

Considering Cable Stretch in Logging Applications

C. Kevin Lyons

ABSTRACT

This paper considers three methods for calculating the un-stretched length of a cable with significant self weight when the final static equilibrium conditions are known. The first method uses an average line tension and Hooke's Law to estimate the un-stretched length. The second method uses a Lagrangian coordinate and Hooke's Law to form an exact equation for the un-stretched length, given the assumption that the cable is linear elastic and the change in length is due to elastic stretch. The third method uses a Lagrangian coordinate; however, construction stretch is included in addition to elastic stretch. The results of this paper indicate the average tension is a suitable surrogate for a tension function that is a function of position when considering elongation to be the result of elastic stretch. When construction stretch is considered, the average tension method also performed well for cables with tensions less than one-third the minimum breaking strength.

Keywords: *catenary, wire cables, elastic stretch, construction stretch, static*

Introduction

The stretch of wire cables is composed of two phenomena, elastic stretch and construction stretch (Bethlehem Wire Rope 2006). Elastic stretch is the actual change in length of the metal elements, which is recoverable in the linear elastic range, and is defined by Hooke's Law. Given the complicated structure of wire ropes, the reported elastic coefficient for a cable is an apparent coefficient that relates the resultant load applied to a cable to the change in length of the cable. Construction stretch is a result of the helical strands compressing the core, which results in a reduction of diameter and an increase in length (Bethlehem Wire Rope 2006). In general, construction stretch is not considered in the structural analysis of cable logging systems. Carson et al. (1982) and Kendrick and Sessions (1991) considered stretch in logging cables to be elastic and to obey Hooke's Law. These authors have not explicitly stated why other sources of cable stretch can be ignored; but, possible reasons are the load applied to the standing rigging during setup is sufficient to remove the construction stretch, and stretch in the running rigging is accounted for by the machine operator during the yarding phase.

Carson (1977) and Irvine (1981) both developed solutions for a cable segment with arbitrary end positions. Carson (1977) used the distance along the stretched cable as the Eulerian (spatial) coordinate, while Irvine (1981) preferred to use the distance along the un-stretched cable as the Lagrangian (material) coordinate. The importance of the solution in Lagrangian coordinates becomes apparent when it is necessary to consider the change in length of the standing rigging due to stretch, which requires knowledge of both un-stretched length and tension. An example of this problem is the p -delta effect in a guyed tower with a fixed base, where small displacements of the top of the tower due to stretch in the guylines reduces the load bearing capacity of the structure.

In the guyed tower problem, Carson et al. (1982) used the Eulerian formulation for the stretched cable length, the tension in the guyline at the top of the tower, and Hooke's Law to calculate the un-stretched length of the guyline during setup, while Kendrick and Sessions (1991) substituted the average end tension. Both of these solutions are approximate since they do not recognize the geometric nonlinearity of the problem, and the tension in the cable varies with position along the cable. The Lagrangian formulation given by Irvine (1981) included the un-stretched length of the guyline explicitly; therefore, when solving for the tension holding the tower in equilibrium during setup, the un-stretched length is also defined.

The objective of this paper is to determine when the approximate solution for the un-stretched length of the cable given by Kendrick and Sessions (1991) is appropriate. This paper will consider three methods for calculating the un-stretched length of a cable with significant self weight, given the final equilibrium conditions. The first method suggested by Kendrick and Sessions (1991) uses an average line tension and Hooke's Law to estimate the un-stretched length. The second method suggested by Irvine (1981) uses a Lagrangian coordinate and Hooke's Law to form an exact equation for the un-stretched length, given the assumption that the cable is linear elastic and the change in length is due to elastic stretch. The third method developed in this paper uses the Lagrangian coordinate as suggested by Irvine (1981); however, construction stretch is included in addition to elastic stretch. The third method is included as an upper bound to the problem.

Un-Stretched Length: Approximate Solution

Note, a list of the symbols used is included in **Appendix 1**. Carson et al. (1982) state the cable equations developed by Car-

The author is Assistant Professor (kevyons@interchange.ubc.ca), Forest Resources Management, University of British Columbia, Forest Sciences Center, 2nd Floor 2424 Main Mall, Vancouver BC V6T 1Z4. This paper was received for publication in January 2007. ©Forest Products Society 2008.

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son (1977) are for the stretched length of the cable. In fact the equations given by Carson (1977) are formed in terms of an Eulerian coordinate system at a time sufficiently distant from the time of loading in order to consider the cable in static equilibrium, and thus do not consider explicitly the stretch of the cable during loading. Kendrick and Sessions (1991) use the equations developed by Carson (1977) to calculate the average end tension in a guyline and Hooke's Law to estimate the un-stretched length of the guyline. Let the stretched length of the cable (L) be known and let T_p be the tension in Eulerian coordinates, then Kendrick and Sessions (1991) define the un-stretched length of an extensible cable (L_o) by:

$$L_o = L \left(1 + \frac{T_p(L) + T_p(0)}{2A_o E} \right)^{-1} \quad [1]$$

where:

A_o = the metallic cross sectional area of the un-stretched cable and

E = the axial elastic coefficient.

When considering the un-stretched length of a cable, it is simpler to use a Lagrangian coordinate system; therefore, it is necessary to write Equation [1] in terms of Lagrangian coordinates. In this problem it is assumed that the final equilibrium condition is known and it is the un-stretched length that is to be determined. Given that L is known, the magnitude of the tension at the supports will be the same in either the Lagrangian or Eulerian formulations at the time when the cable is in static equilibrium. There is a small error in the method used by Kendrick and Sessions (1991), where they use the mass per unit length of the un-stretched cable (M) instead of the mass per unit length of the stretched cable (M_p) in the Eulerian formulation; however, this error is assumed to be negligible. Thus, let

$$T(0) = T_p(0) \text{ and } T(L_o) = T_p(L) \quad [2]$$

where:

T = the tension in Lagrangian coordinates.

Given Equation [2], Equation [1] can be rewritten using the Lagrangian form of the tension function.

$$L_o = L \left(1 + \frac{T(L_o) + T(0)}{2A_o E} \right)^{-1} \quad [3]$$

Un-Stretched Length: Elastic Stretch

Variable definitions will be reset for this section to ensure there is no confusion with the previous section. Irvine (1981) uses the distance along the un-stretched cable (s) as the Lagrangian coordinate, and p is the distance along the stretched cable. In the following, A is the support at $p = 0$ and B is the support at $p = L$, where L is the total stretched length of the cable, and P is an arbitrary point on the stretched cable. The un-stretched length of the cable is L_o , $\lim_{s \rightarrow 0} p = 0$, and $\lim_{s \rightarrow L_o} p = L$ (Fig. 1).

The following derivation is based on methods suggested by Irvine (1981). Consider the free body diagram of the cable segment from A to P (Fig. 1). Here V_A and H_A are the vertical and horizontal components, respectively, of tension at support A and $W = MgL_o$, where M is the mass per unit length of the un-stretched cable and g is acceleration due to gravity. When considering a differential segment of the cable the following can be noted:

$$\begin{aligned} dp^2 &= dz^2 + dx^2 \\ \frac{dx}{dp} &= \cos(\theta) \\ \frac{dz}{dp} &= \sin(\theta) \end{aligned} \quad [4]$$

where:

θ = the angle between the tangent line and the horizontal.

Sum the forces in Figure 1 and use the definitions of the trigonometric functions from Equation [4], then:

$$H_A = T \frac{dx}{dp} \quad [5]$$

$$V_A - W \frac{s}{L_o} = T \frac{dz}{dp} \quad [6]$$

Considering the cable to be one dimensional (that is only having the dimension length) and linear elastic, Hooke's Law can be written as:

$$\frac{T}{A_o} = E \left(\frac{dp}{ds} - 1 \right) \quad [7]$$

where:

$\frac{dp}{ds} - 1$ = the axial stretch,

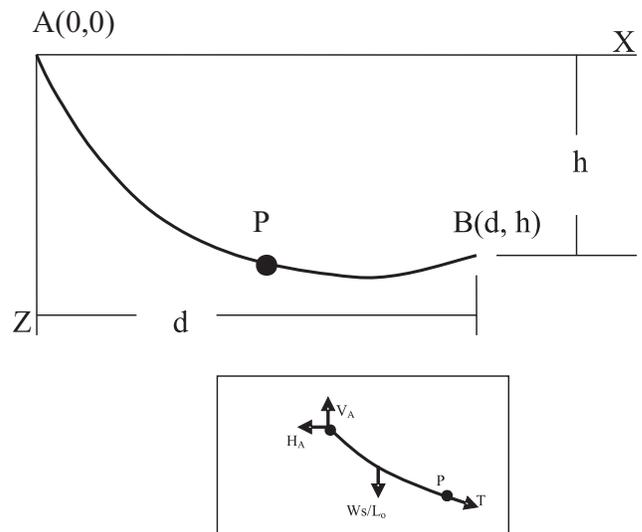


Figure 1. ~ Cable segment diagram.

E = the axial elastic coefficient, and

A_o = the metallic cross sectional area of the un-stretched cable.

In the Lagrangian formulation, T is a function of s . Thus, separate variables in Equation [7] and integrate.

$$p = s + \frac{1}{EA_o} \int T(s) ds \quad [8]$$

Divide both sides of the first part of Equation [4] by dp^2 , substitute definitions of the derivatives from Equations [5] and [6] into this, and solve for $T(s)$.

$$T(s) = \sqrt{H_A^2 + \left(V_A - W \frac{s}{L_o} \right)^2} \quad [9]$$

Substitute Equation [9] into Equation [8] and integrate noting V_A and H_A are independent of s .

$p =$

$$s - \frac{L_o H^2}{4WEA_o} \left\{ \sinh \left[2 \sinh^{-1} \left(\frac{V_A - Ws}{H} \right) \right] + 2 \sinh^{-1} \left(\frac{V_A - Ws}{H} \right) + c \right\} \quad [10]$$

Here H_A has been replaced with H since the horizontal component of tension in a cable segment, subject only to vertical body loads, is constant over the segment.

Use the boundary condition $\lim_{s \rightarrow 0} p = 0$ to solve for the constant of integration (c) in Equation [10] and the boundary condition $\lim_{s \rightarrow L_o} p = L$ to form an equation for L_o .

$L_o =$

$$L \left(1 - \frac{H^2}{4WEA_o} \left\{ \sinh \left[2 \sinh^{-1} \left(\frac{V_A - W}{H} \right) \right] + 2 \sinh^{-1} \left(\frac{V_A - W}{H} \right) - \sinh \left[2 \sinh^{-1} \left(\frac{V_A}{H} \right) \right] - 2 \sinh^{-1} \left(\frac{V_A}{H} \right) \right\} \right)^{-1} \quad [11]$$

Unlike Equation [3], Equation [11] is an exact equation given the assumptions made with regards to the mechanics of a cable in this section and can be used to calculate L_o from L .

There is an additional property of the Lagrangian formulation for cables with significant self weight that is attractive for logging applications. The equations developed by Carson (1977) do not give the position of points along the cable for cables with arbitrary end locations. For most logging applications this is not a concern; however, when intermediate supports are used to maintain skyline clearance, the clearance between the ground and the haulback can be a concern. The Lagrangian formulation as presented by Irvine (1981) provides horizontal and vertical position equations for points along the haulback.

Un-Stretched Length: Elastic and Construction Stretch

Manufactures have provided estimates for construction stretch in the range of 0.25 to 0.5 percent (Bethlehem Wire Rope 2006) and up to 1.0 percent (Wire Rope Corporation of America 2006) for six-strand independent wire rope core steel cable (IWRC). Construction stretch is highly variable and it is not possible to quote exact values; however, for steel core wire ropes Bridon American Corporation (2006a) suggests approximate values for percent elongation that are a function of the factor of safety, where a factor of safety is defined as the minimum breaking strength compared to the applied load (Table 1).

In the analysis of cable stretch Bridon American Corporation (2006a) consider elastic stretch and construction stretch to be additive. Let ϵ_T represent the total change in length divided by the un-stretched length of a differential cable element; ϵ_C represent the change in length due to construction stretch divided by the un-stretched length of a differential cable element; and ϵ_E represent the stretch ratio defined by Hooke's Law, where:

$$\epsilon_T = \epsilon_C + \epsilon_E \quad [12]$$

Considering the data in Table 1, a power function could be used to represent ϵ_C . With the expectation that this function will have to be integrated in order to develop an equation for the stretched length of the cable, a polynomial function is preferred. Fitting a second-order polynomial with zero intercept to the data in Table 1 results in:

$$\epsilon_C = 0.0414 \left(\frac{T(s)}{T_{ult}} \right)^2 \quad (R^2 = 0.9698) \quad [13]$$

where:

$T(s)$ = the tension at the coordinate s and

T_{ult} = the minimum breaking strength of the cable.

Recall $\epsilon_T = \frac{dp}{ds} - 1$ and that ϵ_E is related to tension by Hooke's Law, then combining Equations [12] and [13] and solving for the derivative of p results in:

$$\frac{dp}{ds} = \frac{T(s)}{EA_o} + 0.0414 \left(\frac{T(s)}{T_{ult}} \right)^2 + 1 \quad [14]$$

The change in length due to ϵ_C and ϵ_E are additive because they are linear functions of the un-stretched cable length. Since construction stretch is considered non-recoverable, Equation [14] is only valid for a new cable that is heavily loaded; however, this is acceptable as a worst case scenario to compare the Ken-

Table 1. ~ Percent elongation due to construction stretch (Bridon American Corp. 2006a).

Factor of safety	8:1	5:1	3:1	Heavily loaded
% elongation	0.125	0.25	0.5	1.0

drick and Sessions (1991) approximate method to exact methods of calculating the un-stretched cable length.

Separate variables in Equation [14] and integrate.

$$p = \int \frac{T(s)}{EA_o} ds + \frac{0.0414}{T_{ult}^2} \int T(s)^2 ds + s + c_1 \quad [15]$$

Note $s + \frac{1}{EA_o} \int T(s) ds$ is defined by Equation [10]. Thus, the remaining problem is to find:

$$F(s) = \int T(s)^2 ds \quad [16]$$

Substitute Equation [9] into Equation [16] and integrate.

$$F(s) = H^2 s + V_A^2 s - \frac{V_A W s^2}{L_o} + \frac{W^2 s^3}{3L_o^2} + c_2 \quad [17]$$

Substitute Equations [10] and [17] into Equation [15] and combine the constants of integration into c , then:

$p = s -$

$$\frac{L_o H^2}{4WEA_o} \left\{ \sinh \left[2 \sinh^{-1} \left(\frac{V_A - \frac{Ws}{L_o}}{H} \right) \right] + 2 \sinh^{-1} \left(\frac{V_A - \frac{Ws}{L_o}}{H} \right) \right\} + \frac{0.0414}{T_{ult}^2} \left(H^2 s + V_A^2 s - \frac{V_A W s^2}{L_o} + \frac{W^2 s^3}{3L_o^2} \right) + c \quad [18]$$

Use the boundary condition $\lim_{s \rightarrow 0} p = 0$ to solve for the constant of integration (c) in Equation [18], and the boundary condition $\lim_{s \rightarrow L_o} p = L$ to form an equation for L_o .

$L_o =$

$$L \left(1 - \frac{H^2}{4WEA_o} \left\{ \sinh \left[2 \sinh^{-1} \left(\frac{V_A - W}{H} \right) \right] + 2 \sinh^{-1} \left(\frac{V_A - W}{H} \right) \right\} - \sinh \left[2 \sinh^{-1} \left(\frac{V_A}{H} \right) \right] - 2 \sinh^{-1} \left(\frac{V_A}{H} \right) \right) + \frac{0.0414}{T_{ult}^2} \left(H^2 + V_A^2 - V_A W + \frac{W^2}{3} \right) \quad [19]$$

Nondimensional Comparison

To avoid confusion between the different methods of calculating cable length, let the subscript ∞ indicate variables corresponding to Equation [3], the approximate solution for un-stretched length; the subscript E indicate variables corresponding to Equation [11], the exact solution for un-stretched length given elastic stretch; and the subscript EC indicate variables corresponding to Equation [19], the exact solution given elastic and construction stretch. For example the un-stretched lengths become L_{∞} , L_{oE} and L_{oEC} in Equations [3], [11], and [19], re-

spectively. Two comparisons considering the different estimates of un-stretched length will be made in this section: L_{∞} to L_{oE} and L_{∞} to L_{oEC} .

When comparing L_{∞} to L_{oE} the un-stretched length found using the method suggested by Kendrick and Sessions (1991) is compared to the exact equation when considering the cable to be linear elastic. When considering the cable to be linear elastic, the change in length is relatively small ($< 1\%$) and it is expected that the nonlinear geometric effect will be small. When comparing L_{∞} to L_{oEC} the method suggested by Kendrick and Sessions (1991) is compared to the exact equation when considering cable stretch to be the combined effect of elastic stretch and construction stretch. When considering cable stretch to be the combined effect of elastic stretch and construction stretch, the change in length can be large ($> 1\%$) and is a nonlinear function of tension. Thus, geometric and material nonlinearity could be significant in the second comparison.

Dimensionless variables to be used in the analysis of a cable subject to multiple point loads are suggested by Irvine (1981). The dimensionless variables are useful when considering the general behavior of the system; however, Irvine (1981) uses the un-stretched length to eliminate the length dimension. In this paper since the un-stretched length is of interest, it is preferable to use the stretched length to eliminate the length dimension. The following dimensionless variables will be used in this paper:

$$\phi = \frac{V_A}{W}, \chi = \frac{H}{W}, \beta = \frac{W}{EA_o}, \tau = \frac{T_{ult}}{W}, \psi_{\infty} = \frac{L_{\infty}}{L}, \psi_E = \frac{L_{oE}}{L}, \psi_{EC} = \frac{L_{oEC}}{L} \quad [20]$$

Substitute Equation [9] into Equation [3], then use Equation [20] to form the dimensionless equation for un-stretched length given the method suggested by Kendrick and Sessions (1991).

$$\psi_{\infty} = \left\{ 1 + \frac{\beta}{2} \left[\sqrt{\chi^2 + \phi^2} + \sqrt{\chi^2 + (\phi-1)^2} \right] \right\}^{-1} \quad [21]$$

Substitute Equation [20] into Equation [11] to form the dimensionless equation for un-stretched length when considering the cable to be linear elastic.

$$\psi_E = \left\{ 1 - \frac{\chi^2 \beta}{4} \left[\sinh \left(2 \sinh^{-1} \left(\frac{\phi-1}{\chi} \right) \right) + 2 \sinh^{-1} \left(\frac{\phi-1}{\chi} \right) \right] - \sinh \left(2 \sinh^{-1} \left(\frac{\phi}{\chi} \right) \right) - 2 \sinh^{-1} \left(\frac{\phi}{\chi} \right) \right\}^{-1} \quad [22]$$

Note:

$$\sinh \left(2 \sinh^{-1} \left(\frac{\phi-1}{\chi} \right) \right) = 2 \left(\frac{\phi-1}{\chi} \right) \sqrt{1 + \left(\frac{\phi-1}{\chi} \right)^2}$$

$$\sinh \left(2 \sinh^{-1} \left(\frac{\phi}{\chi} \right) \right) = 2 \left(\frac{\phi}{\chi} \right) \sqrt{1 + \left(\frac{\phi}{\chi} \right)^2} \quad [23]$$

Substituting Equation [23] into Equation [22] produces the simplified version of the dimensionless equation for the un-stretched length of an elastic cable.

$$\Psi_E = \left\{ 1 - \frac{\chi^2 \beta}{2} \left[\left(\frac{\phi-1}{\chi} \right) \sqrt{1 + \left(\frac{\phi-1}{\chi} \right)^2} + \sinh^{-1} \left(\frac{\phi-1}{\chi} \right) - \left(\frac{\phi}{\chi} \right) \sqrt{1 + \left(\frac{\phi}{\chi} \right)^2} - \sinh^{-1} \left(\frac{\phi}{\chi} \right) \right] \right\}^{-1} \quad [24]$$

If the dimensional variables are substituted back into Equation [24], then it is possible to regain the equation developed by Kozak et al. (2006).

Substitute Equation [20] into Equation [19] and take into account Equation [23] to form the dimensionless equation for un-stretched length when considering cable stretch to be a function of both elastic stretch and construction stretch.

$$\Psi_{EC} = \left(1 - \frac{\chi^2 \beta}{2} \left\{ \frac{\phi-1}{\chi} \sqrt{1 + \left(\frac{\phi-1}{\chi} \right)^2} - \frac{\phi}{\chi} \sqrt{1 + \left(\frac{\phi}{\chi} \right)^2} + \sinh^{-1} \left(\frac{\phi-1}{\chi} \right) - 2 \sinh^{-1} \left(\frac{\phi}{\chi} \right) \right\} + \frac{0.0414}{\tau^2} \left(\chi^2 + \phi^2 - \phi + \frac{1}{3} \right) \right)^{-1} \quad [25]$$

To compare the different estimates of un-stretched length, it is necessary to identify ranges for the dimensionless variables β , ϕ , the ratio $\frac{\phi}{\chi}$ and τ . First consider $\beta = \frac{MgL_o}{EA_o}$, and let ρ be the density (mass of metal divided by volume of metal) of the cable.

Then $M = \rho A_o$ and $\beta = \frac{\rho g L_o}{E}$. If ρ is assumed to be similar for

different diameters of cable then the variation in β is dominated by L_o and E . Thus, since E is similar for most cables in the 6 by 19 class, setting a range for L_o will define the range for β . Using data from the 6 by 19 class of IWRC, a common value for ρ is 9300 kg/m³ (Bridon American Corp. 2006b) and E is typically 9.310E10 Pa (Bridon American Corp. 2006c). Selecting a range for L_o of 50 to 300 m produces a range in β of 4.90E-05 to 2.94E-04.

It is possible for V_A to approach zero at A if the slope of the cable approaches zero at that point; therefore, it is possible for ϕ to approach zero. Alternatively, consider a tight relatively short cable subject to a 40,000 N tension, where the slope of the chord is 100 percent, weight per unit length is 17 N/m, and the length of the cable is 50 m. For the short cable example, it is assumed the tension at B is similar to that at A , thus for this example ϕ can be approximated by:

$$\phi \approx \frac{40,000 \sin\left(\frac{\pi}{4}\right)}{(17)(50)} = 33.3 \quad [26]$$

The ratio ϕ/χ is equal to the slope of the cable at support A , and for the short cable example this should be approximately equal to 1.0; however, this could equal zero for a cable with a slope of zero at this point.

Recall τ is the minimum breaking strength of the cable divided by the total weight of the cable. For 3/4 in. 6 by 19 extra extra improved plow steel (EEIP) cable Bridon American Corporation (2006b) lists $M = 1.4$ kg/m and $T_{ult} = 256.3$ kN. Considering un-stretched cable lengths for this cable of 50 m and 300 m, results in τ being 373 and 62, respectively. For 1.0 in. 6 by 19 EEIP cable Bridon American Corporation (2006b) lists $M = 2.4$ kg/m and $T_{ult} = 450.3$ kN. Considering un-stretched cable lengths for this cable of 50 m and 300 m, results in τ being 383 and 64, respectively. The weight per unit length of a cable is strongly correlated to the minimum breaking strength; therefore, τ is relatively independent of the diameter of the cable and is most strongly affected by the length of the cable.

The ratio of Ψ_{∞} to Ψ_E will be constructed as a function of ϕ for 12 different combinations of β , ϕ/χ , and τ that span the range of values considered in the preceding paragraphs. These combinations will be termed Cases. The values used for the dimensionless variables in the 12 cases are listed in Table 2.

Comparing Ψ_{∞} to Ψ_E (Cases 1 to 4) considers the accuracy of using the average tension as opposed to a tension function that is a function of position, and when considering only elastic stretch Ψ_{∞} estimated Ψ_E very well. At worst the ratio of Ψ_{∞}/Ψ_E was 0.99996, which occurred at low ϕ values, and for most ϕ values Ψ_{∞} and Ψ_E were indistinguishable. The approximate solution uses the support loads for the stretched cable when calculating the un-stretched length. This ignores the nonlinear geometric effect that could occur as the cable length changes. Apparently the small change in length due to elastic stretch does not result in a significant nonlinear geometric effect.

Comparing Ψ_{∞} to Ψ_{EC} (Cases 5 to 12) considers the accuracy of using the average tension and elastic stretch as opposed to a tension function that is a function of position and the combined effect of elastic stretch and construction stretch. When considering elastic stretch and construction stretch Ψ_{∞} estimated Ψ_{EC} well for certain variable combinations, but not as

Table 2. ~ Dimensionless variable values.

Case	β	τ	ϕ	ϕ/χ
1	4.90E-05	na	0.2 to 33	0.2
2	4.90E-05	na	0.2 to 33	1.0
3	2.94E-04	na	0.2 to 33	0.2
4	2.94E-04	na	0.2 to 33	1.0
5	4.90E-05	377	0.2 to 33	0.2
6	4.90E-05	377	0.2 to 33	1.0
7	4.90E-05	62	0.2 to 33	0.2
8	4.90E-05	62	0.2 to 33	1.0
9	2.94E-04	377	0.2 to 33	0.2
10	2.94E-04	377	0.2 to 33	1.0
11	2.94E-04	62	0.2 to 33	0.2
12	2.94E-04	62	0.2 to 33	1.0

well for others (Fig. 2). The data in Figure 2 can be divided into four groups:

1. Cases 7 and 11 represent long cables with the slope of the cable at support A nearing zero,
2. Cases 8 and 12 represent long cables with the slope of the cable at support A nearing 100 percent,
3. Cases 5 and 9 represent short cables with the slope of the cable at support A nearing zero, and
4. Cases 6 and 10 represent short cables with the slope of the cable at support A nearing 100 percent.

The worst results occurred in Cases 7 and 11 at higher values of ϕ . To have a high ϕ value and a cable slope nearing zero at support A, requires the slope of the chord of the cable to approach zero.

By varying ϕ/χ a different tension can be defined at support A for a given value of ϕ . Alternatively the tension can be held constant and ϕ/χ , which is the slope of the cable at support A, can be used to define ϕ . Given ϕ and ϕ/χ for a particular tension, the curves in Figure 2 can be used to find a corresponding ψ_∞/ψ_{EC} . The combinations of ϕ and ψ_∞/ψ_{EC} for a dimensionless tension equal to $\tau/3$ are connected by a dashed line in Figure 2. As noted previously since τ is independent of the size of the cable, these results are relatively independent of the size of the cable for a given class of cables.

It is interesting to note when considering a dimensionless tension equal to $\tau/3$ at support A that $\psi_\infty/\psi_{EC} \approx 1.0044$ for all Cases considered (it is not shown in Fig. 2 but for Cases 6 and 10 $\psi_\infty/\psi_{EC} = 1.0043$ at $\phi = 88.4$). If the tension is defined as a different portion of τ , a different value will be found for the ratio ψ_∞/ψ_{EC} and this will be similar for all Cases considered. Thus, it

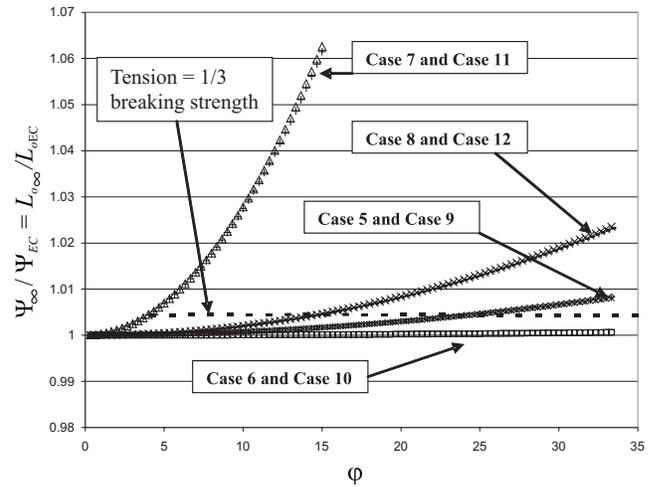


Figure 2. ~ Ratio $\psi_\infty/\psi_{EC} = L_{o\infty}/L_{oEC}$

is the tension in the cable that drives the difference between ψ_∞ and ψ_{EC} , and when the tension is limited to a working load of $\tau/3$, ψ_∞ is approximately 0.44 percent larger than ψ_{EC} .

Conclusions

This paper considers three methods for calculating the un-stretched length of a cable with significant self weight given the final equilibrium conditions. The first method suggested by Kendrick and Sessions (1991) uses an average line tension and Hooke's Law to estimate the un-stretched length. The second method suggested by Irvine (1981) uses a Lagrangian coordinate and Hooke's Law to form an exact equation for the un-stretched length given the assumption that the cable is linear

Appendix 1. – List of symbols

Symbol	Description	Symbol	Description
A	Left hand support of a cable segment	T	Tension in Lagrangian coordinates
A_o	Metallic cross sectional area of the un-stretched cable	T_p	Tension in Eulerian coordinates
B	Right hand support of a cable segment	T_{ult}	Minimum breaking strength of the cable
c	Constant of integration	V_A	Vertical component of tension at A
d	Horizontal distance between supports	W	Total weight of the cable segment
E	Axial elastic coefficient	x	Horizontal Cartesian coordinate
g	Acceleration due to gravity	z	Vertical Cartesian coordinate
H_A	Horizontal component of tension at A	β	Dimensionless weight variable
H	Horizontal component of tension	e_C	Stretch ratio due to construction stretch
h	Vertical distance between supports	ϵ_E	Stretch ratio due to elastic stretch
L	Length of the stretched cable	ϵ_T	Total stretch ratio
L_o	Length of the un-stretched cable	ϕ	Dimensionless vertical tension variable
$L_{o\infty}$	Length of the un-stretched cable defined by Eq. [3]	ψ_∞	Dimensionless length of the un-stretched cable defined by Eq. [3]
L_{oE}	Length of the un-stretched cable defined by Eq. [11]	ψ_E	Dimensionless length of the un-stretched cable defined by Eq. [11]
L_{oEC}	Length of the un-stretched cable defined by Eq. [19]	ψ_{EC}	Dimensionless length of the un-stretched cable defined by Eq. [19]
M	Mass per unit length of the un-stretched cable	ρ	Density of the un-stretched cable
M_p	Mass per unit length of the stretched cable	θ	Angle between the tangent line of the cable and the horizontal
p	Distance along the stretched cable	τ	Dimensionless minimum breaking strength of the cable
P	An arbitrary point on the stretched cable	χ	Dimensionless horizontal component of tension
s	Distance along the un-stretched cable		

elastic and the change in length is due to elastic stretch. The third method developed in this paper uses the Lagrangian coordinate as suggested by Irvine (1981); however, construction stretch is included in addition to elastic stretch. The results of this paper indicated when only considering elastic stretch that the method suggested by Kendrick and Sessions (1991) performed very well, which indicates the average tension is a suitable surrogate for a tension function that is a function of position. When construction stretch is considered, the method suggested by Kendrick and Sessions (1991) also performed well for cables where the tension is less than one-third the minimum breaking strength of the cable.

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