Book Reviews

Fractals in the Earth Sciences

Edited by Christopher C. Barton and
Paul R. LaPointe
Plenum Press, New York
1995, 264 p., US$59.50

Fractals in Petroleum Geology and Earth Processes

Edited by Christopher C. Barton and
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These two symposia were published in the same year, but internal evidence suggests that *Fractals in the Earth Sciences* was prepared first, in about 1991, and was originally supposed to be published as a GSA *Memoir* (it is referenced that way by one of the authors, on p. 203). It has several useful review papers, but there are few references from the 1990s. *Fractals in Petroleum Geology...* apparently came later, and has references up to 1992 (and a few up to 1993). Together, the volumes constitute a useful addition to the literature on applications of fractals to the earth sciences, but the delay in publication is regrettable for such a fast-moving field. Although the first volume concentrates on general reviews and the fractal properties of faults and other fractures, and the second volume has articles on hydrocarbon accumulations, porous media, and stratigraphy, the scope of both volumes is quite large and not confined to the topics I have just listed. Indeed, one might claim that the main distinction is that the second volume includes some important topics not found in the first: notably multifractals, and the use of fractals in simulation.

Few earth scientists have received any formal instruction about fractals. Most are vaguely aware of their existence, generally as pretty pictures, but few have any interest in the application of fractals in the sciences (a graph in the second volume, p. xv, nicely quantifies the way the earth sciences have lagged behind physics and chemistry in fractal studies). The few enthusiasts for research on fractals are often regarded with considerable suspicion by their colleagues (fractals have the distinction of having been mocked by a satirical abstract, published in the AGU *Transactions*!). Partly this may be attributed to the resistance shown to any new branch of science, partly it is a reaction to the exaggerated claims of fractal fanatics, and partly it is due to the perception, only partly mistaken, that most applications of fractals are descriptive, with little foundation in scientific theory. The title of a famous editorial in *Physics Today* was "Fractals: Where's the Physics?". The author of that editorial, Leo Kadanov, however, went on to do research on fractals, and the subject has caught the attention of many other excellent scientists, so fractals are definitely not pseudo-science, or trivia, and some knowledge of the subject ought quickly to become part of the intellectual equipment of most earth scientists.

The first of these two volumes is a good place to start. The first paper is a useful, if somewhat uncritical, review of the whole field by Turcotte and Huang. An article by Pruess suffers from the usual disadvantages of presentations by mathematicians, brevity and a parsimonious use of examples and illustrations, but it will repay careful reading. Sooner or later anyone reading about fractals will run into the Hausdorff measure and dimension: topics highly unlikely to appeal to or be comprehensible to earth scientists. They are given a brief treatment in this paper (which most readers will probably not grasp), followed by a discussion of three practical techniques that may be used to estimate the Hausdorff dimension. Application of these three techniques to estimate the dimension of a straight line (known theoretically to be equal to unity) gives values ranging from 0.85 to 1.06. The value of this paper is that it gives some theoretical insight into what most potential users of fractals have discovered the hard way: it is very difficult to obtain practical estimates of fractal dimension that come within 10% of the "true" value.

There are three main aspects to the scientific (as opposed to mathematical) study of fractals: 1) how do we know that a real object is fractal (and what is the significance of this identification, if it can be made)? 2) how can we best measure the fractal dimension, and are there other scientifically significant measurements that might be used to characterize the object? and 3) how are natural fractals generated?

In practical applications, fractals are objects that show some sort of self-similarity over at least a part of their range of scales. Self-similarity (or self-affinity, if scaling varies in different dimensions, as it generally does in surface profiles) is generally demonstrated by showing straight-line log-log plots. The most famous of these was the first: the length of the coastline of Britain, as a function of the size of the divider "step" used to measure it. But does a power law really *prove* self-similarity? I think not, although the editors, in a paper in the second volume (p.14), state categorically "A fractal distribution is defined where a number of objects *N(r)* with a characteristic linear dimension equal to or greater than *r* satisfies (a power law)." This definition implies that regular geometric objects, including straight lines (see above), and objects randomly located in space, are fractals, which is an extreme view, to put it mildly.

Log-log plots, with straight lines fitted to them by linear regression (a highly dubius procedure, as acknowledged by a few of the more thoughtful practitioners, who go right on doing it) are the stock-in-trade of most fractologists. Sometimes the reaction of any sensitive reader should be to gag on what is being pushed down his throat. An example capable of inducing such a reaction, in an otherwise interesting paper, is found in the paper by Fowler in the first volume (the paper by Meakin and Fowler in the second volume is a far more extensive and interesting treatment of the same types of phenomena). Figure 12.6 on p. 244 presents a picture of an almost perfectly euhedral plagioclase crystal, and a log-log plot produced by a box count of the digitized outline of this single crystal, which it is claimed shows that it has a fractal dimension of 1.1. It is obvious to this reader that: 1) this is not an object that shows self similarity or fractal geometry; 2) that the log-log plot shows only a central region that might possibly be fitted by a straight line; and 3) if it was, the line would have a slope not significantly different from one, which is what one would expect from any Euclidean, non-fractal object. My biased advice is to ignore any correlation coefficients or standard errors presented in support of power-law plots; they are all devoid of real statistical meaning (a valid statistical theory for fractals is just beginning to be developed, notably by the Canadian statistician Colleen Cutler).

Nevertheless, some natural phenomena do show a close approximation to power laws that relate the cumulative number or size of objects to the size of the measuring stick, or the feature being measured. The slope of such plots, interpreted as a fractal dimension, indicates that the dimension is fractional. The best known example, discussed by many writers, is the Gutenberg-Richter law relating the number of earthquakes in a given period and area to their magnitude. Scientists owe to Benoit Mandelbrot the realization that power-law phenomena are much commoner than was once thought, and that they have some very peculiar properties. For example, it is (theoretically) impossible to calculate a meaningful "average" or "variance" for the magnitude of earthquakes; if calculated in the usual way, both statistics are simply a function of the duration and sensitivity of the observations.

A power law differs significantly from statistical distributions, (such as the Lognormal distribution), that have often been used to describe nature frequency distributions. Disputed distributions include the size of floods and other hydrologic phenomena, size distributions (of grains, oil fields, and ore bodies), and stratigraphic thickness distributions. Proving conclusively that an observed distribution is a power law (or any other theoretical distribution) is, however, not an easy task. Natural fractals, in particular, are expected to be self similar, and therefore to show power laws, *only* over a part of their size distribution. Coastlines, for example, have a scale limited at the top by the size of the object (*e.g.,* Britain), and at the bottom by the fact that self-similarity does not extend to small scales, because of the presence of smooth features such as beaches. Yet it is the extremes of the distribution that are most critical in distinguishing power laws from competing statistical distributions.

These problems are discussed in some detail in three papers by Barton and Lapointe, Lapointe, and Crovelli and Barton, in the second volume. I think a reasonable case has been made that petroleum accumulations have a power-law distribution, though I still wonder what this really means about their "self-similarity."

Given that a power law exists, and (I would add) that we have other reasons to conclude that the phenomenon shows self similarity, how can the dimension be measured? The answer is, of course, from the slope of the log-log plot. The most thoroughly explored application is topography, and the problems and limitations of the dozen or so techniques available have become only too apparent in recent years (Malinverno's review in the first volume is excellent, but for a more recent, and more critical, review than any appearing in these two volumes I recommend Klinkenberg's paper in *Mathematical Geology,* v. 26, p. 23-46, 1994). Different methods, all supposedly approximating the Hausdorff dimension, yield different results, and even a single method has difficulty achieving estimates with a precision better than 0.05 (which is not very impressive, when the maximum possible range of values is 1.0). Indeed, it is hard to know just what the accuracy is, since it is hard to produce "standard random fractals" of known fractal dimension: the two commonly used methods are the midpoint displacement and Fourier techniques, and the accuracy of both is suspect. Some well-known methods (*e.g.,* the dividers method) do not work well on exactly self-similar fractals, such as the Koch curve.

One problem with the descriptive use of fractal geometry is that it seems unrewarding to reduce all aspects of topography, for example, to a single dimension. Weissel *et al.* in their paper on the Ethiopean plateau (in the second volume, p. 135) cite Mandelbrot as pointing out that the dimension of an airport runway is the same as that of the Himalayas: a striking, if trivial, observation but one that hardly makes us believe that dimension will tell us much that is scientifically interesting about topography. In fact, their whole paper, although an interesting discussion of the Ethiopean rift flank, seems to me to show that essentially nothing was contributed to our understanding of the region by the (rather half-hearted) attempt to measure the fractal dimension of two selected areas. At the least, one would like to have a richer set of descriptive measures for fractals. Another measure, currently under study but not discussed in these volumes is "lacunarity." More work, however, is being done on the idea that many natural fractals are not adequately described by a single dimension, but need a complete "spectrum of dimensions." Such "multifractals" are discussed in two well-written papers in the second volume, authored by Plotnick and Prestegaard: the first applies the multifractal concept to stratigraphic sequences, and the second to fluctuations of bedload transport in gravel rivers.

Finally, how are natural fractals to be explained? One answer is that they are to be explained, like any other scientific observations, by theoretical models. The range of possible models is already quite large. Some are simple stochastic models, like random walks (and their associated Lévy flights or dusts), or the binomial multiplicative models used to generate multifractals. Others are less simple, but equally stochastic, such as the cellular automata models used to simulate fluid percolation through porous media, or the growth of dendrites and skeletal crystals by "diffusion-limited aggregation." Those who still doubt the scientific potential of fractal studies should read the articles on these two subjects by Feder and Jössang, and by Meakin and Fowler, in the second volume: if these articles do not convince them, then they have probably already aquired a severe case of fractal blindness. Fractal objects ("strange attractors") also appear in the state space of low-dimensional chaotic dynamic systems, but according to Bak and Chen (first volume, p. 233) "The belief that there may be a connection between low-dimensional chaos and fractals is without mathematical foundation." Instead, Bak (see next review) believes strongly that the most common cause of natural fractal objects, including sand avalaches, earthquakes, and many other geological phenomena, is a multidimensional dynamic state poised on the edge of chaos (or catastrophy), which he calls "self-organized criticality."

How Nature Works: The Science of Self-organized Criticality

Per Bak
Copernicus (Springer-Verlag)
New York, 1996, 212 p., US$27.00

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Per Bak is a physicist at Brookhaven, with a sceptical view of scientific institutions, and a low opinion of many of his fellow scientists, apparently especially of geophysicists:

> ... (who) often show little interest in the underlying principles of their science. Perhaps they take it for granted that no general principles apply, and that no general theory...can exist. (p.81)

This book is about a general theory to explain the existence of power laws, like the Gutenberg-Richter law relating the number of earthquakes $N(m)$ with a magitude great than some value m

$$N(m) = am^{-b}$$

A log-log plot of N against m is a straight line with a slope of $-b$. Alternatively, one might plot the magnitude of earthquakes against time: taking a power spectrum of this time series would reveal that the variance in earthquake magnitude was proportional to the frequence (f) raised to some negative power ß. Time series with this type of spectrum are said to show "$1/f$" noise, and have been commonly observed in many fields. $1/f$ noise can be simulated by random walks, and the extended phenomenon was called "fractional Brownian motion" by Mandelbrot and Wallis. One of their pioneer papers about this has been reprinted in the first of the two volumes edited by Barton and LaPointe, and reviewed above. Power laws and $1/f$ noise are now generally thought to be characteristic of fractal objects.

Bak has developed a general theory to explain power laws and $1/f$ noise (and more generally, the complexity of nature): he calls this theory "self-organized criticality" (SOC). Bak argues that complex systems, with many degrees of freedom, that are driven far from equilibrium by the application of some extrinsic but possibly steady force tend

> ...to evolve into a poised "critical" state, way out of balance, where minor disturbances may lead to events...of all sizes... The state is established solely because of the dynamic interactions among individual elements of the system: the critical state is *self-organized*. (p.1-2).

Bak's model for such a system is a sand pile, continually fed by sand added grain by grain (but randomly) close to the apex.

Cellular automata (computer) models of such a system show that it builds up to a critical state, after which avalaching takes place. The timing and size of the avalanches, however, are quite unpredictable, and do not show any natural periodicity; instead, the power spectrum of the time series shows 1/f noise, and the number and size of the avalanches are related by a power law (real sand piles are not as satisfactory, in this respect, as computer ones: see Anita Mehta, ed., Granular Matter, published by Spinger-Verlag, 1994). In Bak's book, he extends the sand-pile model to: earthquakes, cotton prices, extinctions, landscape geometry and evolution, coupled pendulums, turbidite deposition, volcanic eruptions, pulsars, solar flares, evolution (including punctuated equilibria), the brain, and traffic jams. Perhaps you think this ambitious? I can only say that I know of several other published applications that he has omitted.

Most of the topics considered at length in this book are part of the earth sciences; earthquakes, landscape, sedimentation, evolution, and extinction are the major topics. The style is for the most part autobiographical, alternately entertaining and irritating, and at the *Scientific American* level. Bak argues for an approach to complex systems that is necessarily abstract and statistical. He claims that

> ... we must learn to free ourselves from seeing things the way they are!... If... we concentrate on an accurate description of the details, we lose perspective. A theory of life is likely to be a theory of a process, not a detailed account of utterly accidental details of that process... (p.10)

For most geologists, this approach may be one that they have never seriously considered.

My recommendation: read this book, and decide for yourself how valid the approach is. At the very least, it is entertaining to read a book by a physicist who does not believe that meteorite impacts cause extinctions!

The Geology of South Australia: Volume 1. The Precambrian

Edited by J.F. Drexel and A.J. Parker
South Australia Geological Survey
Bulletin 54
1993, hardbound, 242 p., A$75.00, + A$20.00 overseas surface postage and packaging

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This attractive volume provides an up-to-date overview of the Precambrian geology of South Australia (an Australian state roughly comparable in size to British Columbia). Volume 2 will cover the Phanerozoic geology. As mentioned in the introduction, the book is designed to provide the reader with a comprehensive regional account of the products of sedimentation, deformation, metamor-