {http://www.ensu.ucalgary.ca/Papers/Thesis/GL/99.20116.JFRaquet.pdf}

Raquet, J., Lachapelle, G., Melgård, T. Test of a 400 km x 600 km Network of Reference Receiver for Precise Kinematic Carrier-Phase Positioning in Norway. In Proceedings of the 11th International Technical Meeting of the Satellite Division of the Institute of Navigation, Nashville, September 1998a. {http://www.ensu.ucalgary.ca/~lachapel/98GPSJoR1.pdf}

Raquet, J., Lachapelle, G., Fortes, L. P. Use of a Covariance Technique for Predicting Performance of Regional Area Differential Code and Carrier-Phase Networks. In Proceedings of the 11th International Technical Meeting of the Satellite Division of the Institute of Navigation, Nashville, September 1998b. {http://www.ensu.ucalgary.ca/~lachapel/98GPSJoR2.pdf}

Rothacher, M., Mervart, L. Bernese GPS Software, Version 4.0. Astronomical Institute, University of Bern, 1996.

Sun, H., Melgård, T., Cannon, M.E. Real-time GPS Reference Network Carrier Phase Ambiguity Resolution. In Proceedings of the National Technical Meeting of the Institute of Navigation, San Diego, January 1999. {http://www.ensu.ucalgary.ca/~cannon/99NTMHS.pdf}

Townsend, B., Lachapelle, G., Fortes, L. P., Melgård, T., Nørbech, T., Raquet, J. New Concepts for a Carrier Phase Based GPS Positioning Using a National Reference Station Network. In Proceedings of the National Technical Meeting of the Institute of Navigation, San Diego, January 1999, pp. 319-326.

Trimble. GPSurvey Software User's Guide. Surveying & Mapping Division, Trimble Navigation Limited, Sunnyvale, 1996.

UNAVCO. QC v3 Users Guide. University NAVSTAR Consortium, Boulder, 1994.
baseline (30km) for 5 August 1999.