Optimum Spur Road Layout Near A Forest Boundary Line

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ABSTRACT

A simple method to determine optimum spur road length and spacing in the vicinity of a forest boundary line is presented. Sample problems are solved to illustrate the method.

Key Words: *Forest Engineering, Forest Roads, Forest Operations, Logging, Timber Production*

INTRODUCTION

The problem of optimum road and landing spacing for unbounded forest areas has been treated by numerous authors. A summary of the contributions to this problem has recently been published[3]. Nieuwenhuis [1] has added an extra dimension to this classic problem by considering the optimum spur road length and spacing near a forest boundary line. He proposed maximizing the area accessed per unit length of spur road as an optimal solution when road costs are being minimized constrained by a maximum allowable skidding distance, and assuming negligible landing costs. An alternate method leading to optimum spur road length and spacing for minimum road, landing, and skidding costs is presented in this paper.

ANALYSIS

The harvest geometry is shown in Figure 1. The spur road length and spacing that minimizes cost per unit volume for given road cost, landing cost, and variable skidding cost are desired. Since spur road length is (D-Y) and spur road spacing is

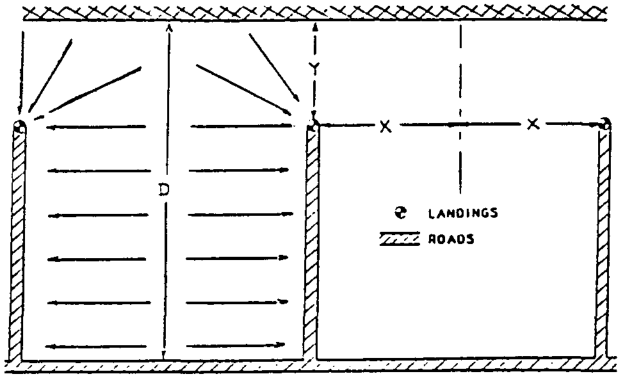

Fig. 1. *Harvest unit geometry with a forest boundary line.*

(2X), the equivalent problem is to find that combination of X and Y that minimizes cost per unit volume. The analysis method is to:

1. Write the cost equation.
2. Set the derivative of cost with respect to $x = X/D$ equal to zero.
3. Set the derivative of cost with respect to $y = Y/D$ equal to zero.
4. Solve the resulting equations simultaneously for optimum x and optimum y.

The total variable cost to harvest the unit is given by:

$$\begin{aligned} C_T &= C_R(D-Y)+C_L + \\ &C_Y(X/2)V(D-Y)(2X)+ \\ &C_Y(ASD)V(2XY) \end{aligned} \quad (1)$$

where
C_T = total variable cost

C_R = road cost per unit length

C_L = landing cost

C_Y = variable skidding cost per unit length

V = volume removed per unit area

ASD = average skid distance, where

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$$ASD = (1/6)\,[2Q + (X^2/Y)\ln((Q+Y)/X) + (Y^2/X)\ln((Q+X))Y],\ [2] \tag{2}$$

$$Q^2 = (X^2 + Y^2) \tag{3}$$

The cost per unit volume is obtained by dividing equation (1) by the total volume removed, 2VXD. Therefore,

$$C_V = (C_Y D/2)\Big[(S_R/D)^2(1-y)/ + (S_L/D)^3/x + x(1-y) + (y/3)\{2q + (x^2/y)\ln((q+y/x)) + (y^2/x)\ln((q+x))/y\ \}\Big] \tag{4}$$

Where C_V = total variable cost per unit volume removed

$$(S_R/D)^2 = (C_R/VC_Y)/D^2$$

$$(S_L/D)^3 = (C_L/VC_Y)/D^3$$

$$q^2 = x^2 + y^2$$

$$x = X/D$$

$$y = Y/D$$

The optimization problem becomes one of determining x and y for given values of (S_R/D) and (S_L/D) that minimize C_V.

For optimum $x = \tilde{x}$, set $dC_V/dx = 0$, resulting in

$$-(S_R/D)^2(1-y)/\tilde{x}^2 - (S_l/D)^3/\tilde{x}^2 + (1-y) + (y/3)(q_1/\tilde{x}) + (y/3)\{(2\tilde{x}/y)\ln((y+q_1)/\tilde{x}) - (y^2/\tilde{x}^2)\ln((\tilde{x}+q_1)/y)\ \} = 0 \tag{5}$$

where $q_1^2 = \tilde{x}^2 + y^2$

Similarly, for optimum $y = \tilde{y}$, set $dC_V/dy = 0$

$$-(S_R/D)^2 / x - x + q_2 + (\tilde{y}^2/x)\ln((x+q_2)/\tilde{y}) = 0 \tag{6}$$

where $q_2^2 = x^2 + \tilde{y}^2$

Require that equations (5) and (6) be simultaneously satisfied to determine $\tilde{x}$ and $\tilde{y}$ that yield minimum C_V. In equation (5) let $y = \tilde{y}$; in equation (6) let $x = \tilde{x}$; also let

$$\tilde{y}/\tilde{x} = \tan\theta \tag{7}$$

Substituting into equation (6) and rearranging,

$$(S_R/D)^2/\tilde{x}^2 = -1 + \sec\theta + \left(\tan^2\theta\right)^* \ln((1+\sec\theta)/\tan\theta) = \theta_1^2 \tag{8}$$

Similarly, equation (5) can be re-arranged and combined with equation (8) to obtain:

$$\theta_2 + \left((1-\theta_1^2)/\theta_1^2\right)/ (S_R/D) - (S_L/D)^3/(S_R/D)^3 = 0 \tag{9}$$

where $\theta_2 = \left(2\tan\theta/3/\theta_1^3\right)^* \left[\theta_1^2 + \sec\theta - 2 + (1/\tan\theta)\ln(\tan\theta + \sec\theta)\ \right]$

Equation (9) is a transcendental, nonlinear equation in the variable θ and the given quantities, S_L/D and S_R/D. An efficient, stable iterative method for solving equation (9) for θ is the method of false positions using θ = 1° and θ = 89° as the first two guesses. Once θ is determined, equation (8) can be used to solve for $\tilde{x}$ and equation (7) can be used to solve for $\tilde{y}$. Optimum harvest unit layout ($\tilde{x}$ and $\tilde{y}$) as a function of given cost values (S_R/D and S_L/D) is presented in Tables 1 and 2 (see page 7).

SOLUTION PROCEDURE

To solve for optimum spur road length and spacing, we propose this direct method:

1. Determine the quantities V, C_R, C_Y, C_L, and D.
2. Calculate $S_R/D = (C_R/VC_Y)^{1/2}/D$ and $S_L/D = (C_L/VC_Y)^{1/3}/D$.
3. Use Table 1 to determine $\tilde{x}$
4. Use Table 2 to determine $\tilde{y}$

5. Calculate the optimum spur road length = $D(1-\tilde{y})$.

6. Calculate the optimum spur road spacing = $2\tilde{x}D$.

SAMPLE PROBLEMS

Case 1. Find optimum layout and minimum cost when:

1. $V = 10$ MBF/acre
   $D = 3000$ ft
   $C_R = \$2.00/\text{ft}$
   $C_L = \$1000$
   $C_Y = \$.01/\text{MBF/ft}$

2. $S_R/D = [(\$2.00/\text{ft})(\text{acre}/10\text{ MBF})(\text{MBF-ft}/\$.01)(43560\text{ft}^2/\text{acre})]^{1/2}/3000\text{ ft}$
   $S_R/D = .311$
   $S_L/D = [(\$1000)(\text{acre}/10\text{ MBF})(\text{MBF-ft}/\$.01)(43560\text{ft}^2/\text{acre}])^{1/3}/3000$
   $S_L/D = .253$

3. $\tilde{x} = .30$

4. $\tilde{y} = .26$

5. Optimum spur road length = 3000(1 - .26) = 2200 feet

6. Optimum spur road spacing = 2(.30)3000 = 1800 feet

The corresponding minimum cost per volume skidded is equation (4):

$$C_V = (\$.01/\text{MBF-ft})(3000\text{ ft}/2)[(.311)^2(.74/.30) + ((.253)^3/.30) + (.30)(.74) + (.26/3)\{2(.40) + ((.30)^2/(.26))\ln((.40 + .26)^2/.30)) + ((.26)2/(.30))\ln((.40 + .30)/.26)\}]$$

$$C_V = \$15.00/\text{MBF}(.239 + .054 + .222 + .087[.80 + .27 + .22])$$

$$C_V = \$/\text{MBF}(3.59 + .81 + 3.33 + 1.68) = \$9.41/\text{MBF}$$

Therefore, the minimum cost corresponding to the optimum harvest layout was \$9.41/MBF. The contribution to this cost was spur road = \$3.59/MBF (38%), landing = \$.81/MBF (9%), skidding to road = \$3.33/MBF (35%), and skidding to landing = \$1.68/MBF (18%). Fixed costs of skidding associated with hooking and unhooking times should be added to \$9.41/MBF, since this figure does not include fixed costs.

Case 2. Illustrating a problem in different units. Find optimum layout and minimum cost when:

1. $V = 200\text{m}^3/\text{ha}$
   $D = 500\text{m}$
   $CR = \$3.00/\text{m}$
   $CL = \$1000$
   $CY = \$.006/\text{m}/\text{m}^3$

2. $S_R/D = [(\$3.00/\text{m})(\text{ha}/200\text{m}^3)(\text{m-m}^3/\$.006)(10^4\text{m}^2/1\text{ha})]^{1/2}/500\text{m}$
   $S_R/D = .316$

   $S_L/D = [(\$1000)(\text{ha}/200\text{m}^3)(\text{m-m}^3/\$.006)(10^4\text{m}^2/\text{ha})]^{1/3}/500\text{m}$

   $S_L/D = .405$

3. $\tilde{x} = .38$

4. $\tilde{y} = .24$

5. Optimum spur road length = 500(1-.24) = 380m

6. Optimum spur road spacing = 2(.38)500 = 380m

The minimum cost is:

$$C_V = (\$.006/\text{m}^3\text{-m})(500\text{m}/2)[(.316)^2(.76/.38) + ((.405)^3/.38) + .38(.76) + (.24/3)\{2(.45) + ((.38)^2/.24)\ln((.45 + .24)/.38) + ((.24)^2/.38)\ln((.45 + .38)/.24)\}]$$

$$C_V = \$1.50/\text{m}^3[.200 + .175 + .289 + .080\{.90 + .359 + .188\}]$$

$$C_V = \$/\text{m}^3(.30 + .26 + .43 + .17) = \$1.16/\text{m}^3$$

CONCLUSION

A simple method to determine optimum spur road length and spacing in the vicinity of a forest boundary line is presented. Sample problems are solved to illustrate the method.

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Table 1. Optimum spur road spacing variable, $\tilde{x}$, as a function of dimensionless cost ratios, S_R/D and S_L/D.

	S_R/D									
SL/D	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.00	0.10	0.19	0.27	0.34	0.39	0.43	-0-	-0-	-0-	-0-
0.10	0.10	0.19	0.27	0.34	0.39	0.43	0.07	0.07	0.07	0.07
0.20	0.13	0.21	0.28	0.35	0.40	0.44	0.16	0.16	0.16	0.16
0.30	0.19	0.25	0.32	0.38	0.43	0.47	0.47	0.26	0.26	0.26
0.40	0.27	0.32	0.37	0.43	0.48	0.52	0.53	0.36	0.36	0.36
0.50	0.37	0.40	0.45	0.50	0.54	0.58	0.61	0.60	0.47	0.47
0.60	0.47	0.50	0.54	0.58	0.63	0.66	0.69	0.70	0.66	0.59
0.70	0.59	0.62	0.65	0.69	0.72	0.76	0.79	0.80	0.79	0.73
0.80	0.72	0.74	0.77	0.80	0.83	0.87	0.89	0.91	0.92	0.89
0.90	0.86	0.88	0.90	0.93	0.96	0.99	1.01	1.03	1.04	1.04
1.00	1.00	1.02	1.04	1.06	1.09	1.12	1.14	1.16	1.18	1.18

Table 2. Optimum spur length variable, $\tilde{y}$, as a function of dimension less cost ratios, S_R/D and S_L/D.

	S_R/D									
SL/D	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.00	0.08	0.17	0.26	0.36	0.46	0.59	1.00	1.00	1.00	1.00
0.10	0.08	0.17	0.26	0.36	0.46	0.59	1.00	1.00	1.00	1.00
0.20	0.07	0.16	0.26	0.35	0.46	0.58	1.00	1.00	1.00	1.00
0.30	0.07	0.15	0.24	0.34	0.44	0.56	0.70	1.00	1.00	1.00
0.40	0.06	0.14	0.23	0.32	0.42	0.53	0.66	1.00	1.00	1.00
0.50	0.06	0.13	0.21	0.30	0.40	0.50	0.62	0.76	1.00	1.00
0.60	0.05	0.12	0.20	0.29	0.38	0.48	0.58	0.71	0.86	1.00
0.70	0.05	0.12	0.19	0.27	0.36	0.45	0.55	0.66	0.79	0.96
0.80	0.05	0.11	0.19	0.26	0.35	0.43	0.53	0.63	0.74	0.87
0.90	0.05	0.11	0.18	0.25	0.33	0.42	0.51	0.60	0.70	0.82
1.00	0.05	0.11	0.17	0.24	0.32	0.40	0.49	0.58	0.67	0.78