

A STUDY OF SEMI-RIGID AND NON-LINEAR BEHAVIOUR OF NAILED JOINTS IN TIMBER PORTAL FRAMES

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SYNOPSIS

In this paper a study is made of the semi-rigid and non-linear behaviour of nailed knee joints in timber frames. A numerical analysis is applied to the non-linear deformation of the joints. The solution is derived using the stiffness method of analysis in which the rotation of each nailed joint is modelled by a series of piece-wise linear relationships, based on the load-deformation characteristics of nailed joints laterally loaded in single shear. The stiffness matrix is corrected at each step, to allow for joint flexibility; and the short-term non-linear deformation of frames, up to ultimate load, is calculated. Experimental verification is made using knee joint specimens and 6 m span timber portals with nailed plywood knee joints. The test results confirm the applicability of the analysis.

Keywords: *timber frames, nailed joints, semi-rigid behaviour, non-linear deformation.*

INTRODUCTION

Traditional approaches to the analysis and design of timber structures idealize the behaviour of connections as either rigid or pinned. These are simply the extreme cases of true joint behaviour. In either of these two conditions, the forces and displacements obtained are unreliable and do not represent the actual structural behaviour, leading to either conservative or under-designed members and joints. In mechanically connected timber structures semi-rigid joints probably occur more frequently than in other common structural materials.

Research into the semi-rigid behaviour of other structural materials such as steel is extensive; and in that, connections have received a great deal of attention from numerous investigators. Several experimental studies have been conducted to establish moment-rotation relationships for different connection types. Attempts have also been made to develop mathematical models to represent joint flexibilities [1,2,3]. Several models representing linear, bilinear, and trilinear approximations have been adopted [4,5]. Exponential and polynomial functions representing connection characteristics have also been used [6,7].

The semi-rigid and non-linear behaviour of the nailed joints in timber structures have been neglected by most previous investigators. Brynildsen and Booth [8] made a study of semi-rigid joints on W-braced trusses by introducing flexibility coefficients, which provided the basis for subsequent investigations. Further work on the semi-rigid behaviour of W-braced trusses was carried out by Reardon [9]. He studied the effects of axial semi-rigidity at the connections of top and bottom chords on the heel joints of W-braced trusses. An analysis of truss-plate connections was carried out by Foschi [10]. He incorporated the non-linear load-deformation characteristics of the connection by assuming it as "continuous" due to the high density of the teeth, where the connector properties were derived from those of a single tooth. A study of the stability and reliability of semi-rigid timber trusses was carried out by Gizejowski and Mansell [11]. Their study was concerned with the effects of additional member forces resulting from the deflected profile of the structure as well as the effects of intermediate rotational stiffnesses of the nailed-plate joints. Hirai [12] made a study of the deformation of the semi-rigid frames with nailed gusset plates. He based his investigation on the work of Itani and Obregon [13], who made a non-linear analysis of the racking resistance of nailed shear walls using principles of minimum potential energy. Their analyses were dependent on the non-linear load-slip relationship of nailed connections loaded in single shear. They showed that in order to obtain a true representation of displacements, non-linear analysis is needed.

In this paper a method of analysis is described for the prediction of non-linear load-displacement characteristics of timber structures incorporating semi-rigid nailed connections. The method utilizes a stepwise linear approximation using the stiffness method of analysis for plane-frame structures with

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semi-rigid joints. It builds on the work of the above investigators [8,12,13] and has been carried out for structures subject to short duration loading. Although creep is an important aspect of timber engineering, it has not been considered here as it involves completely different problems meriting a separate study.

MOMENT-ROTATION BEHAVIOUR

The non-linear moment-rotation ($M-\phi$) behaviour of a semi-rigid joint can be idealized as a series of stepwise linear relationships of gradient R_1, R_2, \dots, R_n , as shown in Figure 1. If at a general loading increment j , the angle of rotation of the semi-rigid joint changes from ϕ_{j-1} to ϕ_j caused by change in rotational moment from M_{j-1} to M_j ; then the incremental moment-rotation of the joint may be defined as:

$$\delta M = R_j \cdot d\phi \quad (1)$$

where,

$$R_j = (M_j - M_{j-1}) / (\phi_j - \phi_{j-1})$$

ROTATIONAL RIGIDITY OF A NAILED JOINT

Consider a nail (i) subjected to a single shear force of (f_i) in a nailed plywood to timber joint as shown in Figure 2 undergoing a slip of (S_i). The non-linear load-deformation response of the nailed joint can be idealized as a series of stepwise linear relationships of gradients $k_{1i}, k_{2i}, \dots, k_{ni}$. A general

load-slip relationship of a nailed joint subjected to single shear may be expressed as ($f_i = k_i \cdot S_i$); where k_i is the slip modulus of the nail (i) depending on the nail type and size and timber species factors. The slip modulus could also be further expressed as:

$$k_i = f(k_1, k_2, k_3, \dots) \quad (2)$$

where, k_1 = factor for environment,
 k_2 = factor for duration of loading,
 k_3 = factor for interaction of elements,
 i.e. grain orientations, friction etc.

From Figure 2, if at a general loading increment j slip of the joint changes from $S_{i(j-1)}$ to S_{ij} caused by the change in applied shear force in the nail (i) from $f_{i(j-1)}$ to f_{ij} , then the incremental load-slip relationship of the nailed joint can be defined as:

$$\delta f_i = k_i \cdot \delta S_i \quad (3)$$

where,

$$k_i = (f_{ij} - f_{i(j-1)}) / (S_{ij} - S_{i(j-1)})$$

Now consider a nailed gusset joint as shown in Figure 3. In this joint load P , acting on the free end of the cantilever, induces a rotational moment of M at the centre of rotation of the nail group (as denoted by c). The rotational moment M acting on the centre of joint rotation includes effects of shear and axial forces. For a joint with n nails and assuming that the torsion formula $M = \Sigma F.r$, [14] can be applied for a nail group; thus, the change in rotational moment δM at a loading increment j , can be expressed as:

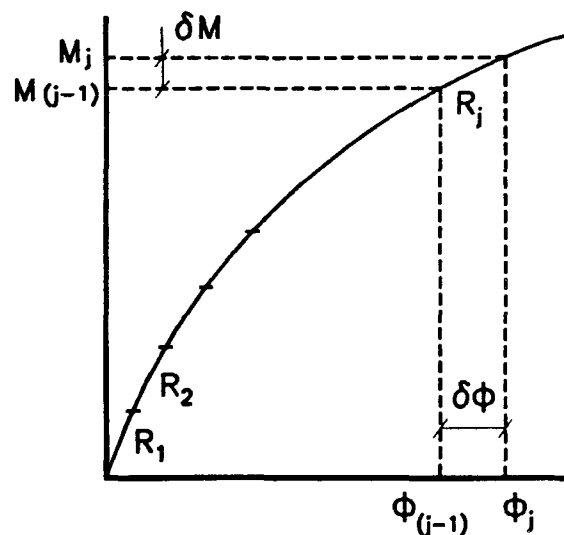
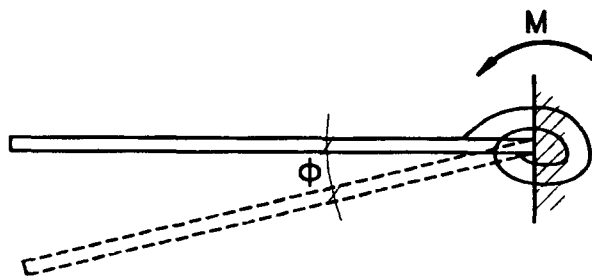


Figure 1. Moment-rotation behaviour of a semi-rigid joint.

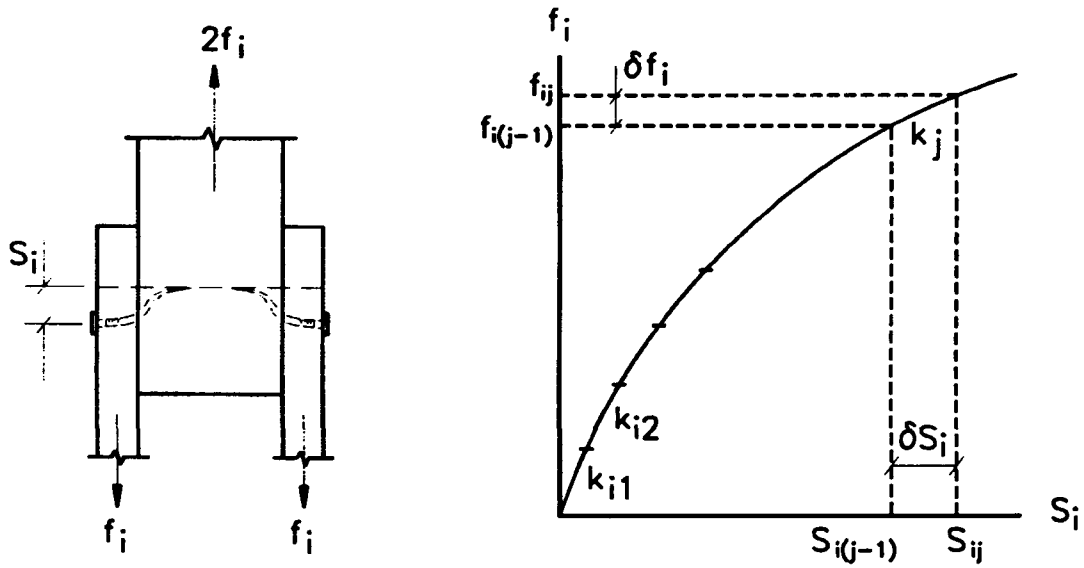


Figure 2. Load-slip behaviour of a nailed joint.

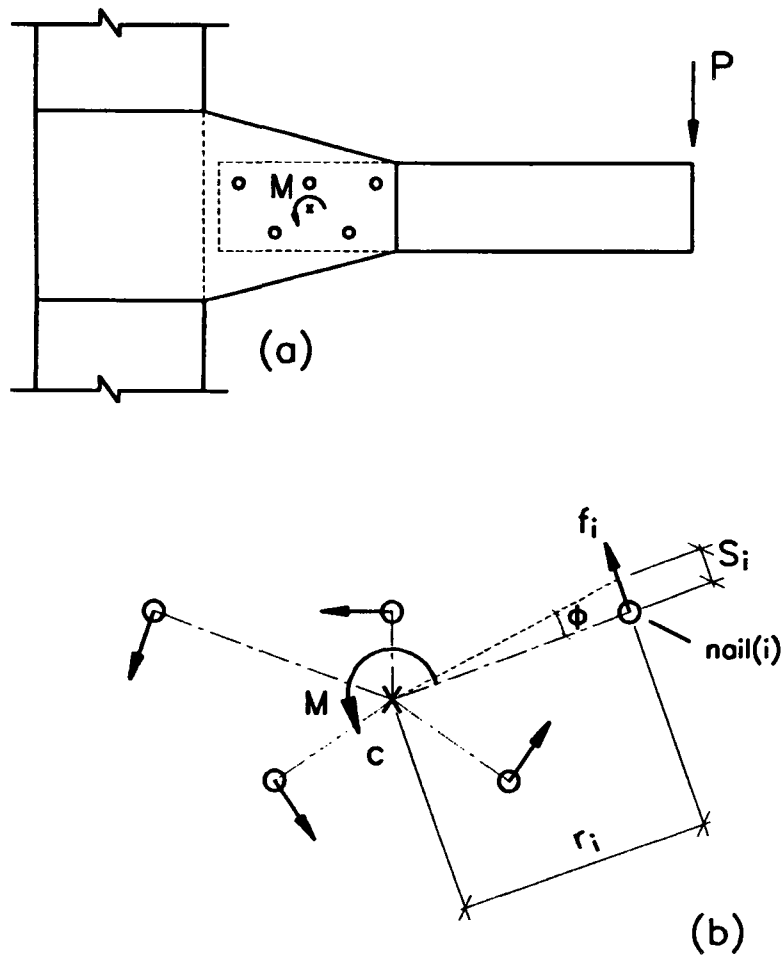


Figure 3. An assumed nailed joint.

$$\delta M = \sum_{i=1}^n \delta f_i \cdot r_i \quad (4)$$

where r_i is the distance between the nail (i) and the centre of rotation of the joint (i.e. nail group). From Figure 3b if the angle of rotation $\delta\phi$ is small, then:

$$\delta S_i = \delta\phi \cdot r_i \quad (5)$$

by substituting for δS_i in equation (3), therefore:

$$\delta f_i = \delta\phi \cdot k_i \cdot r_i \quad (6)$$

by substituting δf_i in equation (4), hence:

$$\delta M = \delta\phi \sum_{i=1}^n k_i \cdot r_i^2 \quad (7)$$

By equating equations (1) and (7) the rotational rigidity of the joint, at a general loading increment j , can be expressed as:

$$R_j = \sum_{i=1}^n k_i \cdot r_i^2 \quad (8)$$

The above equation is based on the assumption that the centre of rotation of the nail group will remain unchanged throughout the non-linear deformation of the joint. In a study of the performance of nailed gusset joints in timber portal frames by Kermani and Lee [15], it was shown that in up to 70% of the ultimate loads the centre of rotation of the nail groups remained more or less stationary and occurred close to their geometric centre.

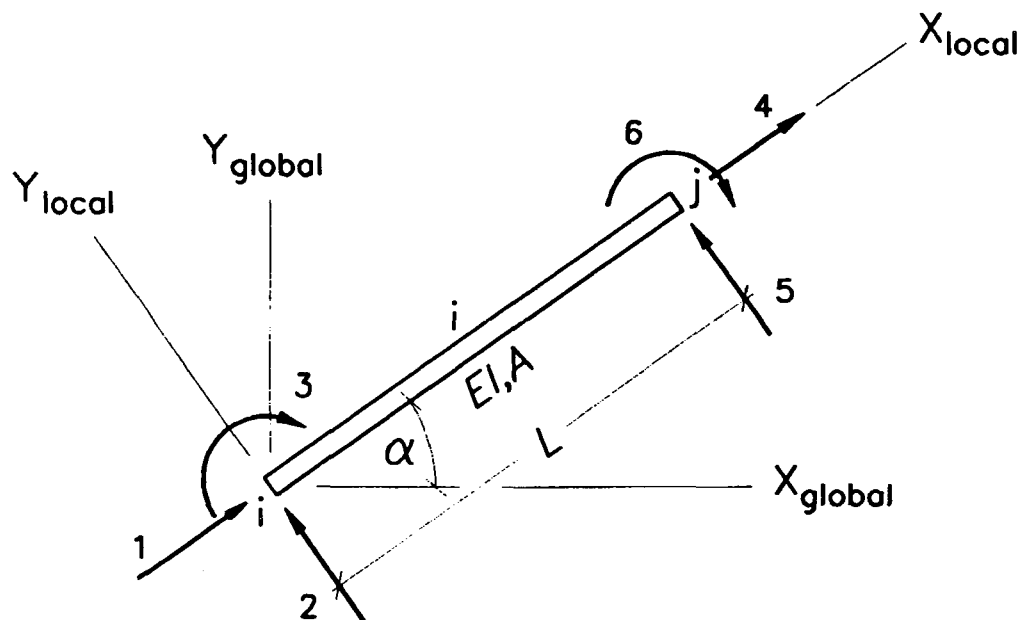


Figure 4. Structural element with local and global co-ordinates.

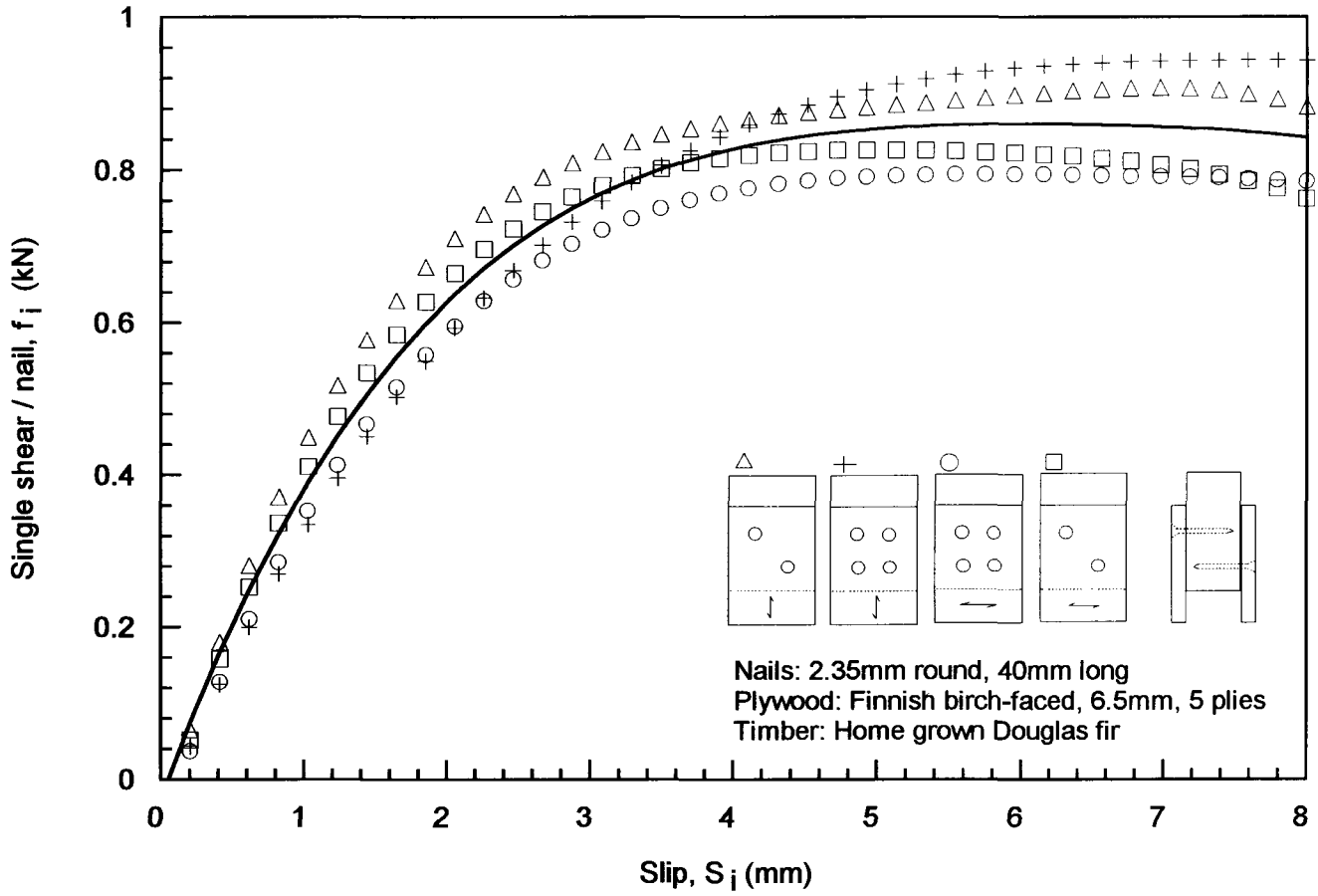


Figure 5. Load-slip behaviour of nailed plywood-timber joints loaded laterally in single shear.

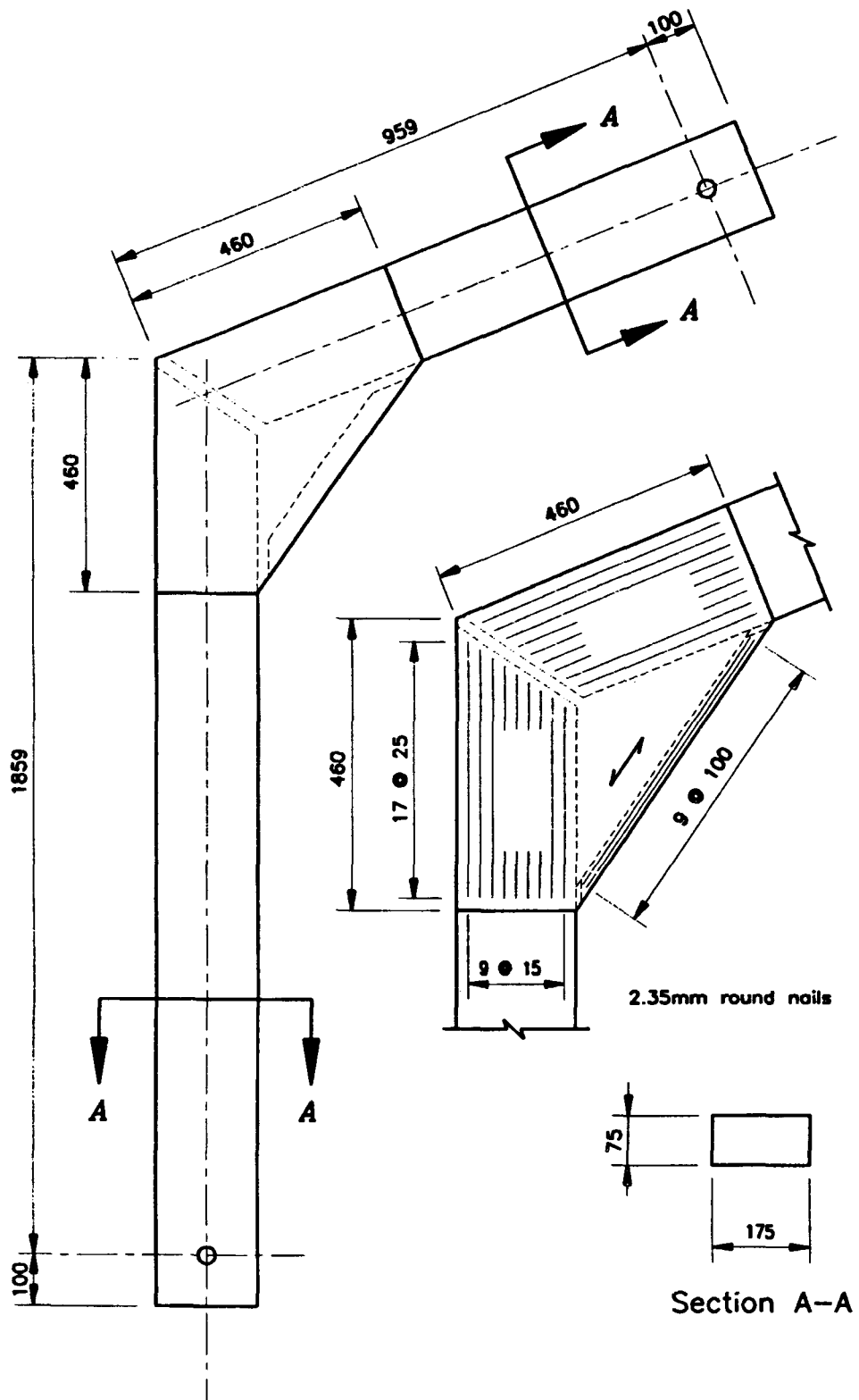


Figure 6. Details of the knee joint specimens.

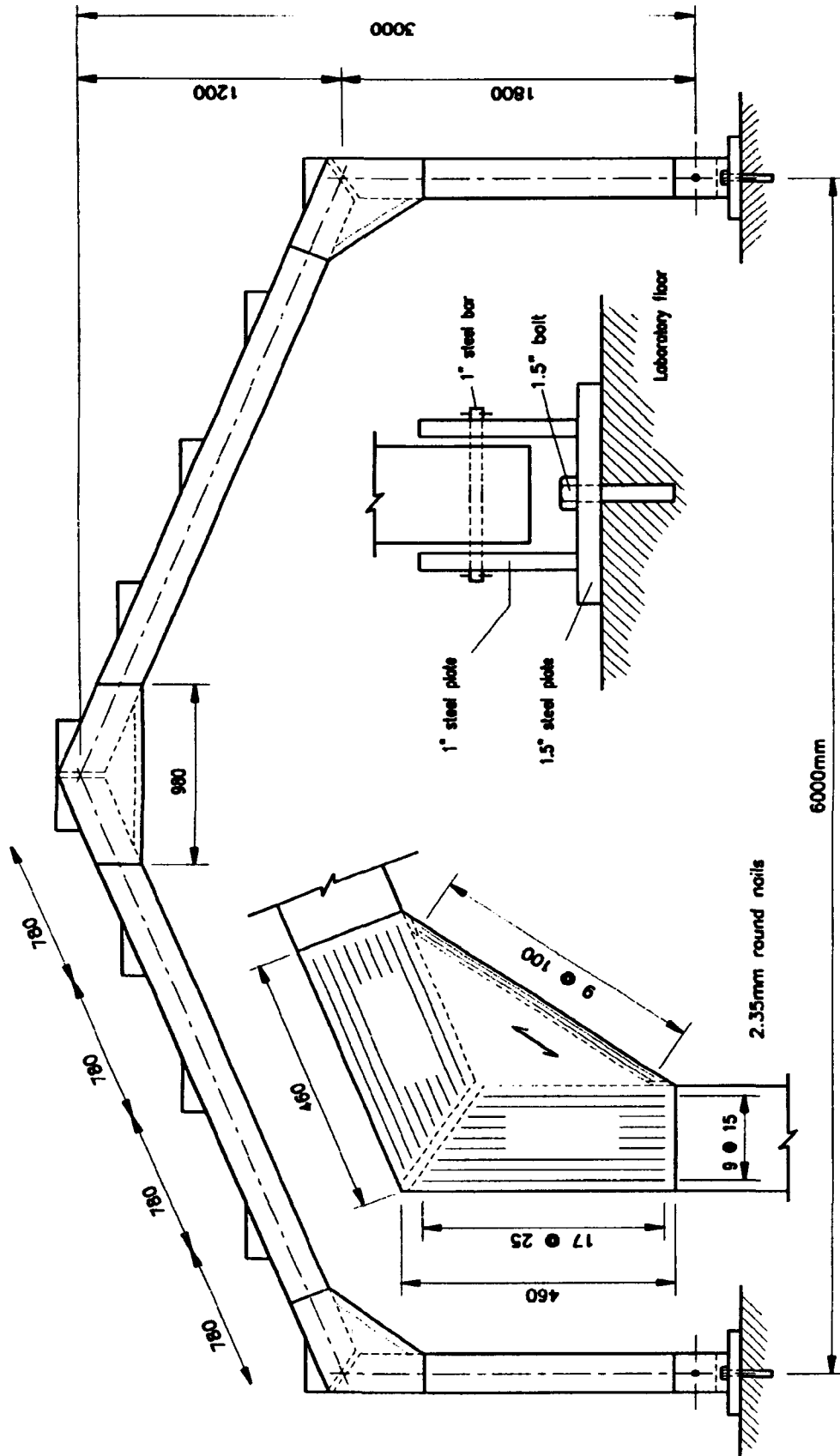


Figure 7. Details of the two-hinge portal.

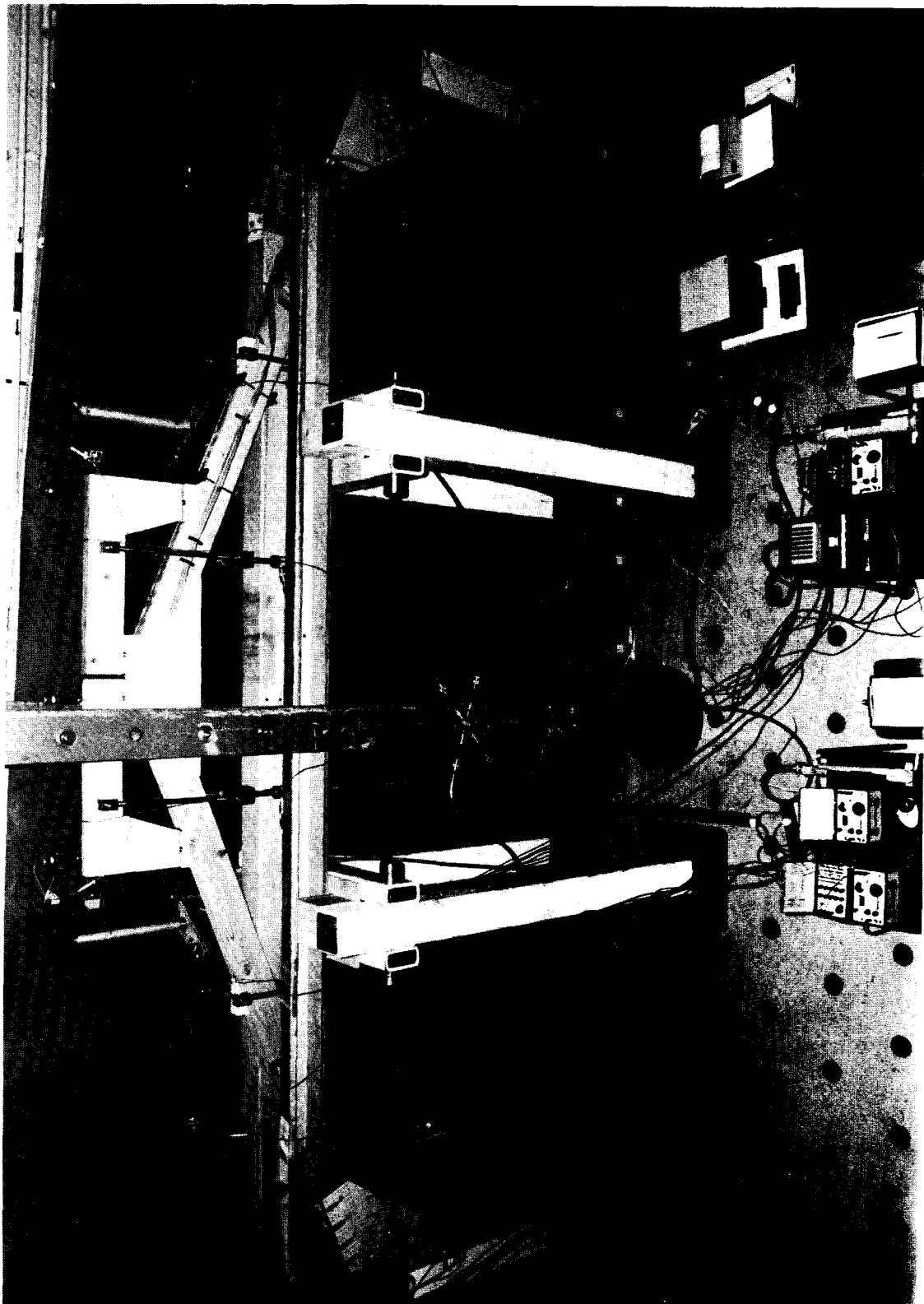


Figure 8. A typical test set-up for the portal frames is shown.

STIFFNESS METHOD FOR ANALYSIS OF SEMI-RIGIDLY JOINTED PLANE-FRAMES

The stiffness method is a well established method of structural analysis, being described in a number of research reports and text books over the past few decades. In this method, the forces {P} acting at the nodes of a structure are related to the nodal displacements {Δ}, in the form of:

$$\{P\} = [K]\{\Delta\} \tag{9}$$

Where the structural stiffness matrix [K], a square matrix equal in order to the number of degrees of freedom in the structure, is assembled from the stiffness matrices [K_e] of the individual elements. The elastic element stiffness matrix [K_e], which contains terms that are functions of the nodal translations and rotations in the local co-ordinate system, for the structural element shown in Figure 4, is given by:

$$[K_e] = \begin{bmatrix} \frac{AE}{L} & & & & & & & & \\ & \frac{12EI}{L^3} & & & & & & & \\ 0 & & \frac{6EI}{L^2} & & \frac{4EI}{L} & & & & \\ & & & & & & & & \\ -\frac{AE}{L} & & 0 & & 0 & & \frac{AE}{L} & & \\ & & & & & & & & \\ 0 & & -\frac{12EI}{L^3} & & -\frac{6EI}{L^2} & & 0 & & \frac{12EI}{L^3} \\ & & & & & & & & \\ 0 & & \frac{6EI}{L^2} & & \frac{2EI}{L} & & 0 & & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \quad \text{symmetrical} \tag{10}$$

The effects of semi-rigid connections at the ends of a structural element can be incorporated in the analysis by modifying the standard stiffness matrix [K_e] of the element. Since a stepwise linear method is adopted, the stiffness of the connection can be modelled as the constant of the linear spring. This makes the method described by Monforton and Wu [5] applicable to the solution procedure compiled in this study. In this method two dimensionless parameters γ_i and γ_j, designated as "rigidity factors," are introduced where

$$\gamma_i = \frac{L}{L + \frac{3EI}{R_{end\ i}}} \quad \text{and} \quad \gamma_j = \frac{L}{L + \frac{3EI}{R_{end\ j}}} \tag{11}$$

The values of γ depend on the known rotational rigidity (8) at the respective ends of the member (i.e. i or j) and on its geometric and elastic properties. It varies from zero for a frictionless pinned connection to unity for a fully rigid one.

Hence the modified stiffness matrix [K_e]_m, which incorporates the rotational stiffness of the connections at the ends of a structural element, can be expressed in the following form:

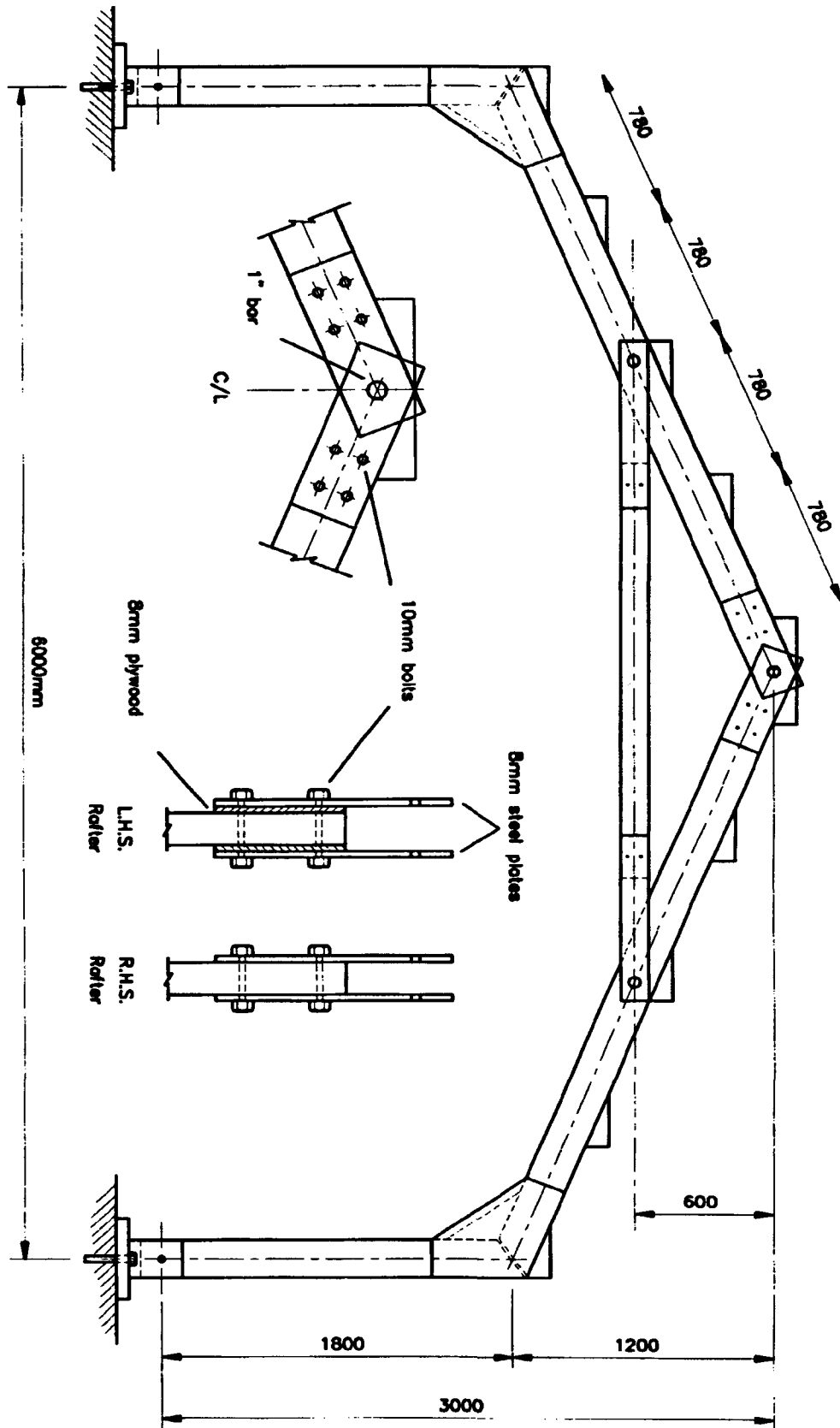


Figure 9. Details of the three-hinge portal with a tie-bar pinned across mid-rafters.

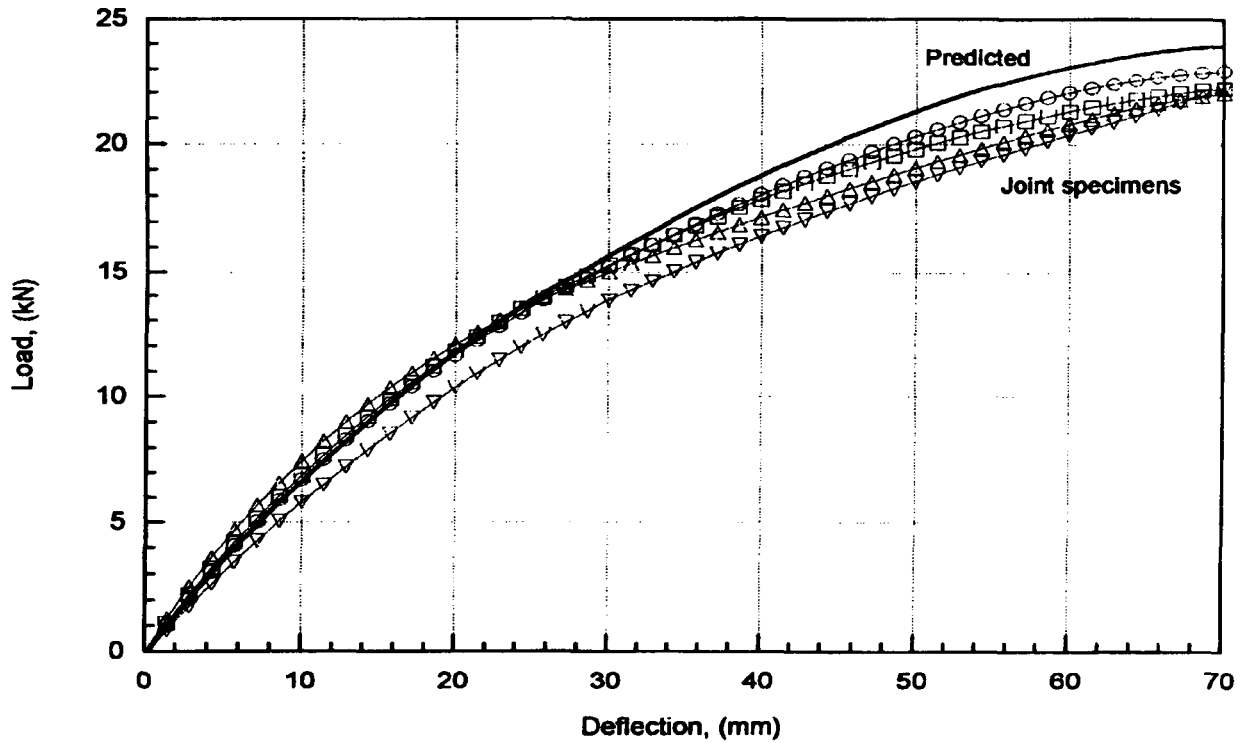


Figure 10. Comparison of the predicted and experimental deflections of the knee joint specimens.

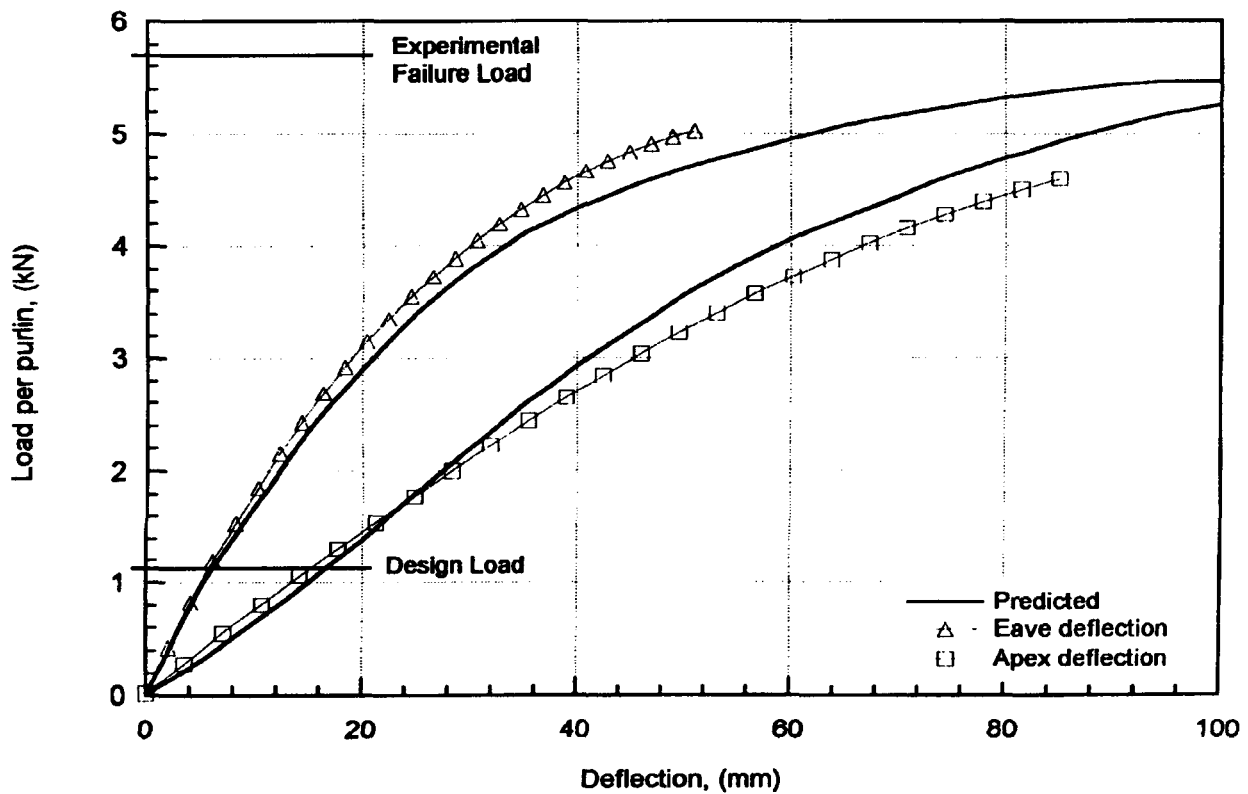


Figure 11. Comparison of the predicted and measured load-deformation behaviour of the two-hinge portal.

$$[K_e]_m = \begin{bmatrix} \frac{AE}{L} & & & & & \\ 0 & \frac{12EI}{L^3} \left(\frac{\gamma_i + \gamma_j + \gamma_i + \gamma_j}{4 - \gamma_i \gamma_j} \right) & & & & \\ 0 & \frac{6EI}{L^2} \left(\frac{\gamma_i (2 + \gamma_j)}{4 - \gamma_i \gamma_j} \right) & \frac{4EI}{L} \left(\frac{3\gamma_i}{4 - \gamma_i \gamma_j} \right) & & & \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & & \\ 0 & -\frac{12EI}{L^3} \left(\frac{\gamma_i + \gamma_j + \gamma_i + \gamma_j}{4 - \gamma_i \gamma_j} \right) & -\frac{6EI}{L^2} \left(\frac{\gamma_i (2 + \gamma_j)}{4 - \gamma_i \gamma_j} \right) & 0 & \frac{12EI}{L^3} \left(\frac{\gamma_i + \gamma_j + \gamma_i + \gamma_j}{4 - \gamma_i \gamma_j} \right) & \\ 0 & \frac{6EI}{L^2} \left(\frac{\gamma_j (2 + \gamma_i)}{4 - \gamma_i \gamma_j} \right) & \frac{2EI}{L} \left(\frac{3\gamma_i \gamma_j}{4 - \gamma_i \gamma_j} \right) & 0 & -\frac{6EI}{L^2} \left(\frac{\gamma_j (2 + \gamma_i)}{4 - \gamma_i \gamma_j} \right) & \frac{4EI}{L} \left(\frac{3\gamma_j}{4 - \gamma_i \gamma_j} \right) \end{bmatrix} \quad \begin{matrix} \\ \\ \\ \\ \\ \\ \end{matrix} \quad \begin{matrix} \\ \\ \\ \text{symmetrical} \\ \\ \\ \\ \end{matrix} \quad (12)$$

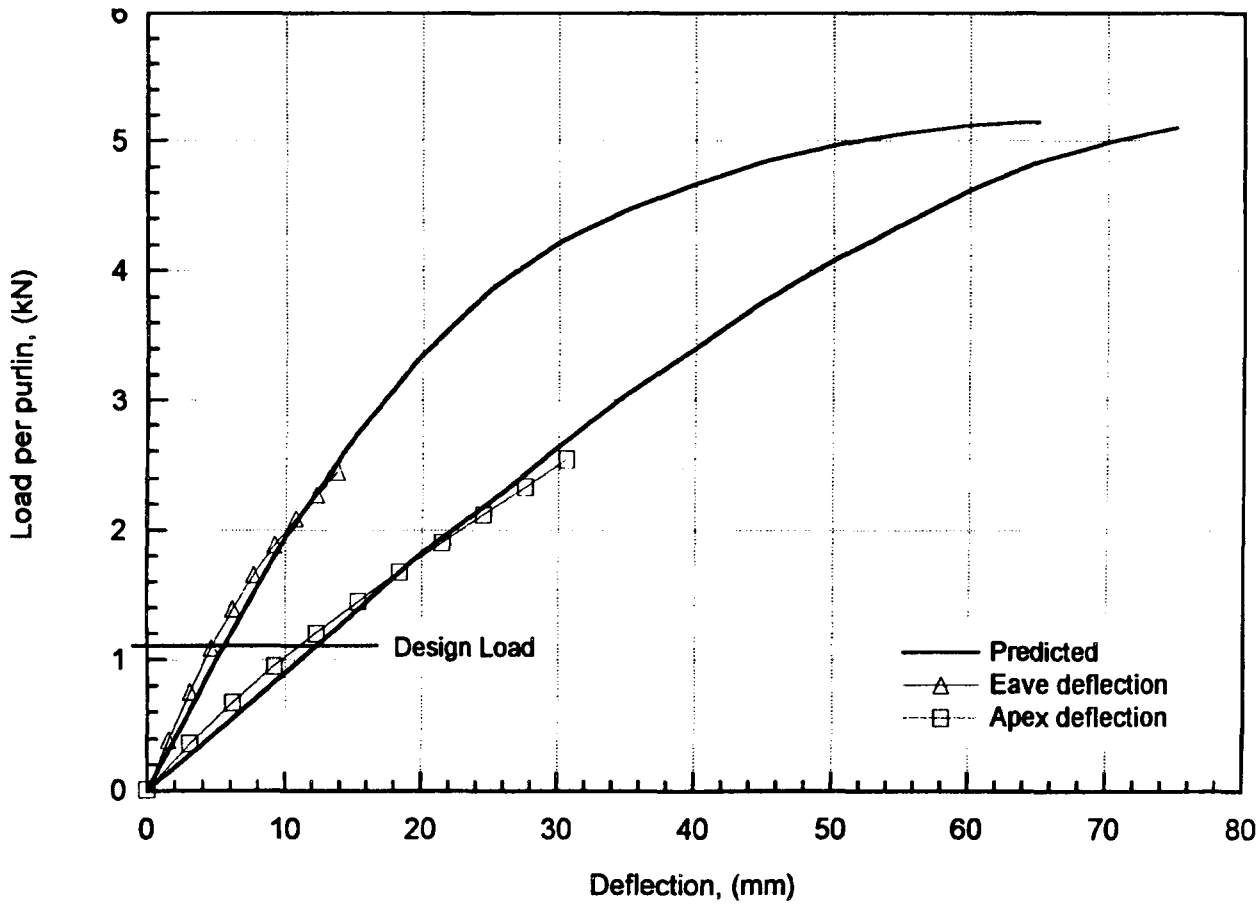


Figure 12. Comparison of the predicted and measured deflections of the three-hinge portal with a tie-bar pinned across mid-rafter.

$[K_e]_m$ the modified member stiffness matrix in terms of the local co-ordinate system, can then be expressed in terms of a global co-ordinate system using the transformation matrix $[T]$ as:

$$[K_m] = [T]^T [K_e]_m [T] \quad (13)$$

where $[K_m]$ is the member stiffness matrix in terms of the global co-ordinates and

$$[T] = \begin{bmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

in which c and s represent cosine and sine of angle a as indicated in Figure 4.

The member stiffness matrices $[K_m]$ can all be assembled to form the overall structure stiffness matrix $[K]$ giving the stiffness equations for the overall structure as defined by equation (9).

NON-LINEAR ANALYSIS

The non-linear response of a semi-rigidly connected frame is a result of the non-linear nature of the connection and the secondary effects of large deflections. The member stiffness matrix, equation (15), includes non-linear coefficients which depend on connection stiffness. As a result, the overall structure stiffness matrix formed by superimposing non-linear member stiffness matrices has a non-linear nature and hence a non-linear solution is necessary.

With reference to the moment-rotation behaviour of nailed joints, described earlier, and assuming that the centre of rotation of the joint or nail group remains at its initial position throughout the non-linear deformation of the frame, the moment carrying capacity and the rotational rigidity of the joint can be calculated.

For a connection with n number of nails distanced r_i from the geometric centre of the nail group undergoing a moment-rotation behaviour as shown in Figure 1, the rotational rigidity of the joint

R_j is expressed by equation (8); and the moment capacity of the joint, at loading increment j , can be expressed by:

$$M_j = \left\{ \sum_{i=1}^n (f_{i(j-1)} + f_{ij}) r_i \right\} - M_{j-1} \quad (15)$$

Therefore, if the non-linear load-slip behaviour of a nailed joint is approximated by a series of successive segments of lines (i.e. piece-wise linear curves) it is possible to determine the limiting moment M_j at the loading increment of j with the appropriate rotational rigidity of R_j . Where the limiting moment may be defined as the maximum magnitude of moment carried by the joint, at loading increment j , above which the rotational rigidity R_j will change to $R_{(j+1)}$.

The non-linear analysis was carried out as a series of linear analyses employing this technique. A computer program was written for the non-linear analysis of structures with semi-rigid joints incorporating nailed connectors, using the method described. In order to carry out non-linear analyses, the load-slip relationship of the nail connections was required. This was accomplished by performing a series of single shear load-slip tests on four groups of nailed plywood to timber specimens, see Figure 5. Thirty-seven samples were tested to establish the mean load-slip curve shown in this figure.

EXPERIMENTAL VERIFICATION

To verify the accuracy of the analysis procedure and method implemented in the computer program, a series of joint specimens and portal frames, recently tested [15,16], were considered.

Four moment resisting portal knee joints and two full portal frames (a two-hinge and a three-hinge portal) incorporating nailed plywood gusset joints were constructed. The joint specimens were designed as part (knee joint) of a 6 m span two-hinged timber portal frame, in accordance with BS5268:Part 2:1988 [17]. The dimensions of a typical knee joint tested are shown in Figure 6. The effective leg lengths shown were chosen in order to produce, when loaded, the correct combination of forces and moments as those produced in the actual portal knee joints.

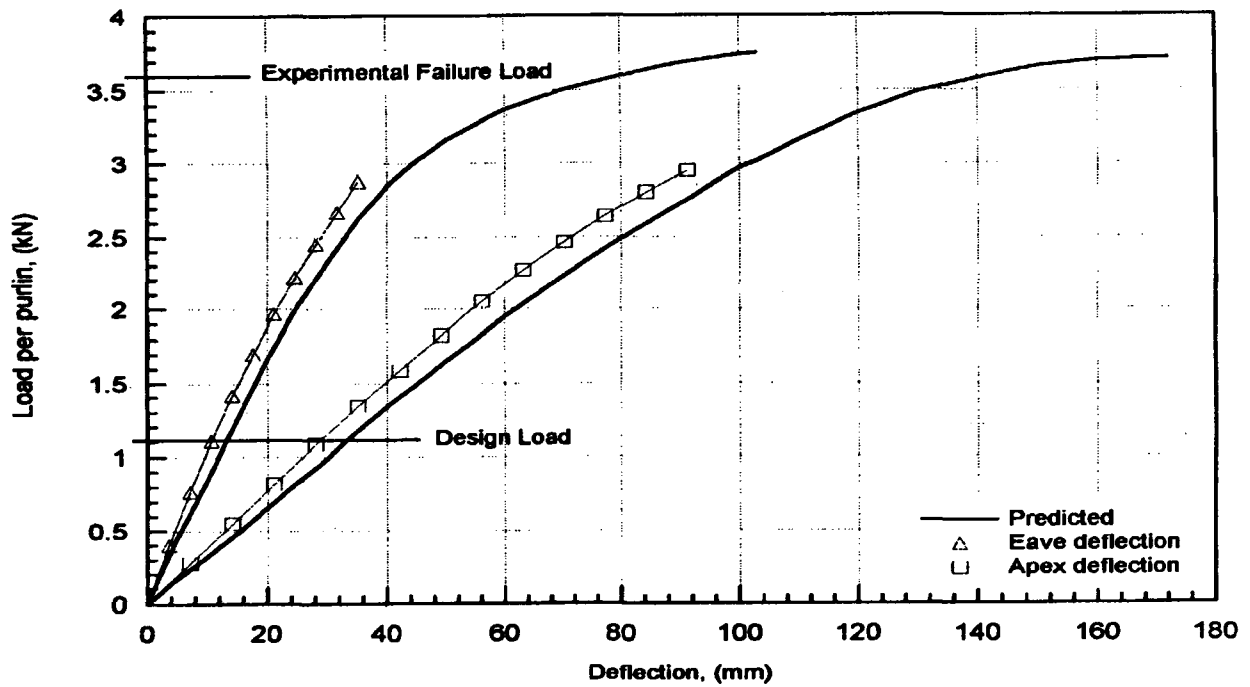


Figure 13. Comparison of the predicted and measured deflections of the three-hinge portal.

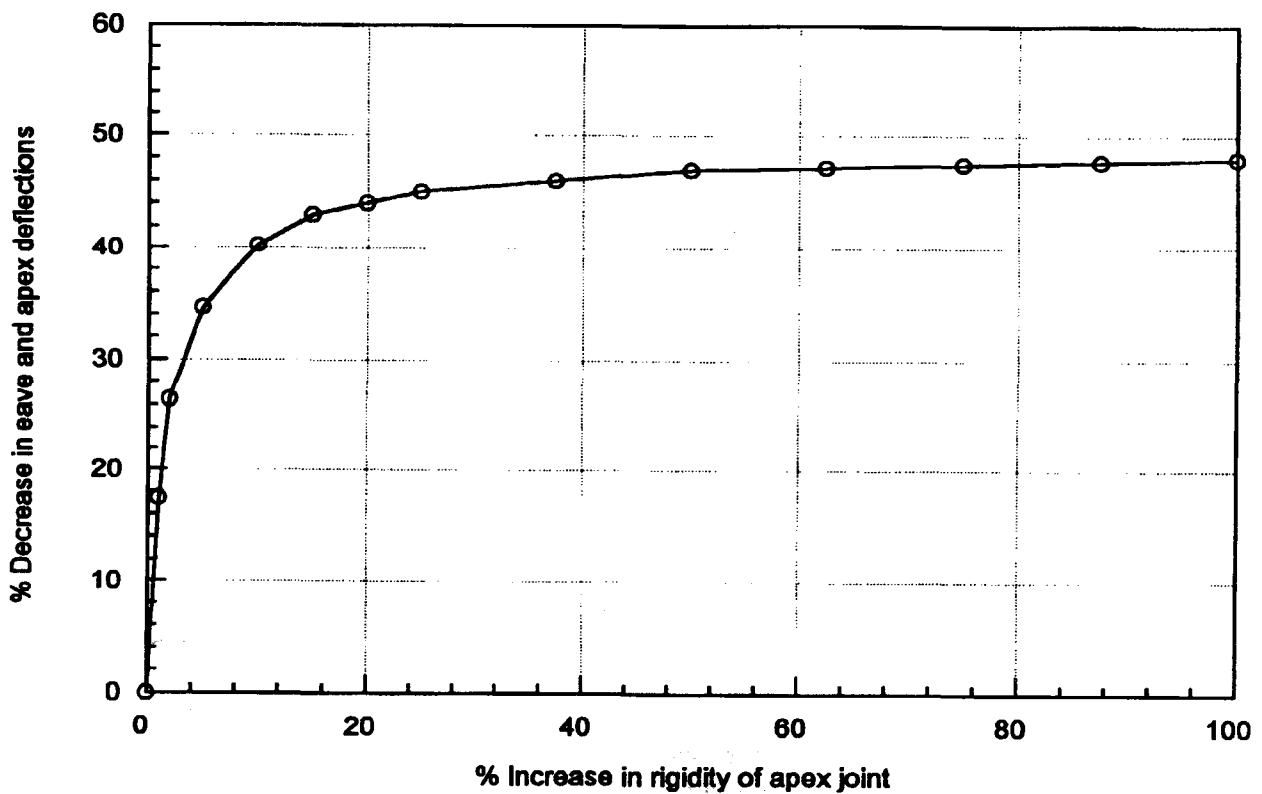


Figure 14. Effect of increase in rotational rigidity of apex joint in structural behaviour of the three-hinge portal.

The portal frames were constructed as a half scale of a 12 m span portal. The fabricated details of the two-hinge portal are shown in Figure 7. The Design Load (working load) was 1.5kN/m on plan.

The rafters and legs of the portals were sawn from 75mm x 250mm British-grown Douglas fir with moisture content of 14±2% and the haunch gussets were cut from Finnish birch-faced (6.5mm thick, 5 plies) plywood obtained from a local timber yard. The gusset plate nailing pattern was derived by assuming a pattern and then determining its moment and shear resistance. By trial and error it was possible to arrive at a nail pattern, Figure 6, in which the load on the average nail did not exceed the safe lateral strength, for the size used, as specified in BS5268: Part 2:1988.

The available headroom in the laboratory made it possible to test the frames in their normal vertical positions. The feet of the portals were pinned to the laboratory floor. Lateral supports with rollers were provided on both sides of the frames along apex gussets and at approximately mid-height of rafter members. The frictional resistance of the test assembly was found to be negligible. In Figure 8 a typical test set-up for the portal frames tested is shown.

The first portal was constructed as a two-hinge portal frame, with no moment of resistance at either base. As this was the first frame to be tested, considerable caution was exercised until the various control and measuring systems were proven. This meant that the frame was subjected to several preliminary tests prior to application of the Design Load, deflection, and strength tests (i.e. 2.5 x Design Load) and hence, the load was increased until the portal failed by tearing of the right hand eave joint gusset at 5.2 x Design Load.

The second portal was constructed as a three-hinge portal frame, with pinned joints at feet and apex. The eave joints and solid members of this portal were similar to those of the two-hinge portal frame. The hinge joint at apex was assembled by marrying the two halves of the portal frame using four mild steel plates bolted together and to the rafter members (two for each rafter leg in a sandwich construction).

The experimental work on the three-hinge portal was carried out in the following three stages.

1. The portal was tested with a tie-bar pinned

across the rafter members at mid-height between eaves and apex, see Figure 9. After application of the strength test, loads were removed and the frame was allowed to recover.

2. The portal was again tested for strength, this time with a tie-bar pinned across the rafter members at a position with 1/4 height from apex and 3/4 height from eaves; and hence the frame was allowed to recover.
3. The portal was tested on its own (with no tie-bar) to failure.

The order in which the experiments were carried out was designed to minimize the possibilities of any damage to the portal frame under deflection and strength tests; and to obtain maximum possible data from the experimental work. This was based on the results of the elastic-linear analyses of the three-hinge portal frame with a tie-bar pinned at various positions across rafter members, using a plane-frame computer program. Analyses showed that the deflection of the three-hinge frame (with no tie-bar across rafters) was on average three times that with a tie-bar pinned across mid-rafter sections and twice that with a tie-bar pinned to rafter members at a level 1/4 height from apex and 3/4 from eaves.

In Figures 10 to 13 the predicted and measured deflections of the four knee joint specimens, the vertical deflection of the apex and the horizontal deflection the eave joints of the two-hinge and the three-hinge portal frames with and without tie-bar are compared. In order to avoid damage to data acquisition equipment at high loading levels (at about 80% of the predicted ultimates), the deflection measurements were stopped and transducers were removed. Full details of the experimental results are given elsewhere [15,16].

From the above figures it can be seen that the predicted load-deformation behaviour including failure loads agree closely with the experimental results.

APPLICATION OF SEMI-RIGID BEHAVIOUR OF THE JOINTS IN DESIGN-ANALYSIS

The advantage of a design utilizing semi-rigid joints was examined on several trial frames and is demonstrated by the example given in Figure 14. In this example a three-hinge timber portal frame with

material properties and dimensions similar to the frames reported earlier was analysed. Several linear analyses were carried out, under the Design Load, and each time the rigidity of the initially pinned apex joint was increased until it became fully rigid, and the forces and deformations at the joints were calculated.

The effects of percentage increase in rotational rigidity of the apex joint from being pinned to fully rigid (0 to 100%) on the percentage decrease in lateral deflections of the eaves and vertical deflection of the apex joints are shown in Figure 14.

This figure shows that if an assumed pinned apex joint possesses 5% rigidity, the total deflections of eaves and apex joints and also magnitude of moments induced at the eave joints would decrease by 35%. At the other extreme, if an assumed fully rigid apex joint possesses 50% rigidity instead of 100%, the total deflections at eaves and apex and also moments induced at eave joints will be increased by only 2%. This highlights that a better understanding of the effects of semi-rigidity of the connections can provide a more rationally based, less conservative design.

Design Procedure

A design procedure which would conveniently handle the flexibility of the nailed timber connections may be described as follows:

1. Analyse the frame, assuming rigid connections, to determine the axial loads, shears and moments in all members and joints of the frame.
2. Utilise this information to determine tentatively the beam, column and joint sizes.
3. For connection sizes and materials defined, use the load-slip characteristics of nailed joints, laterally loaded in single shear (obtained from experimental work or available data) in equations (4) and (8) and then in (15), to determine limiting moments and rotational rigidities at each loading increment.
4. Insert this information and member properties of the tentative sections into a computer program for a non-linear analysis. Plot the load-deformation relationship of the appropriate joints up to working loads or failure loads.

5. Using appropriate factored loads, check that the ultimate limit-state or serviceability limit-state (i.e. deflection) criteria are satisfied. If necessary alter the size of the members and joints and repeat the above procedure from step 2.

SUMMARY

In this paper a systematic and powerful method for the analysis and design of timber portal frames is described. The method allows a simple but realistic representation of the semi-rigid and non-linear behaviour of the nailed connections to be made in the analysis-design process.

The semi-rigid behaviour of nailed connections in the timber portal frames was simply and effectively represented by the non-linear load-deformation characteristics of the simple nailed joints laterally loaded in single shear. Using the non-linear analysis, the ultimate load carrying capacity of the structures with nailed connections was effectively predicted.

The semi-rigid connections cause considerable effect on the distribution of forces and in particular moments in frames. In general, the overall maximum moments decrease in beams and increase in columns compared with rigidly connected frames. These effects would cause significant changes in the design of members of a timber frame. Connections of variable rigidity may be designed to obtain more economical sections by balancing of member sizes and connection moments.

NOTATIONS

A	cross-sectional area of element
[C]	correction matrix, a (6 x 6) matrix
E	modulus of elasticity of element
f_i	shear force in nail (i)
ϕ	joint rotation
Y_i/j	fixity factor at end i or j of element
I	second moment of area of element
[K]	structural stiffness matrix in global co-ordinate system
$[K_e]$	element stiffness matrix in local co-ordinate system
$[K_e]_m$	modified element stiffness matrix
$[K_m]$	element stiffness matrix in global co-ordinate system

k_i	slip modulus of nail (i) at a loading increment j
L	length of element
M	applied moment in a joint
{P}	applied load vector, a (n x 1) matrix
R_j	rotational rigidity of a joint at a loading increment j
r_i	distance from nail (i) to the centre of rotation of the nail group
S_i	slip of nail (i) at a loading increment j
[T]	transformation matrix.

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