A Self-tipping Wood-chip Dumper

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ABSTRACT

Current wood-chip dumping systems consist of a platform, pivoted at one end, that is raised by expensive telescopic hydraulic cylinders with high energy requirements. A new concept for a self-tipping dumper is described that, theoretically, requires no energy to tip and empty a loaded B-Train Chip Van having a total mass exceeding 60 ts. The platform is pivoted near its centre and advantage is taken of the shift in centre of gravity between the loaded and empty vehicle to tip the platform to 60 degrees and then return it to the horizontal.

Keywords: *harvesting, transportation, tipping plat- form, computer simulation.*

INTRODUCTION

Pulp mills in Canada currently receive approximately 50% of their raw wood fibre supply in the form of chips produced as a byproduct of the lumber and allied products industry. The balance is delivered to mills in the form of roundwood that is then debarked and chipped on site.

Recent developments in mobile chippers have made chipping in the forest economically feasible. The chips produced in this way are comparable in quality to those produced by mill-yard chippers. All "waste" (bark, branches, etc.) is left in the forest, a desirable feature both economically and environmentally. The replacement of log trucks on public highways by enclosed chip vans will also enhance the forest industry's image in the public eye. Roundwood will continue to be delivered but, at least in some regions of Canada, the proportion of wood delivered as chips could rise to as much as 80% before the end of this century. Nearly all chip deliveries to the mill are made by road in semi-trailers. These are designed primarily for hauling on public highways and must, therefore, meet all provincial size and weight regulations. As wood chips have a relatively low density, the present chip trailers, while conforming with size restrictions, may not be able to take full advantage of the weight limits. "Straight-box" semi-trailers, because of their length, may also be less suited for off-highway transportation on the bush roads where they will be increasingly required to travel. These concerns have resulted in the development of trailers specifically designed for transporting pulp chips. One such development is the B-Train Chip Van.

B-TRAIN CHIP VAN

The B-Train Chip Van was designed to maximize payload [1,4]. It consists of two box-trailers connected by a triple-axle (tridem) group that allows some flexibility in the spacing between the trailers. Because wood-chip loads have a relatively low density, volume capacity is maximized (within provincial regulations) to bring the payload as close as possible to the legal weight limit by lowering the floor of each trailer between the axles to form a "belly". Ground clearance of these bellies is greater for vehicles designed for use on forest roads than for those designed primarily for public road hauling. Pressure transducers indicate to the operator when each axle group is loaded to the legal limit [4]. Ground clearance can be adjusted to the legal height limit as the trailers are filled. In some versions, the trailers slide together to close the gap for dumping; in others, the gap is closed by a system of side doors. Figure 1 gives a general configuration for a B-Train Chip Van in position on a dumping platform. The mass of the vehicle and its load are proportionally allocated to the four axle groups (components) in the manner shown. Also illustrated are the approximate locations of the centres of gravity (CG) above each axle group for the loaded and empty vehicle. Distances are measured from the "backend" of the platform and the height of the CG of each load component is measured from the platform's upper surface.

CURRENT DUMPING SYSTEMS

Current dumping systems consist of a steel platform that is pivoted, or hinged, at one end (Figure 2). The loaded chip van is driven onto this platform, which is then raised by hydraulic cylinders mounted on either side of the platform and midway along its

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	— Balance point, X_{θ} —		Piston		
Tipping	Loaded	Empty	Loaded	Empty	Piston stroke
angle, θ (degrees)	(m)	(m)	(kN)	(kN)	(m)
0	11.67	13.11	-15392	26718	0.00
10	11.35	13.01	-29812	24329	0.35
20	11.01	12.90	-43769	21787	0.70
30	10.62	12.78	-57146	18896	1.04
40	10.15	12.64	-69827	15349	1.37
50	9.51	12.44	-81695	10571	1.69
60	8.53	12.14	-92624	3251	2.00

Table 1. Balance points for loaded and empty vehicles, piston forces, and length of piston stroke for a B-Train Chip Van on a 30-tonne platform.

Table 2. Piston forces required to raise and lower a B-Train Chip Van on a modified dumping platform (counterweight = 25 ts).

Tipping angle, θ (degrees)	Balance point (Loaded) (m)	Piston stroke (m)	Piston force required to raise loaded vehicle (kN)	Force required to lower empty vehicle (kN) ¹	
0	9.55	0.00	2898	-6245	
10	9.33	0.52	3096	-1379	
20	9.09	1.00	3269	4491	
30	8.82	1.44	3445	11857	
40	8.49	1.81	3686	22039	
50	8.05	2.12	4210	39571	
60	7.36	2.34	6473	94749	

¹Negative values indicate the forces required to pull the platform down; positive values indicate that the platform will drop unaided.

length. A "back-stop" raised at the rear of the trailer retains the vehicle on the platform. As the platform is raised, the chips are discharged from the rear of the trailer into a pit or hopper from which they are subsequently transported to the appropriate chip pile. Under some conditions, the platform must be tipped to an angle of 60 degrees to ensure complete discharge of the load. After discharging its load, the vehicle drives forward off the platform. In another model, simpler than this "drive-through" version, the loaded chip van is reversed

onto the platform until the rear contacts a fixed back-stop. After discharge, the vehicle moves forward. This simpler version is less expensive, but the forces required to raise the platform are the same and productivity is lower.

Because the combination of a loaded B-Train and the dumper platform can exceed 90 ts, considerable force is needed to raise the platform and, because the extension may exceed 10 m, expensive heavy-duty telescopic cylinders are required. Williams [3] has estimated that the annual energy costs for this type of dumper could be considerably in excess of \$50 000. Other unloading methods, such as side dumping or bottom discharge, could reduce the energy requirements and the need for expensive hydraulic systems. However, both of these methods would require major changes to the design of the chip vans or result in loss of payload. Mills would also have to operate two unloading systems during the phase-in period.

The objective of the present study was to investigate a new chip-dumper concept that would be less expensive to manufacture and more energy efficient in use. It takes advantage of the variation in the location of the centres of gravity of loaded and empty chip vans.

THE CONCEPT

In Figure 1, a B-Train Chip Van is shown on a tipping platform that is 25 m long, 0.5 m deep, and approximately 3 m wide. The mass of the platform, considered to be centred at its midpoint, is 30 tonnes. Rather than pivoting at its "back-end", the platform will be pivoted in the region of its balance point located at a distance X_{θ} from the back-end. The value of X_{θ} changes as the platform tips (because the CG of the components are not located on the horizontal axis of the platform) and will also be different for the loaded and empty vehicles.

Values of X_{θ} at different tipping angles can be determined by taking moments. The formula for calculating the moment of each component (axle group and platform CG) is:

$$M_{i} = W_{i} \cdot [(x_{i} - X_{\theta}) \cdot \cos \theta - (y_{i} + r/2) \cdot \sin \theta$$
(1)

where M_i = moment of component i (kN.m)

W_i = mass centred at the CG of component i (kg)

 $x_i = distance from "back-end" of platform (m)$

y_i = height of CG above platform (m)

 θ = angle of tip (degrees).

 X_{θ} = distance of the balance point from the back-end (m)

r = depth of the platform (m)

The value of X_{θ} will be such that $M_i = 0$. By transposing, it can be shown that:

$$X_{\theta} = [\Sigma W_{i} x_{i} \cos \theta - \Sigma W_{i} (y_{i} + r/2) \sin \theta] / \Sigma W_{i} \cos \theta$$
$$= [\Sigma W_{i} x_{i} - \Sigma W_{i} (y_{i} + r/2) \tan \theta] / \Sigma W_{i} \qquad (2)$$

Table 1 gives the values of X_{θ} at 10-degree intervals as the loaded vehicle is tipped from the horizontal to an angle of 60 degrees and for the empty vehicle as it is returned to the horizontal. The initial balance

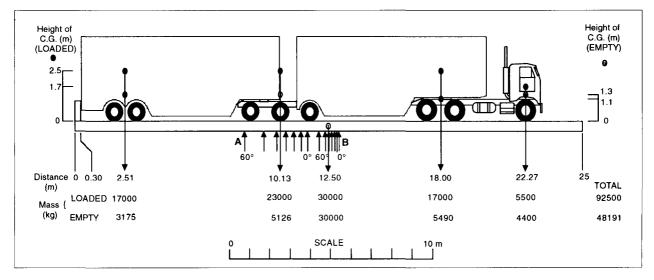


Figure 1. Specifications for a B-train chip van on a 25-m tipping platform of mass 30 ts. The arrows at**A** indicate the range in balance points as the platform tips with the loaded vehicle from 0 to 60 degrees. The arrows at **B** indicate the corresponding range for the empty vehicle. (Based on information supplied by Peerless Page Industries Ltd., Penticton, B.C.)

point for the loaded vehicle is at 11.67 m but, as the platform tips, this moves toward the back-end until, at an angle of 60 degrees, it is at 8.53 m. This assumes that the load remains intact; in practice, chips begin to leave the van as soon as it starts to tip but the bulk of the load is discharged *en masse*, usually before the platform reaches 60 degrees. At 60 degrees the balance point for the empty vehicle on the platform is at 12.14 m but this moves to 13.11 m when the platform returns to the horizontal position. These ranges are indicated in Figure 1.

In current chip dumpers the platform is pivoted, or hinged, at the back-end and raised by hydraulic cylinders located on either side near the midpoint. If the platform is now pivoted at a distance greater than 11.67 m from the back-end, the platform should tip of its own accord to dump the load. However, if the platform has to be tipped to 60 degrees, it will not automatically return to the horizontal unless the pivot is located at a distance less than 12.14 m.

There are a number of practical problems associated with this concept that have to be addressed before it could be put into practice. The first of these is that the rate of tipping must be controlled or damage to the platform and vehicle could result. Control could be hydraulic in a manner similar to that used in existing dumpers. The main difference would be that the system would be much simpler and less expensive. A smaller cylinder would be required and, because the extension (or stroke of the piston) would be considerably reduced, telescopic cylinders would no longer be required. The hydraulic system would be "passive", in that it would no longer be required to raise or lower the platform but would solely control the speed at which it tipped. If the pivot point is located at 12 m and the cylinder at 14 m, the forces on the cylinder are given in Table 1. For the loaded vehicle, the forces are all negative, indicating that the platform would tip up of its own accord. Conversely, the forces are all positive for the empty vehicle, indicating that the platform would return to the horizontal unaided. The method of calculating the forces is given in the Appendix. The length of the piston stroke is also given in Table 1.

A second problem is the narrow range (11.67 -12.14 m) within which the pivot must be located if self-dumping is to be completed. This range could be extended if the pivot point could be moved towards the back-end as the platform tips. The simplest way of achieving this would be to "roll" the platform over a curved surface, as is shown schematically in Figure 3. It is then possible to locate the initial (horizontal) pivot at any point between 11.67 and 13.11 m. Because the mass of the loaded vehicle is likely to be more variable than that of the empty vehicle, the pivot point should be located closer to the initial balance point of the empty vehicle (say, at 12.5 m). The cylinder could still be located to the right of the pivot point (say at 14.50 m) but, if this is done, the maximum piston extension (2.95 m) will be greater than for the fixed-pivot case cited above (2 m). The cylinder should, therefore, be mounted on the platform at a point a little closer to the back-end than the balance point at 60 degrees (e.g., at 9 m the maximum extension will be 2.56 m). The anchorage for the piston will then be located at some distance below the platform. The calculation of coordinates of points on the curved surface and the location of the piston anchorage is given in the Appendix.

Another problem is that, even if the same vehicle configuration is used at all times, there will be variations in the mass of the load that may alter the balance points. This factor would become more critical if, as would invariably be the case in practice, different vehicle configurations have to be unloaded. Electronic sensors in the approach ramp to the platform could measure the load on each axle and a computer could calculate the balance point. The backstop could then be automatically repositioned so that unaided tipping could still take place. Another solution would be to have a moveable counterweight on the platform that could be automatically adjusted to ensure that the balance points were correctly located. However, the cost of constructing such systems could be prohibitive. By adding a pump and motor, the hydraulic control system could be used to supply power as needed to raise and lower the platform. Because the forces required are much less than in the rear-hinged dumpers, the cylinders can be still smaller and considerably less expensive.

A possible configuration for a "roll-over" dumper is shown in Figure 4. The platform is prevented from slipping as it is tipped over the curved surface on the base by the curved restraining support member (the two curved surfaces must correspond). This support member is anchored to the base of the curved surface and passes through a slot in the platform to a bridging member that connects to the on-ramp. As it tips, the platform slides down the forward surface of the restraining member. This configuration also lends itself to an alternative means of controlling tipping: a low-powered electric or hydraulic motor could be

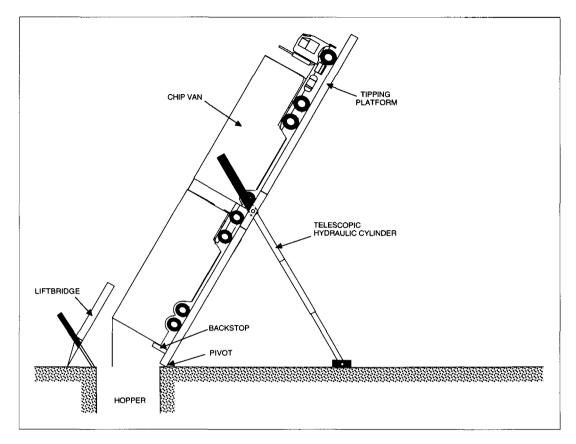


Figure 2. Diagram of a typical chip dumper in current use.

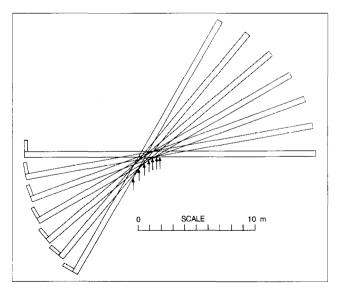


Figure 3. Method of moving the pivot point as the platform tips by "rolling"it over a curved surface.

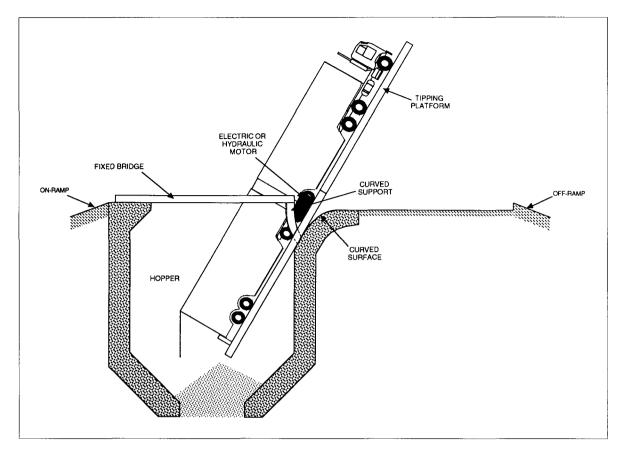


Figure 4. Chip dumper that uses the "roll-over" principle to move the pivot point as the platform tips.

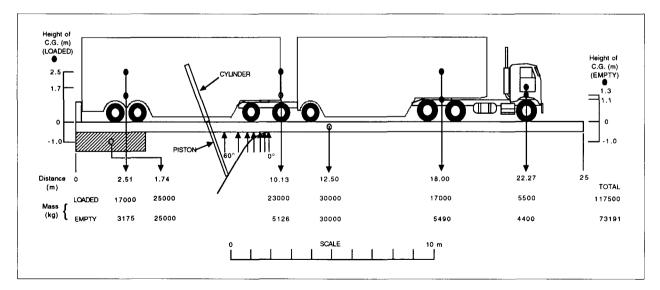


Figure 5. B-Train Chip Van on a modified dumping platform with a 25-t counterweight at the back-end. The arrows at **C** indicate the range in balance points as the loaded vehicle tips.

mounted on the platform that, by means of a pinion, would engage a rack located on the forward surface of the restraining member. This motor would also supply supplementary power for raising or lowering the platform if required. A somewhat similar concept for moving the pivot point with a curved surface was used by Schmidt [2] for side-dumping rail cars.

A disadvantage with these self-tipping designs is that, when fully tipped, the back-end of the platform may extend 10 m or more below its horizontal position. The cost of the extra excavation, or ramp building, to permit this may exceed the reduction in dumper cost. Where dumper installation is being done in conjunction with the construction of a new mill, there may be sufficient material suitable for ramp building and the extra cost can be absorbed. On some sites, advantage can also be taken of local topography to reduce costs.

By attaching a counterweight to the underside of the back-end of the platform, the balance point can be shifted towards the back-end and the depth of the platform at full tipping angle can be reduced. A 25-t counterweight would require 10.42 m³ of concrete (assuming 1 m³ of concrete has a mass of 2400 kg). A modified dumper with such a counterweight is shown in Figure 5 and the calculated balance points are given in Table 2. The initial balance point for the loaded vehicle is moved approximately 2 m closer to the back-end at 9.55 m and moves to 7.36 m when fully tipped. For the platform to tip unaided, the pivot point would have to be located slightly to the right of the initial pivot point. However, the balance point for the empty vehicle is, at all tipping angles, closer than 9.55 m to the back-end and so a force would be required to return it to the horizontal position. The most efficient solution would be to locate the pivot at a point between the loaded and empty balance points. This should be done in such a way that the maximum force required to push the loaded vehicle up is approximately equal to the maximum force required to pull the empty vehicle down to the horizontal position. Table 2 shows the forces involved in raising the loaded vehicle to the fully tipped position and returning the empty vehicle when the initial pivot point is at 9.48 m and the hydraulic cylinder is at 6.48 m from the back-end. A positive value indicates that a force is required to raise the platform; negative values indicate that no force is required to raise the platform or, for lowering, a force is required to pull the platform down. It can be seen that the maximum force required to

raise the platform (from its initial position) is 6473 kN while the maximum to lower it (from the fully tipped position) is 6245 kN. The forces involved will also depend on the location of the cylinder on the platform; the farther it is from the pivot point, the longer the maximum stroke of the piston, and the less the forces required to raise and lower the platform.

The greater the mass of the counterweight, the closer will the pivot points be to the back-end and the less the extra depth of excavation or height of ramp building. To examine the effects of different counter-weights and different locations of the hydraulic cylinder, a series of computer simulations (Tests 1-9) were conducted. These are summarized in Table 3, which gives examples of the calculated maximum piston forces involved, the maximum piston stroke, and the additional depth of excavation for counter-weights of 25, 50, and 100 t (Tests 5-9). These can be compared with the data for a non-weighted platform (Tests 2-4) and a currently used platform that is pivoted at the back-end (Test 1).

Test 1 indicates some of the disadvantages of the present design of dumper are a maximum force to raise the platform of over 86 000 kN and a maximum piston stroke at 60 degrees of 12.5 m. The main advantage, of course, is that with the pivot at the back-end the platform does not drop below its horizontal position and no additional excavation is required. Another disadvantage with the "drive-through" version of this dumper is that a lift-bridge is required for the vehicle to cross the hopper as it approaches the ramp. This may be 3-4 m in length and is not required where the platform is pivoted away from the back-end.

The main disadvantage with the self-tipping dumper can be seen to be the extra depth of excavation (or approach ramp building) required to accommodate the back end of the platform (Test 2). "Rolling" the platform as it tips (Test 3; see Figure 3) only reduces this depth from 10.39 to 10.12 m. At the same time, however, rolling increases the maximum piston extension from 2 to 2.95 m when the cylinder is located 2 m from the initial pivot point towards the front-end of the platform. Mounting the cylinder 3.5 m from the pivot towards the back-end (Test 4) reduces the piston stroke to 2.56 m. The platformmounted motor and rack and pinion system, illustrated in Figure 4, would appear to be the best method of controlling tipping and providing auxiliary power if required.

Table 3. Results of computer simulations with different platform configurations.

Test no.	Counter- weight (kg)	Initial Final (horizontal) (60 degrees) pivot point pivot point (m) (m)		Location of	— Maximum piston — force		piston d	Maximum depth ³
			hydraulic cylinder (m)	Raising loaded ¹ (kN)	Lowering empty ² (kN)	(m)		
1	0	0.00	0.00	12.50	86337	27011	12.50	0.00
2	0	12.00	12.00	14.00	-15392	1877	2.00	10.39
3	0	12.50	9.36	14.50	-8326	14433	2.95	10.12
4	0	12.50	9.36	9.00	-22010	8383	2.56	10.12
5	25000	9.48	9.48	11.48	4348	-10559	2.00	8.21
6	25000	9.48	7.29	6.48	6473	-6245	2.34	7.71
7	25000	9.50	7.31	6.50	4724	-6733	2.34	7.73
8	50000	8.06	6.69	6.56	46342	-48762	1.12	6.69
9	100000	6.00	6.00	8.00	49001	-41912	2.00	5.20

NOTE: All distances are measured from the back-end of the tipping platform.

¹ Negative values (italicized) indicate that the platform will tip up unaided.

² Positive values (italicized) indicate that the platform will return to the horizontal position unaided.

³ Maximum depth of the back-end of the platform below its horizontal position.

The remaining tests are for the modified platform with a counterweight. For all of these, the balance points for the empty vehicle are closer to the back-end than for the loaded vehicle so that force is required to either raise or lower (or both) the platform. With a 25-t counterweight and a fixed pivot (Test 5), the additional depth is reduced to 8.21 m but, as with all examples of this type of platform, forces are required to both raise and lower the platform. "Rolling" the platform (Tests 6 and 7) can reduce the additional depth requirement by approximately half a metre. Fine adjustments to the location of the initial pivot point, made by trial and error, can approximately equalize the forces for raising and lowering. This, as previously noted, is probably a desirable feature. Another 0.5 m reduction in depth is obtained with the 50-t counterweight (Test 8) but there is a marked increase in force requirements. The "roll-over" method cannot be used with the 100-tonne counterweight as the balance point (6.58 m) at 60 degrees is further from the back-end than the initial balance point (6.51 m). With the pivot point fixed at 6 m, the depth is reduced to 5.19 m. Increasing the counterweight may not be practical because of the corresponding increase in force requirements. The additional stress on the platform at the pivot point may also be a problem.

CONCLUSION

The self-tipping wood-chip dumper concept is a theoretical possibility that warrants further study and development. No attempt has been made in this paper to consider the engineering design required to build such a dumper but implementation of the basic concept should not present a problem as most of the features are present on existing dumpers. The addition of a counterweight places extra stresses on the platform. A safety device must be installed to prevent the platform from tipping as the vehicle drives onto the platform. Doubtless practical solutions can be found to these problems, but the cost of the solutions may more than offset any savings in other areas.

Because of the narrow range within which the pivot point must be located, and the variations in load size and chip van model that dumpers will be expected to handle, it must be realized that auxiliary power has to be available. However, by adopting some of the principles outlined here, the system used can be simpler and cheaper, both to manufacture and to operate. The greatest disadvantage at the moment is the requirement for additional excavation or ramp building, the cost of which may more than offset any savings in dumper construction.

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APPENDIX

1. Calculation of Curved Surface for "Roll-over" Tipping

It is not possible to develop a function that will describe how the curved surface varies with tipping

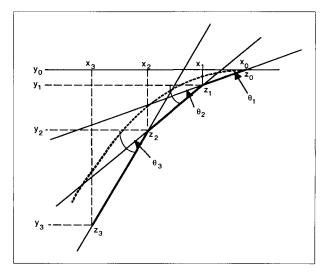


Figure 6. Calculation of points on the curved surface for the "roll-over" method of tipping the platform.

angle θ in a continuous fashion in terms of horizontal (x-) and vertical (y-) coordinates. Rather, this must be done in steps as is illustrated in Figure 6 in which the variables are defined as follows:

- x_0,y_0 (and z_0,y_0) = coordinates of the initial pivot (horizontal) point;
- θ_{i} = tipping angle ($0 \le \theta_{i} \le \theta_{max}$ where θ_{max} is the maximum tipping angle);

 x_i, y_i = coordinates of the pivot point for θ_i ; and

 $z_0 - z_i$ = distance, along the "curved" surface, of the pivot point for θ_i from the initial pivot point.

The values of x_0 and of z_i for each θ_i are known and $\theta_0 = 0$. The problem is to calculate the values of each x_i and y_i that correspond to the z_i as follows:

 $\begin{array}{l} x_{1} = x_{0-1} z_{0} - z_{1})\cos\theta_{1} \\ y_{1} = y_{0-1} z_{0} - z_{1})\sin\theta_{1} \\ x_{2} = x_{1-1} z_{1} - z_{2})\cos\theta_{2} \\ y_{2} = y_{1-1} z_{1} - z_{2})\sin\theta_{2} \\ \text{etc.} \end{array}$

In general:

 $\begin{array}{l} x_{i=} x_{i-1-i} (z_{i-1} - z_i) \cos \theta_i \\ y_{i=} y_{i-1-i} (z_{i-1} - z_i) \sin \theta_i \end{array}$

If the incremental increase $\Delta\theta$ in the value of θ becomes small, the points generated by the $x_i y_i$ lie along a smooth curve. The shape is affected by the value of $\Delta\theta$ that is used to generate the curve. In Figure 6, for illustrative purposes, the value of $\Delta\theta$ was 20 degrees; the "smooth" curve is the shape of the surface when $\Delta\theta$ is two degrees.

2. Calculation of Tipping Forces

The two configurations that have to be considered when developing the formulae for calculating the forces required to tip (and lower) the platform are shown in Figure 7. Figure 7A shows the situation when the hydraulic cylinder is mounted on the platform forward (to the right) of the initial pivot point and the piston is anchored to the horizontal base of the dumper. In this case, the platform is raised by "pushing up" on its front end. The closer the cylinder mounting point is to the pivot point, the greater the force required to raise the platform but the shorter the piston stroke. In Figure 7B, the cylinder is mounted back (to the left) of the initial pivot point and, where the "roll-over" method of tipping is used, the piston anchor point is on the

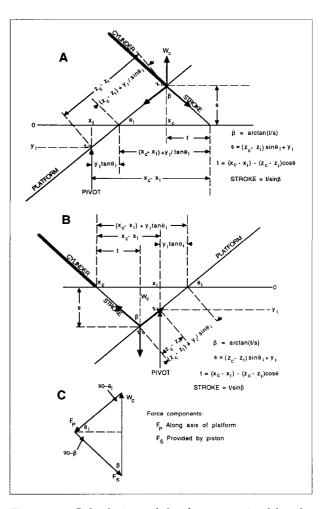


Figure 7. Calculation of the force required by the piston to raise the platform: **A** - Cylinder mounted on the platform to the right (i.e. towards the frontend of the platform) of the pivot, piston anchored on the horizontal base of the dumper; **B** - Cylinder mounted on the platform to the left (i.e. towards the back-end) of the pivot point, piston anchored on the curved surface of the dumper; and **C** - Force diagram. NOTE: All y-values below the O-line in **A** and **B** are negative.

curved surface but farther from the initial pivot point than the final pivot point (z, in Figure 6). In this case, the platform is raised by "pushing down" on its back-end.

For the platform to be in equilibrium, the sum of the moments, $\sum M_i$, must be equal to (but opposite in sign) to M_c , the moment at z_c , the mounting point for the cylinder. The mass, W_c , acting at this point will be equal to $M_c/(x_c - x_i)$. W_c acts vertically downwards with a force of W_c kN. Part of this force, F_{p} , is directed

along the platform to the pivot point but, to maintain equilibrium, a force has to be applied by the piston. The force diagram (Figure 7C) shows the direction of these forces for situation **A** (for situation **B**, the direction of the arrows is reversed). The force required by the piston, F_s , can be calculated as follows:

$$\frac{F_{s}}{\sin(90 - \theta_{i})} = \frac{W_{c}}{\sin(90 - \beta + \theta_{i})} = \frac{F_{p}}{\sin\beta}$$
(Law of sines)

Then:

$$F_{s} = \frac{W_{c} \cdot \sin(90 - \theta_{i})}{\sin(90 - \beta + \theta_{i})} = \frac{W_{c} \cdot \cos \theta_{i}}{\cos(\beta - \theta_{i})}$$

As θ_i increases in value, W_c and $\cos \theta_i$ decrease and, although β decreases, the value of F_s decreases also. Thus the greatest force is that which is required to raise the platform from its initial horizontal position.