A Method for Determining Offtracking of Multiple Unit Vehicle Combinations

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ABSTRACT

The required road width around curves on forest roads is largely determined by the difference in wheel paths between the inside front tractor wheel and the inside rear trailer wheel. This difference, known as offtracking, is a function of the vehicle and road geometry. This paper presents a method for determining the offtracking of fixed and variable length multiple unit vehicle combinations travelling over forest roads. The computational method numerically integrates the differential equations which compute the path (tractrix) that the rear of a vehicle follows from a given steering curve. A unique three-point solution method is used to determine the instantaneous center of rotation for trailers in the vehicle combination. The method is shown to have good agreement with experimental data. It is suitable for use on microcomputers for single and multiple curves. A microcomputer program, OFFTRACK, was developed using this methodology.

Keywords: *Forest Roads, Road Design, Logging Trucks, Offtracking*

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INTRODUCTION

When a tractor trailer combination enters a curve, the centerline of the trailer axle begins to trace a path that is different fromthe centerline of the tractor front axle. This difference in paths, which alsocoincides with the difference in paths between the inside front tractor wheel and the inside trailer wheel, is called the offtracking of the vehicle. Offtracking largely determines travelway width around curves on forest roads.This paper describes a method that can be used to determine offtracking of multiple unit vehicle combinations that use Ackerman type steering. All axles are constrained perpendicular to the longitudinal axis of the vehicle. The approach is suitable for low speed vehicle operation. Vehicle dynamics and tire mechanics are not considered. A microcomputer program, OFFTRACK, has been developed that will simulate offtracking of six different vehicle combinations commonly found on forest roads [9]. The path of front and rear overhanging objects such as yarder towers or tree length logs is also computed. Notation that is used is given near the end of this paper.

Tractrix Equation

The mathematical description of the path that the rear axle of a vehicle follows from a given steering curve for low speed maneuvers is called the tractrix of the steering curve [3]. A steering curve is defined as the set of geometrics that guides a vehicle unit. For a multiple unit vehicle combination, a tractrix is also formed by the path of the hitch point for each trailer. The path of the hitch point becomes the steering curve of the trailer whose rear axle will follow the tractrix of the hitch point. The problem of determining offtracking for multiple unit vehicle combinations is solved by finding the solution of a consecutive series of tractrix curves. It is difficult to obtain a closed form solution for the tractrix curve. However, a numerical integration can be easily done for each tractrix. The steering curve for each trailer hitch point is determined through a simple curve fitting technique.

The method presented here represents the vehicle combination in what is known as a bicycle wheel model [8]. Each axle group is represented by a single wheel at the centerline of the longitudinal axis of the vehicle unit. Tandem axles are represented by a single wheel at the centerline of the tandem axle group. The bicycle wheel model may not be appropriate for vehicle units that have axles within a group that are spaced relatively far apart [6]. For tandem axles that are spaced on 1.2 to 1.5 meter centers, the bicycle wheel model is adequate.

For a single unit vehicle combination, the formulation of the general tractrix problem begins with the definition of the tractrix angle α The tractrix angle is defined as the angle between the vehicle unit heading and the path heading. For a curve, the path heading is the heading of a line that is tangent to the curve at the location of the centerline of the front axle (Figure 1).

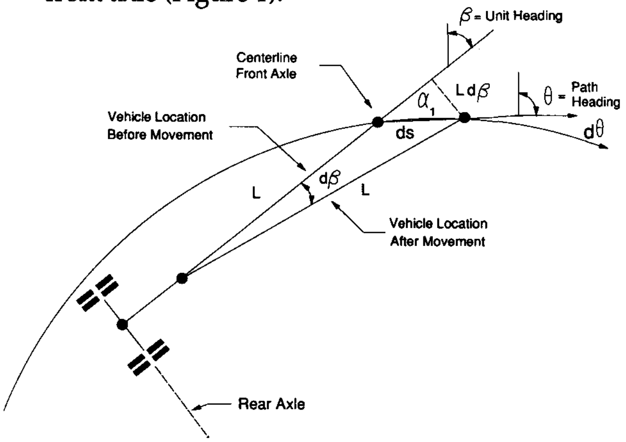


Figure 1. General Tractrix Development

The general tractrix equation is formulated by moving the centerline of the front of the vehicle unit ahead along the steering curve a small distance *ds*, Figures 1 and 2. The rear of the vehicle unit will then begin to follow the direction β that the vehicle unit was headed previously [2]. The incremental distance *ds* should be no more than 0.30 meters or the assumptions of small angle theory will not apply [8]. For very small movements, the geometry will approach the situation as shown in Figure 2.

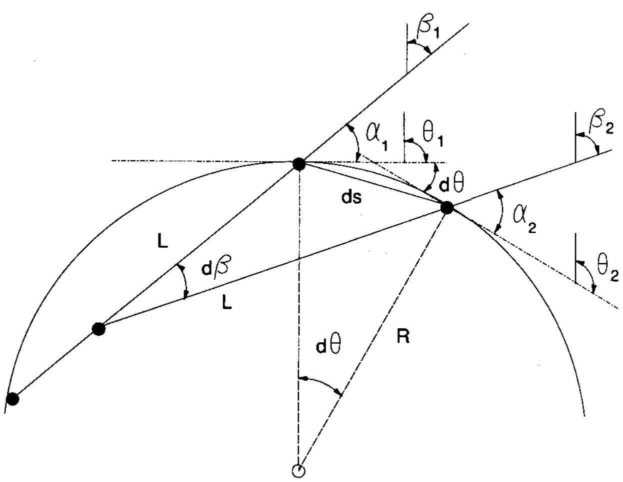


Figure 2. Tractrix Development

The relationship between the change in tractrix angle, $d\alpha$, vehicle unit length, L, instantaneous steering curve radius, R, and the current tractrix angle, α_1, can be derived from Figures 1 and 2 as follows:

$$d\theta = \frac{ds}{R} \qquad (1)$$

For very small angles of $d\theta$:

$$Ld\beta \approx ds \sin \alpha_1 \qquad (2)$$

From Figure 2, $d\beta$ is defined as:

$$d\theta = \theta_2 - \theta_1$$

$$\theta_1 = \theta_2 - d\theta \qquad (3)$$

$$\theta_1 = \beta_1 + \alpha_1 \qquad (4)$$

$$\theta_2 = \beta_2 + \alpha_2 \qquad (5)$$

Setting equation (4) equal to equation (3) yields:

$$\beta_1 + \alpha_1 = \theta_2 - d\theta$$

$$\theta_2 = \beta_1 + \alpha_1 + d\theta \qquad (6)$$

Setting equation (6) equal to equation (5) yields:

$$\beta_1 + \alpha_1 + d\theta = \beta_2 + \alpha_2$$

$$(\beta_1 - \beta_2) + (\alpha_1 - \alpha_2) + d\theta = 0$$

$$d\beta = d\theta - d\alpha \qquad (7)$$

Substituting equation (7) into equation (2) yields:

$$Ld\theta - Ld\alpha = ds \sin \alpha_1 \qquad (8)$$

For a circular curve, substituting equation (1) into equation (8) yields:

$$\frac{Lds}{R} - Ld\alpha = ds \sin \alpha_1$$

$$d\alpha = ds \left[\frac{\frac{L}{R} - \sin \alpha_1}{L} \right] \qquad (9)$$

Equation (9) is the differential equation to obtain the next tractrix angle when the steering point of the vehicle unit is moved up on the curve.

Equation (7) is used to update the heading of the vehicle unit. A standard latitude and departure coordinate system is used to record hitch, axle, and wheel locations. Adoption of a sign convention allows equation (9) to be used for simulation of road segments with reversing curves. The sign convention used is defined as clockwise curves have a positive radius while counter-clockwise curves have a negative radius in the direction of travel.

For all vehicle units except the lead unit, equation (9) is not absolute. This is because the input radius is not of a circular curve but that of a tractrix. However, if the incremental distance *ds* is kept small it will approach a circular curve. The equations presented above are similar to those presented by Della-Moretta and Cisneros [2] for a single unit. The methodology developed here uses equations (7) and (9) for multiple unit vehicle combinations by linking the units through a series of steering curves.

Instantaneous Center of Rotation

When a multiple unit vehicle combination travels through a curve, it is an assemblage of rigid bodies each rotating about their own instantaneous center (Figure 3). If the instantaneous center of each vehicle unit can be determined, then the instantaneous radius of the steering curve for each vehicle unit can be found. This instantaneous radius is required for numerical integration of equation (9) for each vehicle unit. Equation (9) also requires the distance *ds* that the hitch point has moved. This can be found from the coordinates of the current and previous hitch location as shown in Figure 4.

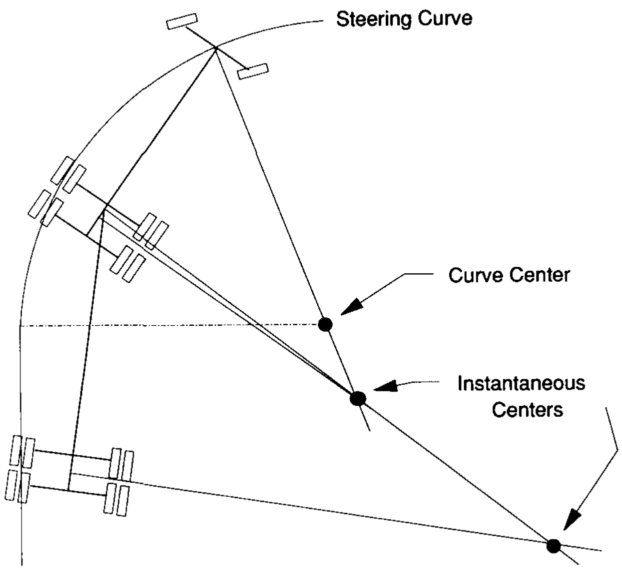


Figure 3. Instantaneous Centers of Rotation

The instantaneous radius of the steering curve can be found directly from the coordinate locations of the hitch point. The current and the last two coordinate locations of the hitch point can be used to approximate a circular segment. For uniform movements of the hitch point, the geometry will approach that of Figure 4.

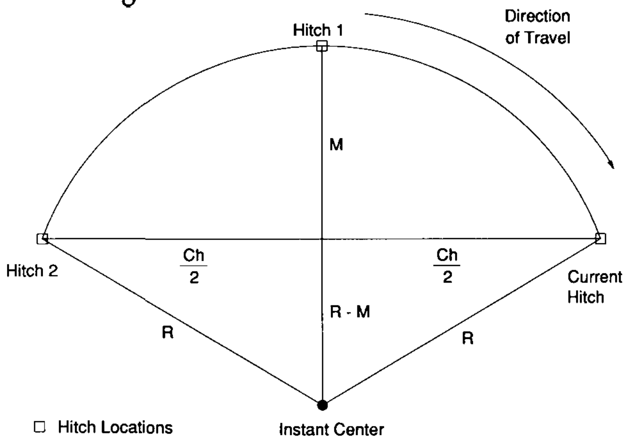


Figure 4. Instantaneous Radius by Three Points on Curve

The instantaneous radius is found as:

$$R = \frac{M}{2} + \frac{Ch^2}{8M} \qquad (10)$$

The middle ordinate M is found by finding the shortest distance from the previous hitch location (Hitch 1) to the chord by using standard analytical geometry. The chord distance Ch is found by using the coordinates of the current and second previous (Hitch 2) hitch locations.

The sign of the instantaneous radius is also needed for equation (9). The sign can be determined by noting which side of the chord the previous hitch point (Hitch 1) is located on.

Variable Length Vehicle Units

Log truck trailers that are commonly used in the western United States do not have a constant length when the vehicle combination negotiates a curve. Friction, developed between the bunks and the logs, binds the trailer to the tractor. The geometry of a variable length log truck trailer is shown in Figure 5. The bunk to bunk distance, L_3, remains constant while the trailer length increases by means of a sliding compensator. The instantaneous length of the trailer, L_4, can be determined by using the headings of the tractor, $ß_1$, and trailer, $ß_2$, at a given point in time.

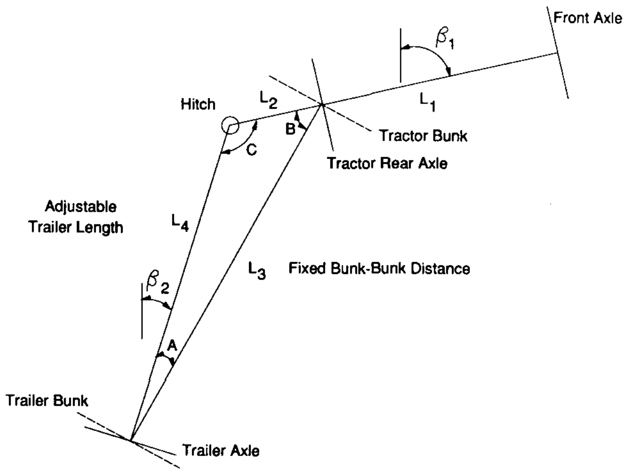


Figure 5. Variable Length Trailer

The adjustable trailer length L_4 is found by:

$$C = 180° - (\beta_1 - \beta_2)$$

$$A = \sin^{-1}\left[\frac{L_2 \sin C}{L_3}\right]$$

$$B = 180° - (A + C)$$

$$L_4 = \sqrt{L_2^2 + L_3^2 - 2L_2L_3 \cos B} \qquad (11)$$

Front and Rear Load Overhangs

Front and rear overhang can limit vehicle passage where cut slopes or other obstacles exist at the edge of the roadway. Large front overhang such as a yarder tower on a rubber tired undercarriage is shown in Figure 6.

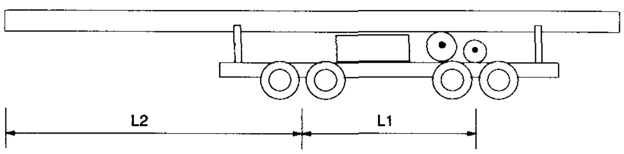


Figure 6. Typical Load Overhang

The coordinates of a load overhang path can be determined at any point from the heading ß of the vehicle unit that it is carried on and the distance that it extends from the front or rear of the vehicle unit.

Comparison with Experimental Data

The predictions from the OFFTRACK program were compared to experimental measurements from several field studies and scale model simulators. The speeds of the test vehicles in the actual equipment tests were not given in the literature [1, 5]. A description of each test situation and the results follow.

1) Log Truck Field Test

A test was conducted with a loaded log truck by the USDA Forest Service in cooperation with Champion International at an unpaved log yard in Bonner, Montana [1]. The test was conducted by marking curves with radii of 15.24 to 38.10 meters at centerline and central angles of 19 to 180 degrees.

The truck was driven around each curve with the driver attempting to keep his left front tire on the mark. The path of the rear most trailer tire was noted as the wheel track left in the dirt yard and the offtracking recorded. The authors noted that measurements could be in error by as much as 0.15 meters. The truck used in the test had the dimensions as shown in Figure 7. A comparison of the offtracking from the OFFTRACK program and the field study for a 15.24 meter radius curve at centerline is given in Table 1.

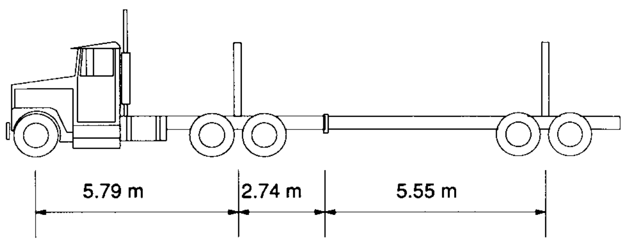


Figure 7. Test Log Truck Dimensions

Table 1. Comparison Of Offtracking (meters) Between USDA Log Truck Field Test And The OFFTRACK Program For A 15.24 Meter Radius Curve.

		Central Angle Ahead of P.C. (deg)							
Source	Curve Central Angle	0	19	30	46	64	90	116	180
Field Test	19	.55	.64						
OFFTRACK		.64	.70						
Field Test	30	.67	1.13	1.07					
OFFTRACK		.70	1.10	.98					
Field Test	46	.67	1.40	1.52	1.13				
OFFTRACK		.73	1.40	1.46	1.19				
Field Test	64	.67	1.40	1.74	1.74	1.34			
OFFTRACK		.67	1.37	1.58	1.68	1.34			
Field Test	90	.67	1.40	1.74	1.77	1.92	1.46		
OFFTRACK		.67	1.37	1.62	1.83	1.92	1.46		
Field Test	116	.67	1.40	1.74	1.77	1.95	1.83	1.46	
OFFTRACK		.67	1.37	1.62	1.80	1.95	2.04	1.52	
Field Test	180	.67	1.40	1.74	1.77	1.95	1.83	2.07	1.43
OFFTRACK		.67	1.37	1.62	1.80	1.95	2.04	2.07	1.55

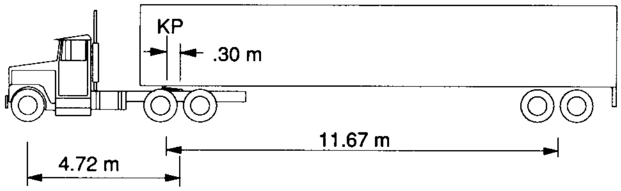


Figure 8. Caltrans Semi Tractor Trailer

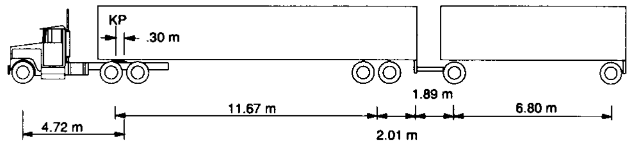


Figure 9. CalTrans Rocky Mountain Double

The data in Table 1 indicate a good correlation between the field test and the OFFTRACK program. Most of the measurements agree within 0.10 meters and the maximum deviation is 0.21 meters.

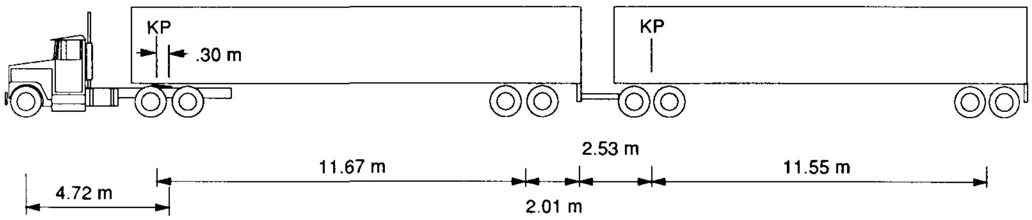


Figure 10. CalTrans Turnpike Double

2) CalTrans Operational Test

The California Department of Transportation (CalTrans) conducted offtracking tests of a semi tractor trailer, Rocky Mountain Double, Turnpike Double, and Triple combinations in 1984 [5] (Figures 8 to 10). The tests were conducted with Viking Freight Systems, Inc. of Santa Clara, California in a paved yard. A line was painted for 18.29, 24.38, and 30.48 meter radius curves over a 180 degree central angle. The driver entered the curve on a tangent, placed the left front tire on the painted line through the curve and then left on a tangent. The path of the right rear most tire was continually marked on the pavement. The OFFTRACK program was used to simulate the semi tractor trailer, Rocky Mountain Double, and the Turnpike Double. The dimensions of the vehicles tested are shown in Figures 8 to 10. KP in the figures is the kingpin of a trailer. All vehicles had a trailer width of 2.59 meters and a tractor front axle tread width of 2.06 meters.

The field test recorded swept width as the difference in radii measuredto the left front wheel and the right rear wheel from the center of the curve.

Measurements were recorded in 30 degree increments of central angle through the curve. A comparison of the OFFTRACK program results with the CalTrans study for singles and doubles combinations is given in Tables 2 through 4.

Most of the swept width measurements in Tables 2 through 4 agree within 0.30 meters and the maximum deviation is 1.16 meters. In most cases the OFFTRACK program over predicts the swept width.

3) Scale Model Tests

Scale models were also used to test the methodology presented here. Thepredicted swept widths from the OFFTRACK program were compared to the Drafting Model Simulator (DVS) [4] and the

Table 2. Comparison Of Swept Width (meters) BetweenCal-Trans Field Test And The OFFTRACK Program For A 18.29 Meter Radius Curve.

		Central Angle Ahead of P.C. (deg)						
Source	Vehicle	0	30	60	90	120	150	180
Field Test OFFTRACK	Semi Tractor Trailer	3.69 3.96	5.61 5.76	6.55 6.67	7.16 7.16	7.44 7.47	7.62 7.41	6.49 6.16
Field Test OFFTRACK	Rocky Mtn. Double	4.60 4.88	6.86 7.01	8.08 8.26	8.75 8.96	9.17 9.33	8.81 8.96	7.19 7.34
Field Test OFFTRACK	TurnPike Double	5.46 6.31	8.60 9.23	10.61 11.16	11.83 12.34	12.28 12.74	11.70 12.22	9.57 10.24

Table 3. Comparison Of Swept Width (meters) Between CalTrans Field Test And The OFFTRACK Program For A 24.38 Meter RadiusCurve.

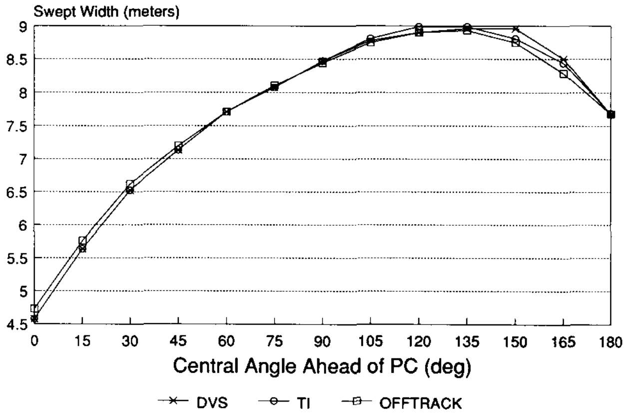
		Central Angle Ahead of P.C. (deg)						
Source	Vehicle	0	30	60	90	120	150	180
Field Test OFFTRACK	Semi Tractor Trailer	3.26 3.50	4.91 5.06	5.55 5.67	5.82 5.91	5.97 6.00	6.03 6.00	4.94 4.97
Field Test OFFTRACK	Rocky Mtn. Double	3.81 4.08	5.82 5.97	6.52 6.77	6.92 7.10	7.13 7.25	6.92 7.13	5.12 5.67
Field Test OFFTRACK	TurnPike Double	4.39 4.91	7.01 7.44	8.26 8.67	8.87 9.33	9.27 9.48	8.96 9.36	6.22 7.38

Table 4. Comparison Of Swept Width (meters) Between CalTrans Field Test And The OFFTRACK Program For A 30.48 Meter Radius Curve.

		Central Angle Ahead of P.C. (deg)						
Source	Vehicle	0	30	60	90	120	150	180
Field Test OFFTRACK	Semi Tractor Trailer	3.54 3.26	4.75 4.63	5.06 5.03	5.18 5.15	5.15 5.18	5.27 5.18	4.18 4.36
Field Test OFFTRACK	Rocky Mtn. Double	3.72 3.69	5.49 5.39	5.88 5.91	5.94 6.07	6.00 6.13	6.04 6.10	5.00 4.88
Field Test OFFTRACK	TurnPike Double	4.39 4.30	6.55 6.55	7.32 7.44	7.59 7.77	7.74 7.92	7.71 7.80	5.82 6.07

and the Tractrix Integrator (TI) [7]. The test consisted of a 14.63 meter radius curve on the outside front wheel through a 180 degree central angle with tangents on both ends. The vehicle used was a tractor trailer with a 5.49 meter tractor wheelbase, 10.97 meter trailer length and the fifth wheel directly over the centerline of the tractor tandems. The test results shown in Figure 11 show a good correlation with the maximum deviation being 0.21 meters. Comparisons of swept widths using the DVS for multiple road segments of compound and reverse curves were within 0.15 meters.

Figure 11. Comparison of Scale Models to OFFTRACK



DISCUSSION AND CONCLUSIONS

The OFFTRACK program simulations provided results within 0.15 meters of both scale model simulators and the log truck field measurements. There is however an over estimation of swept widths for the doubles combinations in the CalTrans test. Program results compare favorably except for the shorter radius curves with the turnpike doubles combination where overestimates of swept width of 0.61 meters are made. This disparity is probably due to slip and scrubbing of the tires or possibly misalignment of axles [6]. The predicted offtracking for the field test on the log truck in a unpaved yard was very close to measured. This appears to support the influence of tire mechanics because of the lower coefficient of friction associated with the unpaved yard versus the paved yard. The over prediction of swept width in the CalTrans test data is also consistent with that reported by several authors when comparing scale model simulators to actual equipment tests [1, 7]. In general, the methodology presented appears to predict offtracking with reasonable accuracy for use as a road design tool.

Copies of the OFFTRACK program can be obtained by sending a self addressed mailer with a MS-DOS formatted diskette to Program Leader, Advanced Training Group, USDA Forest Service, Room 267, Peavy Hall, Oregon State University, Corvallis Or 97331.

NOTATION AND DEFINITIONS

Unless otherwise noted all angles are in radians and distances are in meters.

α	Tractrix angle. Angle between the vehicle unit heading and a tangent to the steering curve.
$d\alpha$	Change in the tractrix angle after movement.
α_1, α_2	Tractrix angle before and after movement respectively.
θ	Path heading, measured as an azimuth.
$d\theta$	Change in the path heading after movement.
θ_1, θ_2	Path heading before and after movement respectively measured as an azimuth.
β	Vehicle unit heading, measured as an azimuth.
$d\beta$	Change in the vehicle unit heading after movement.
β_1, β_2	Vehicle unit heading before and after movement respectively measured as an azimuth.
R	Instantaneous radius of a steering curve.
ds	Incremental distance that the front of a vehicle unit is moved along it's steering curve.
L	Length of a vehicle unit.
M	Middle ordinate of the curve defined by three consecutive locations of a hitch point.

Ch — Chord distance between the first coordinate pair and third coordinate pair of a hitch point.

A, B, C, — Interior angles formed by the center line of the logs, trailer, and truck frame from the tractor rear axle to the hitch point for a variable length trailer.

Swept Width — The difference in radii measured to the outside front wheel and the inside rear wheel from the center of a curve.

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