The Effect of Logging Compartment Division on Harvesting Pricing

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Abstract

Volumetric productivity in harvesting varies as a function of the properties of the trees. Consequently, the harvesting price paid to the contractor tends to vary. However, pricing appears to depend not only on the properties of trees, but also on the division of any logging site into compartments. Such an effect depends on the statistical properties of any compartment. In the case of an exponentially distributed trunk volume, an upper incomplete gamma function appears to describe the relationship between the harvesting price and the productivity function. In the case of Weibull-distributed trunk volume, the relationship is given in terms of the ratio of two gamma functions. In the case of left-truncated Weibull-distribution, a ratio of two upper incomplete gamma functions appears. Exponential distribution may induce a deviation in excess of 10%, whereas a Weibull distribution with nonzero mode value induces at most 4%. Unification of compartments increases the harvesting price.

Keywords: Logging compartment size, probability density, trunk volume distribution.

Introduction

Length being considered as a linear measure of scale, volume becomes cubic. Time consumption in delimbing depends primarily on tree length, but the volume of the tree on the cube of tree length. Consequently, time consumption per volume unit is approximately proportional to tree length in a power of -2. In other words, volumetric productivity in delimbing is approximately proportional to the trunk volume in the power of 2/3. In general, harvesting expense per volume unit tends to depend on tree volume in a power that is greater than -1 but less than zero (Kuitto et al., 1994; Brunberg, 1997; Eliasson & Lageson, 1999; Suadicani & Fjeld, 2001; Reza Ghaffarian et al., 2007; Rottensteiner et al., 2008; Mizaras et al., 2008).

From many viewpoints, it is rather important that harvesting operations offered to contractors become priced according to the features of the stand affecting productivity. Underpricing operations may induce financial difficulties to the contractor unlucky enough to harvest sites of low productivity. Overpricing operations induces problems to the forest farmer or the concession owner. Harvesting expenses being the greatest expenses in forestry, the cost structure in harvesting affects — or at least should affect — any decision made in silviculture.

A practical difficulty is that the properties of trees are not usually known before harvesting begins. Such information is gathered in the course of harvesting. Consequently, it is common to apply a price list, where the harvesting price is given as a function of tree volume. More specifically, the average trunk volume of the trees harvested within a compartment becomes determined during harvesting. The volumetric harvesting price has been predetermined as a function of the average trunk size, according to an agreement between the contractor and his customer.

Harvesting productivity typically depends on tree size. More specifically, the volumetric productivity in harvesting a trunk depends on the volume of the trunk. However, pricing of a harvesting operation typically is based on the *average* trunk volume of trees within a compartment. Consequently, the harvesting price does not only become determined by the productivity function. It also depends on the distribution of trunk volumes within the logging compartment.

Harvesting productivity issues fall within the discipline of forest engineering. Pricing of operation also involves forest economics or industrial economics. The economics of harvesting operations, however, is rather specifically related to productivity issues. The tradition of forest engineering research appears to be rather experimental. Empirical productiv-

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ity functions often are too complicated to be useful in further analysis (Kuitto et al., 1994; Brunberg, 1997). Forest engineers probably are not accustomed to simplifying formulae enough to make them useful in analysis. This may be the reason that mathematical approaches into productivity appear to be largely missing in the literature (Brunberg, 1997; Lageson, 1997; Affenzeller & Stampfer, 2007; MacDonald, 2007; Pan et al., 2007; Ovaskainen et al., 2008).

The objective of this paper is to study harvesting pricing mathematically. In particular, the effect of the distribution of trunk volumes within a logging compartment on the harvesting price will be investigated.

Justified by experimental investigations reviewed above, we discuss the productivity in harvesting as a power-law function of the trunk volume. A single power law does not necessarily describe productivity over a complete range of trunk volumes. Any empirical productivity law however can certainly be composed of a sequence of power-law functions.

The harvesting price is discussed as a power-law function of the average trunk volume within the compartment, the exponent only experiencing a sign reversion. Generality of results is aspired, and thus the exponent naturally is not fixed. We intend to study the harvesting price in relation to the outcome of the productivity function. The outcome will depend on the distribution of trunk volumes within any logging compartment. A few different kinds of distributions will be discussed, and the effect of the parameters characterizing any distribution explored.

Statistical Effects on Compartment Harvesting Price

Let us approximate the volumetric productivity in harvesting as a power law function of trunk volume v as

$$\frac{dV}{dt} = Q\left(\frac{v}{v_0}\right)^{x} \quad (1).$$

In eq. (1), the constant Q naturally corresponds to volumetric productivity in the case of trunk volume v being equal to an arbitrary reference volume v_0. In order to retain most general applicability of the results, we do not fix the value of the exponent x. The volumetric harvesting price presumably leans to the productivity function. The differential harvesting expense with respect to harvested volume then is

$$\frac{dE}{dV} = \frac{d(th)}{dV} = \frac{h}{Q}\left(\frac{v}{v_0}\right)^{-x} \quad (2),$$

where h is the time unit expense of the harvesting system, and t is time. The differential harvesting expense per harvested volume unit is a function of the trunk volume, or $\frac{dE}{dV} = \frac{dE}{dV}(v)$.

On the other hand, the total harvesting price within a site divided to compartments *k, l, ..., m* can be written

$$VH = \sum_{j=k}^{m} V_j H_j \quad (3),$$

where V_j is the volume of trunks within compartment j, and H_j is the harvesting price per volume unit within the compartment. The total volume of the trunks within any compartment of course can be written in terms of the number of trunks multiplied by the mean volume

$$VH = \sum_{j=k}^{m} n_j \bar{v}_j H_j \quad (4).$$

The harvesting price H_j presumably is a function of the mean trunk volume, or $H_j = H(\bar{v}_j)$. It is assumed here to be a power-law function of the arithmetic mean trunk volume

$$H_j = \frac{h}{Q}\left(\frac{\bar{v}_j}{v_0}\right)^{-x} \quad (5).$$

We find that the harvesting price function in Eq. (5) closely resembles the differential harvesting expense function of Eq. (2). However, the difference is that Eq. (5) is using the arithmetic mean of trunk volume within the compartment, whereas Eq. (2) discusses the trunk volume of an individual tree.

On the basis of the differential expense function of Eq. (2), an estimate for the harvesting expense within a compartment can be produced as

$$E = \int_V \frac{dE}{dV} = \int_n \frac{dV}{dn}\frac{h}{Q}\left(\frac{v}{v_0}\right)^{-x} = \int_n v\frac{h}{Q}\left(\frac{v}{v_0}\right)^{-x} \quad (6),$$

where n refers to the number of trunks within the compartment. Thus the harvesting price according to Eq. (5), in relation to expense according to Eq. (6) is

$$\frac{(VH)_j}{E} = \frac{n(\bar{v}_j)^{1-x}}{\int_n v^{1-x}} = \frac{(\bar{v}_j)^{1-x}}{\overline{v^{1-x}}} \quad (7).$$

The inverse of Eq. (7) is

$$\frac{E}{(VH)_j} = \frac{1}{n}\int_n \left(\frac{v}{\bar{v}_j}\right)^{1-x} = \overline{\left(\frac{v}{\bar{v}}\right)^{1-x}} \quad (8).$$

The second derivative of Eq. (8) with respect to the exponent x is

$$\frac{1}{n}\int_n \left(\frac{v}{\bar{v}}\right)^{1-x}\left[\ln\left(\frac{v_i}{\bar{v}}\right)\right]^2 \quad (9),$$

which is non-negative. Eqs. (7) and (8) yield unity with exponent x values zero and unity. The productivity law of Eq. (1) may reasonably reach values between these extremes. Within this range, Eq. (8) must thus be less than unity, and its inverse Eq. (7) greater than unity.

Eq. (8) can be written

$$\frac{E}{(VH)_j} = \int_0^\infty \left(\frac{v}{\bar{v}}\right)^{1-x} p(v)dv = \int_0^\infty \left(\frac{v}{\bar{v}}\right)^{1-x} p(\frac{v}{\bar{v}})d\frac{v}{\bar{v}} \quad (10).$$

This form allows us to insert a few density functions $p(\frac{v}{\bar{v}})$ for the normalized trunk volume $\frac{v}{\bar{v}}$, and to study how the ratio $\frac{E}{(VH)_j}$ or its inverse $\frac{(VH)_j}{E}$ becomes affected.

Results for a Few Density Functions

Let us first discuss an exponential distribution, where

$$p(\frac{v}{\bar{v}}) = \exp(-\frac{v}{\bar{v}}) \quad (11).$$

Further defining y≡2-x, we can identify a gamma function

$$\frac{E}{(VH)_j} = \int_0^\infty \left(\frac{v}{\bar{v}}\right)^{y-1} \exp(-\frac{v}{\bar{v}})d\frac{v}{\bar{v}} = \Gamma(y) = \Gamma(2-x) \quad (12).$$

The exponent x running from zero to one, we have to study the values of the gamma function within the range 1…2. Both of these end points correspond to gamma function value of unity. In the middle of the range the value of the gamma function is 0.886. Exponent x value of 2/3 corresponds to gamma function with argument 4/3, which yields 0.893.

In reality, the density function of the volume of trees to be harvested hardly may extend to zero. In such a case the exponential density function may be given as $p(\frac{v}{w}) = \exp(-\left(\frac{v}{w} - \frac{a}{w}\right))$, for values $v \geq a$. The mean trunk volume for this distribution is $\bar{v} = a + w$. We will now discuss the trunk volume normalized with the scaling parameter: $V = \frac{v}{w}$, $A = \frac{a}{w}$, $\bar{V} = A + 1$.

Eq. 8 may then be rewritten as

$$\frac{E}{(VH)_j} = \overline{\left(\frac{V}{\bar{V}}\right)^{1-x}} = \int_A^\infty \left(\frac{V}{A+1}\right)^{1-x} \exp(-(V-A))dV = \frac{\overline{V^{2-x}} - A^{2-x}}{(2-x)(A+1)^{1-x}} \quad (13).$$

We find from Eq. (13) that the limit A->0, $(1-x)\overline{V^{-x}}$ tends to a gamma function according to Eq. (12). Apart from this limit, we are able to say

$$\overline{V^y} = y\overline{V^{y-1}} + A^y \quad (14).$$

In the right-hand part of Eq. (13), x must differ from 2. Another choice in the integration in parts (Eq. (13)) would raise limitation $x \neq 1$, which would be more detrimental in the present case. Eq. (13) can be rewritten as

$$\frac{E}{(VH)_j} = \frac{\exp(A)}{(A+1)^{y-1}} \int_A^\infty V^{y-1} \exp(-V)dV = \frac{\exp(A)}{(A+1)^{y-1}} \Gamma(y, A) = \frac{\exp(A)}{(A+1)^{1-x}} \Gamma(2-x, A) \quad (15),$$

where we find an upper incomplete gamma function. Such a function can be evaluated in terms of the corresponding gamma function and the lower incomplete gamma function (Arfken & Weber, 2001). However, the most straightforward way is to evaluate Eq. (13) numerically for the appropriate values of A and x. The result is shown in Fig. 1. We find from Fig. 1 that harvesting expense in relation to harvesting price, which is given by Eq. (12) in the case of an exponential distribution extending to zero, approaches unity as a function of the lower limit of the volume distribution A.

The exponential distribution obviously can be used to describe trees to be harvested in low thinning, where predominantly small trees are removed. Such a distribution cannot necessarily describe the trees to be removed in final felling. Another kind of distribution must be discussed.

A Weibull distribution with density function $p(v) = \frac{k}{\lambda}\left(\frac{v}{\lambda}\right)^{k-1} \exp(-\left(\frac{v}{\lambda}\right)^k)$ appears to be able to describe quite a few phenomena in Nature (Weibull, 1951; Yang et al., 1978). Using this distribution, Eq. (8) becomes

$$\frac{E}{(VH)_j} = \int_0^\infty \left(\frac{v}{\bar{v}}\right)^{1-x} p(v)dv = \frac{\Gamma\left(\frac{1-x}{k}+1\right)}{\left[\Gamma\left(\frac{1}{k}+1\right)\right]^{1-x}} \quad (16).$$

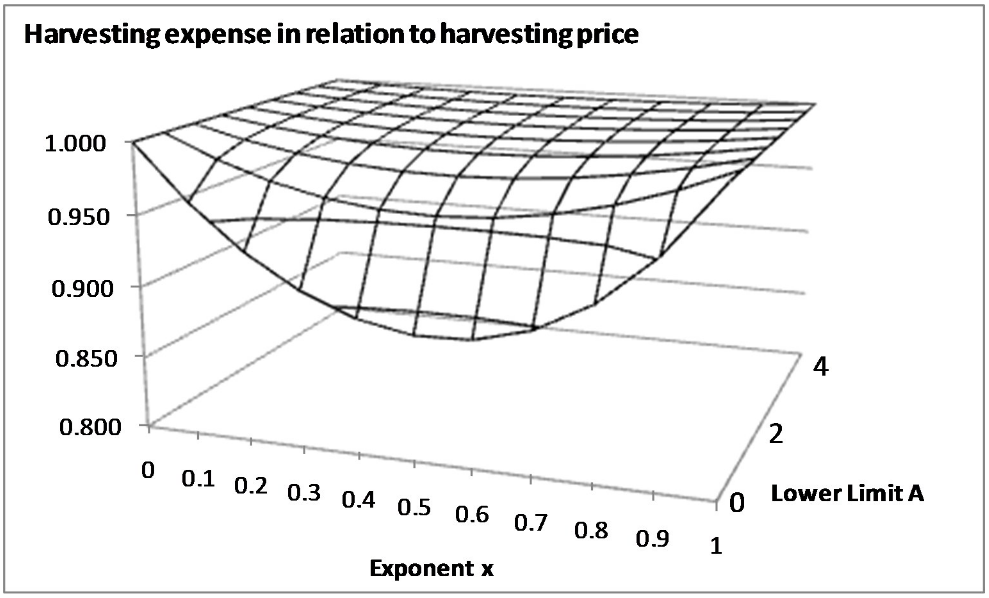

Figure 1. Harvesting expense in relation to harvesting price, with exponentially distributed trunk volume, according to Eq. (13).

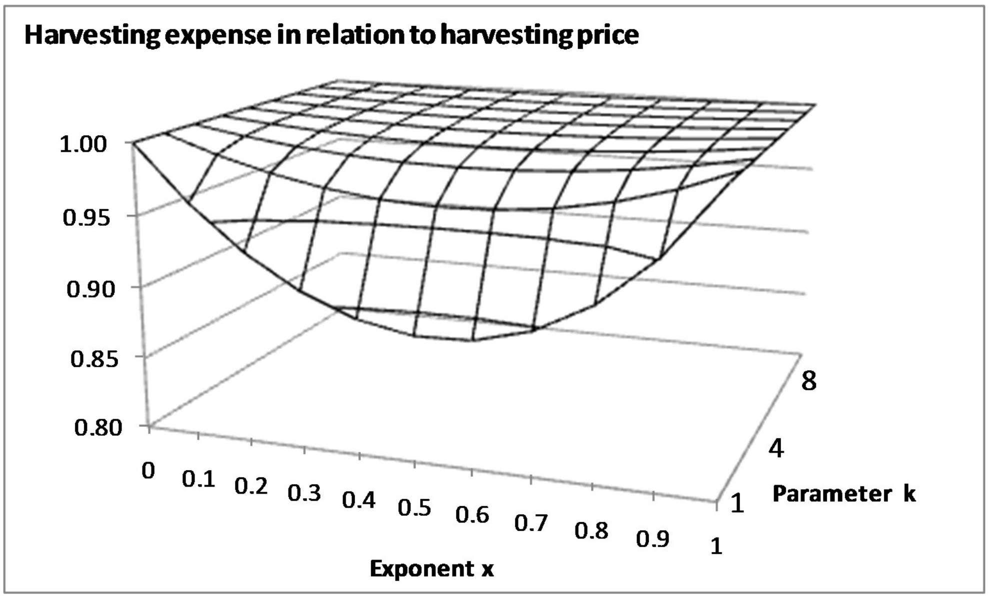

Figure 2. Harvesting expense in relation to harvesting price, with Weibull-distributed trunk volume, according to Eq. (16).

Harvesting expense in relation to harvesting price, according to Eq. (16), is shown in Fig. 2. We find that the ratio approaches unity as a function of the distribution parameter k.

With low values of k, the Weibull distribution shows a large probability density close to zero. In that sense, it resembles the exponential distribution. However, this changes when k approaches 2, in which case zero observations have vanishing probability, and the normalized standard deviation corresponds to 0.52. In general, the normalized standard deviation of the Weibull distribution is

$$\frac{\sigma}{\mu} = \frac{\left[\Gamma\left(\frac{2}{k}+1\right) - \Gamma^2\left(\frac{1}{k}+1\right)\right]^{1/2}}{\Gamma\left(\frac{1}{k}+1\right)} \qquad (17).$$

Figure 2 is redrawn as a function of the normalized standard deviation in Fig. 3, where k runs from 2 to 10, and $\frac{\sigma}{\mu}$ correspondingly from 0.52 to 0.12. We find, then, in Fig. 3, the deviation between the harvesting price and the productivity function hardly exceeds 4%.

One might argue that the distribution of trunk volumes to be harvested does not necessarily extend to zero, neither in the case of Weibull-distributed trunk volumes. This would lead to a left-truncated Weibull distribution (Zutter et al., 1986; Maltamo et al., 2004; Palahí et al., 2006), with density

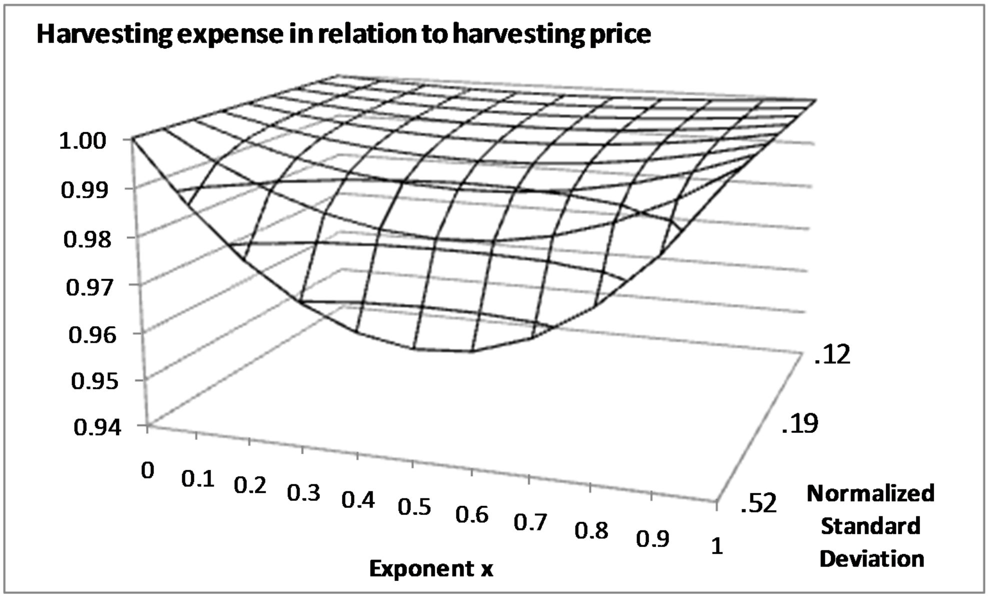

Figure 3. Harvesting expense in relation to harvesting price, with Weibull-distributed trunk volume, as a function of the normalized standard deviation.

function $p(v)=\frac{k}{\lambda}\left(\frac{v}{\lambda}\right)^{k-1}\exp\left[\left(\frac{a}{\lambda}\right)^{k}-\left(\frac{v}{\lambda}\right)^{k}\right]$ for values $v \geq \mathrm{a}$. Using this distribution, Eq. (8) becomes

$$\frac{E}{(VH)_j}=\int_0^\infty\left(\frac{v}{\bar{v}}\right)^{1-x}p(v)dv=$$

$$\left\{\exp\left[\left(\frac{a}{\lambda}\right)^{k}\right]\right\}^{x}\frac{\Gamma\left(\frac{1-x}{k}+1,\left(\frac{a}{\lambda}\right)^{k}\right)}{\left[\Gamma\left(\frac{1}{k}+1,\left(\frac{a}{\lambda}\right)^{k}\right)\right]^{1-x}} \tag{18}$$

Discussion

We found that when harvesting price is being determined on the basis of mean values of trunk properties within a compartment, the price not only depends on site properties, but also on the division of the site into compartments. In other words, such a harvesting price is not invariant to the division into compartments.

The wider the distribution of trunk properties within a compartment, the greater the harvesting price is. Thus, the greater (larger) the compartments are, the greater (higher) the price is, which may deviate more than 10% from the outcome of the productivity function. Consequently, unifying two compartments with different properties increases the harvesting price.

Four particular trunk volume density functions have been investigated. For those including an exponential factor, the deviation between the harvesting price and the productivity function becomes characterized by a gamma function. In the case of a left-truncated trunk volume distribution, an upper incomplete gamma function appears.

The exponential distribution is actually a special case of the Weibull distribution, and the complete Weibull distribution is a special case of the left-truncated Weibull distribution. Thus Eqs., (12), (13), (15) and (16) are special cases of Eq. (18), which suffices for determining the relationship of the harvesting price to the outcome of the productivity function.

It would be interesting to consider what would happen to harvesting price if trunk volume distributions other than the ones discussed above were used. Gaussian distribution would be unsuitable since the domain would include negative values of trunk volume. The same applies to the Gumbel distribution. The Rayleigh distribution is a special case of the Weibull distribution. The Fréchet distribution might be applicable, as well as the Type-2 Gumbel distribution and the Gamma distribution. The density function of any of these useful distributions contains an exponential factor, and leads to a gamma function characterizing the relationship between the harvesting price and the productivity function.

There is actually a large number of continuous probability density functions containing an exponential factor, thus producing a gamma function when inserted into Eq. (10). Among these, there is a family of Chi-distributions, the Erlang distribution, the folded normal distribution, the half-normal distribution, the inverse Gaussian, the Lévy distribution, and the Maxwell-Bolzmann distribution.

In this paper, the volumetric productivity in harvesting has been approximated in terms of a simple power law function of trunk volume, as given in Eq. (1). In reality, productivity may be a more complicated function of tree size. Productivity increment as a function of increase in trunk size may cease with large trees (Suadicani & Fjeld, 2001; Rottensteiner et al., 2008). In other words, the exponent x appearing in Eq. (1) is not universally independent of trunk size. From the viewpoint of the applicability of the results of this paper, the exponent x must be selected so that it characterizes the tree volume dependency of productivity in the vicinity of the mean trunk volume within any compartment.

Conclusions

Harvesting pricing appears to depend not only on the properties of trees, but also on the division of any logging site into compartments, provided pricing is based on average properties of trees. In the case of an exponentially distributed trunk volume, an upper incomplete gamma function appears to describe the relationship between the harvesting price and the productivity function. In the case of a Weibull-distributed trunk volume, the relationship is given in terms of the ratio of two gamma functions. In the case of a left-truncated Weibull-distribution, a ratio of two upper incomplete gamma functions appears. Exponential distribution may induce a deviation in excess of 10%, whereas the Weibull distribution with nonzero mode value induces at most 4%. Unification of compartments increases the harvesting price.

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