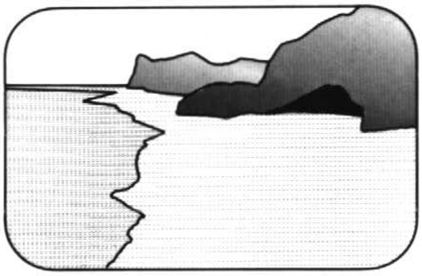

Ocean Tides

Christopher Garrett
Department of Oceanography
Dalhousie University
Halifax, N.S.

> "The great Master of Philosophy drowned himself, because he could not apprehend the Cause of Tydes; but his Example cannot be so prevalent with all, as to put a Period to other Mens Inquiries into this Subject." Richard Bolland, 1675.

This mythical account of Aristotle's death was readily accepted by equally frustrated investigators in the 17th century (Deacon, 1971). Any sensible theory of tides had to await Newton's theory of gravitation, published in his *Principia Mathematica* in 1687. Laplace made major contributions towards the end of the 18th century, and further advances came in the Victorian era, when Lord Kelvin and others devised methods of analysis and prediction of sea level essentially the same as those used today.

The study of tides stagnated during the first half of this century, but has recently undergone a major revival, spurred by technological advances, by the greatly increased number of physical oceanographers seeking fruitful lines of research, and by appreciation of the interaction of ocean tides with other phenomena in oceanography and the earth sciences.

Measurement

The standard tide gauge used in harbours records the height of a float in a "stilling well", connected to the outside water by an orifice small enough to suppress wind-generated waves. In some areas this is being replaced by a pressure transducer, still connected to a pen producing a graph of water level (or bottom pressure = density × gravity × water depth + atmospheric pressure) against time on a strip-chart recorder. But such gauges provide information only about tides at the coast.

Offshore tide gauges measure pressure on the sea floor, employing a variety of different pressure sensors. Some (such as the Canadian gauge shown in *Fig. 1*) are limited to the comparatively shallow water (up to 200 m or so) on the continental shelves, while others are designed for use in the deep ocean, at depths of four km or more. The Cadillac of deep-sea tide gauges is the capsule designed by Snodgrass (1968), which records on computer-compatible tape the frequency of oscillation of a pressure-sensitive quartz crystal, with a sensitivity equivalent to a change in sea level by one mm.

Deployment of these gauges has led to useful measurements of deep-sea tides in a few places, but the processes of equipment loss and changing interests have so far prevented any large scale mapping of deep-sea tides, though this is technically possible.

Laser altimetry from satellites may eventually provide the best way of measuring deep-sea tides, though at present the accuracy is only a couple of metres, largely due to uncertainty in the orbit of the satellite. It should also be borne in mind that a satellite will measure the change in distance of the sea surface from the earth's centre. This is made up of the change in water depth, plus the displacement of the sea floor. These are comparable in some areas.

Figure 1
Offshore tide gauge developed by Tides and Water Levels, Marine Sciences Directorate, Department of the Environment. The inner tube senses pressure changes which are recorded internally on punched paper tape. (Bedford Institute of Oceanography).

Analysis and Prediction

The prediction of tides in a harbour is carried out basically by splitting a record of sea level there into a number (typically 40 or so) of different sine waves, the frequencies of which are dictated by astronomy and are mainly centred around one and two cycles per day (*Fig. 2*). Extrapolating these sine waves into the future and reconstituting them into a single time series provides a prediction of sea level which is published in Tide Tables.

The dominant four semi-diurnal constituents are marked in Figure 2 with their usual symbols. M_2, S_2 are the principal lunar and solar semi-diurnal constituents. N_2 is the larger lunar elliptic constituent; it beats with M_2 over a lunar month to

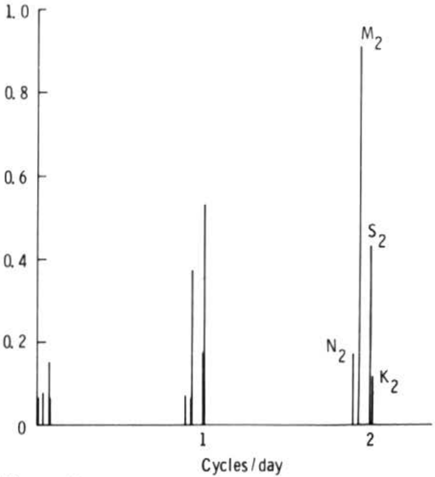

Figure 2
Relative amplitudes of the dominant constituents in the tidal forcing. Latitudinal dependence is different for each of the three bands, near 0, 1, 2cpd, but the same within each band. These, and many other frequencies with smaller forcing amplitudes, may appear in a record of sea level, though in different proportions. Signals at linear combinations of the driving frequencies may also appear, due to nonlinear processes in the ocean.

produce a force that is modulated according to the distance of the moon from the earth. K_2 differs from M_2 by two cycles per lunar month and from S_2 by two cycles per year. It has a part that modulates each in accordance with the movement of the moon and sun north and south of the equator.

Tidal analysis and prediction involves details of considerable complexity (Godin, 1972), and is usually entrusted to a government agency. (Considerable confusion and embarrassment was caused in January 1974 when a US prediction of record tides on the Eastern seaboard led to dire warnings of flooding in the Maritimes. The situation was not improved by the non-availability of Canadian tide tables, the publication of which had been delayed by errors and printers. In fact the predicted high tide for January 9 and 10 at Halifax was 7.8 feet above chart datum, only 1.4 feet above the average high tide of 6.4 feet. It is important to bear in mind that sea level is a combination of tides and meteorologically induced changes, and that the latter can be considerably more important than minor changes in the astronomical tides. The highest sea level ever recorded in Halifax was 10.1 feet during an onshore gale on February 23, 1967. On January 9 and 10, 1974 the predicted 7.8 feet was barely exceeded.)

Numerical Modelling

The laws of fluid motion are well-known, and it should surely be possible to solve these in finite-difference form on a big computer, subject to the "boundary condition" that there be no flow across the coastline, to determine the world-wide distribution of ocean tides. Unfortunately this is not the case. Difficulties arise in representing mathematically the dissipative processes occurring in turbulent flow in shallow seas, but the main difficulty stems from the apparent sensitivity of the problem. For example, minor changes in the representation of the coastline produce major changes in the computed tides (Hendershott, 1973). This has led some investigators to replace the above boundary condition, of no flow across the coastline, with the requirement that the tidal elevation at the coast should equal the observed value. Such models (*Fig. 3*) probably give a fair representation of deep-sea tides, but problems of sensitivity remain, and of course their predictions constitute a type of "dynamical interpolation" rather than a full solution to the problem.

Yet another alarming difficulty in obtaining a numerical solution for the global ocean tide (Hendershott, 1972) arises from the following factors:
1) The solid earth is elastic and yields to the tidal forces in a simple football-like manner (Melchior, 1966; Lambert and Bower, 1974), with an amplitude of tens of centimetres. This movement of the sea floor must be allowed for in the tidal equations.
2) The bulge of this earth tide exerts a gravitational attraction on the water.
3) The "loading" of the earth by the ocean tide produces a further elastic distortion, and again the movement of the sea floor and gravitational attraction of the displaced material must be incorporated into the tidal equations. 4) The ocean tide interacts with itself through self-attraction, i.e., high tide in one area exerts a gravitational attraction on the water everywhere else.

Given a reasonable model of the earth from seismic data, the first two factors above can be dealt with easily, and in fact merely amount to an effective reduction of some 31 per cent in the forcing terms. But the third and fourth factors imply that the tides depend on the tides! Mathematically, the equations become integro-differential rather than differential, which is even nastier than it sounds. Hendershott (1972) has shown that while the forces associated with loading and self-attraction, partially allowed for in Figure 3, are considerably less than the direct tidal forces, their effect in some areas is comparable.

Numerical modelling is also performed on a smaller scale for gulfs and other coastal areas. In such cases the boundary conditions are usually taken to be a requirement of no flow across the shoreline, and prescription that the tidal elevation at the seaward boundary, where the area of the model meets the open ocean, should equal the observed (or guessed) value there. For small areas the direct astronomical forces and the complications of earth tides may be ignored. Models of this type generally work well, though some juggling of friction coefficients is often required

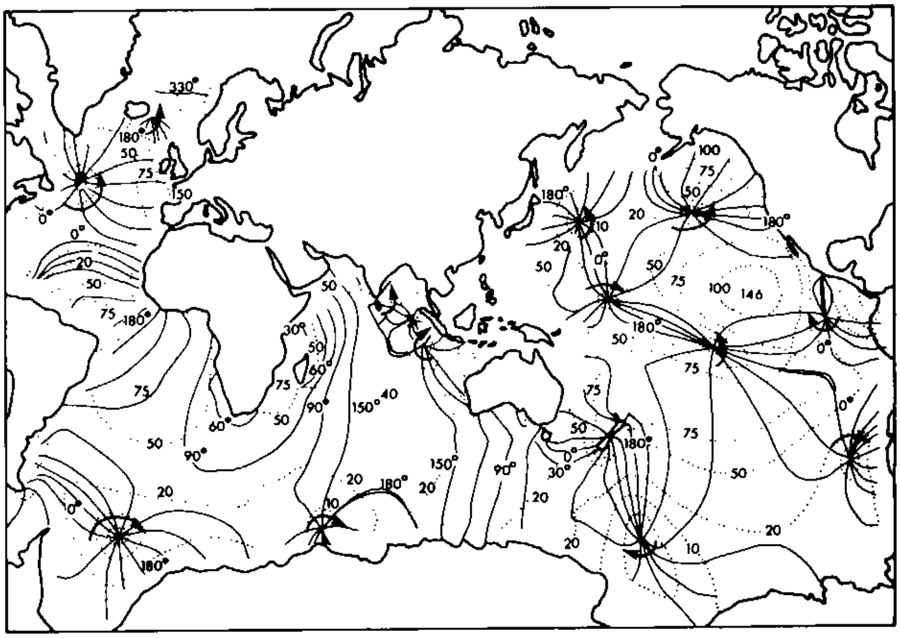

Figure 3
Hendershott's solution for the global M_2 tide (redrawn from Hendershott, 1972). Dotted lines are contours of equal amplitude (cm) and solid lines are contours of equal phase lag. Note the "amphidromic points" marked with arrows, about which the tidal wave appears to rotate.

for good reproduction of observed coastal elevations.

The main value of local models is in deducing tidal currents. Their value in other truly predictive situations is in some doubt. A problem that sometimes arises is that of predicting changes in tidal regime that would be brought about by changes in the geometry of an area, whether by a tidal power dam in the Bay of Fundy, a causeway from New Brunswick to Prince Edward Island, or some other grandiose engineering venture. The tradition is to rerun a numerical model of the area assuming that the offshore tidal elevation will not change. But this is clearly wrong; if the tides in the area are composed of a wave travelling in and a reflected wave travelling back out, the reflected wave, and hence the tides at the seaward boundary of the model, will certainly be altered. The problem is a complex one, and while reasonably well understood in principle, a practical numerical scheme for predicting changes in tidal regime, due to large-scale human intervention, has not yet been developed.

Both for global and local modelling of the tides, as in so many other fields, computer size is not the only consideration. Some very basic scientific problems remain.

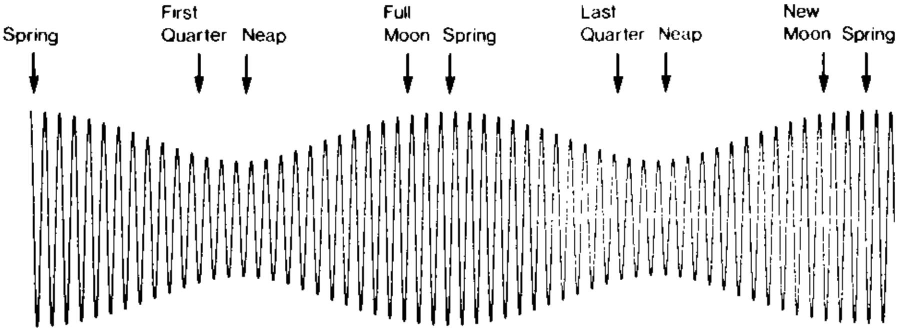

Figure 4
Spring tides do not usually occur until a day or two after full or new moon; the delay is called the 'age' of the tide.

The Age of the Tide

A puzzling tidal phenomenon which has long attracted attention concerns spring and neap tides. In many places there are two roughly equal high waters each day, and the amplitude of this semi-diurnal tide varies with the phase of the moon, being large near new or full moon (spring tides) and small near the moon's quarters (neap tides). The standard explanation of this is that at new or full moon the tidal influence of sun and moon combine, and vice versa at the moon's quarters. But in fact spring tides in most places do not occur until after new or full moon (*Fig. 4*). This curious delay of the modulation of the semi-diurnal tidal response behind the modulation of the forcing function is typically a day or two, and is known as the 'age' of the tide. It was first recorded in A.D. 77 by Pliny the Elder, who attributed it to "the effect of what is going on in the heavens being felt after a short interval, as we observe with respect to lightning, thunder and thunderbolts". In 1799 Laplace pointed out that the age of the semi-diurnal tide can be attributed to a difference in phase lag of the lunar and solar tides behind their respective forcing functions, a positive age corresponding to a greater phase lag for the solar tide. (The phase lag of a given tidal constituent is the lag of the sinusoidal response behind the sinusoidal forcing. It is usually expressed in degrees, with 360° corresponding to one period.) An age of two days arises from the solar semi-diurnal tide S_2 lagging by 49° more than the lunar semi-diurnal tide M_2, a big difference for a frequency difference of only 3½ per cent.

Frequency Sensitivity

To illustrate further this sensitivity of ocean tides to forcing at slightly different frequencies, *Figure 5* shows the admittance of the tide at Halifax and Saint John (*Fig. 6*) to the main semi-diurnal constituents. The admittance is a complex number giving the amplitude ratio to, and phase lag behind, the forcing function.

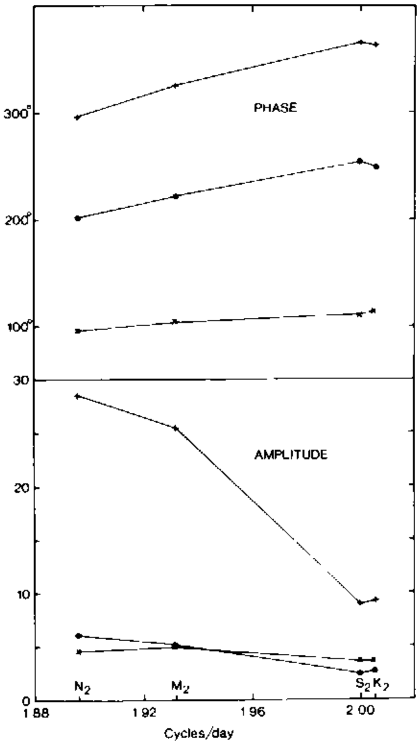

Figure 5
Admittance for semidiurnal tides at Halifax, ●, and Saint John, +, and the ratio Saint John/Halifax, ×.

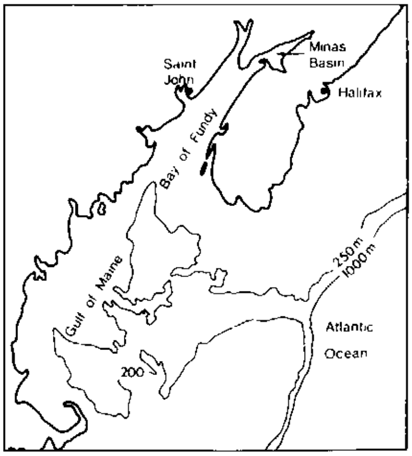

Figure 6
Location of Halifax and Saint John (see Fig. 5) and Minas Basin (see Fig. 10).

The relative amplitudes of N_2, M_2, S_2, K_2 in the astronomical forcing are illustrated in Figure 2. But they appear in different ratios and with different phase lags in actual sea level, as illustrated by the admittance curves in Figure 5. There is a rapid decrease of admittance amplitude with increasing frequency and an increase of phase lag with increasing frequency, as already mentioned in connection with the age of the tide. This increasing phase lag is reminiscent of the behaviour of a simple harmonic oscillator near resonance, and a similar interpretation holds in the oceans.

Indeed, a current view of ocean tides is that the oceans are fairly close to resonance at tidal frequencies, and less dissipative than was once thought (about 20 per cent of the energy of the semi-diurnal tide may be dissipated in each cycle). The tides would probably be much bigger, were it not that the astronomical forcing is rather poorly matched spatially to the way in which the oceans would like to respond.

The admittance of Saint John divided by that of Halifax (*Fig. 5*) may also be interpreted in terms of resonance of the Bay of Fundy and Gulf of Maine system (Garrett, 1972), on the assumption that Halifax tides are representative of the North Atlantic tides that constitute the input to the system. Some allowance has to be made for the effects of nonlinear bottom friction, but the period of the system then seems to be about 13½ hours. The large tides in the Bay of Fundy (*Fig. 7*) may be partly explained in terms of the proximity of this natural period to the tidal period.

Figure 7

Fish weir at Halls Harbour in the Bay of Fundy. Note the sea weed caught near the top of the net at high tide. The maximum tidal range in the upper reaches of the Bay of Fundy is about 51 feet. (Photo from N.S. Communications and Information Centre.)

Internal Tides

While the surface of the sea moves up and down with the tides with a typical amplitude of about one metre, surfaces of constant density in the interior of the ocean may undergo tidal oscillations at least ten times greater than this. The basic mechanism for the generation of these large internal tides, in an idealised example, is illustrated in *Figure 8*. The surface tide tries to move the whole water column to and fro, onto and off the continental shelf. Gravitational restoring forces associated with the density stratification oppose this. Suppose that the ocean is composed of a warm light surface layer on top of a cold heavy lower layer, and that the interface is at the same depth as the shelf. If the density difference is enough to stop the deep water being moved up onto the shelf by the surface tide, then relative to the surface tide the edge of the shelf acts like a piston, moving in and out with the tidal frequency. As the density contrast between the two layers is much less then that between the ocean and the atmosphere, the restoring forces are less at the interface than at the surface, and the interface crinkles up with much shorter, larger waves than the surface.

This is a greatly oversimplified description, but internal tides generated at the edge of the continental shelf and other topographic features in the ocean do produce a significant part of the vertical and horizontal motions in the ocean. Their wavelength is typically just a few tens of kilometres. A significant feature of internal tides is their high degree of variability, associated with changes in stratification of the water column. In this they differ from the regular, predictable, surface tides. Forrester (1974) has described a particularly well documented measurement of internal tides in the St. Lawrence estuary, and suggests that they may be the cause of seasonal variations in tidal currents, as well as variations in the ice pressure experienced by ships in winter.

The role of internal tides in the energy budget for surface tides, and in the vertical mixing of the oceans, is still a matter of debate.

Tidal Friction

Astronomers tell us that the moon is not quite where it ought to be according to the dynamics of a frictionless system. The value they give for the secular acceleration of the moon is doubled occasionally (like the age of the universe) but is currently about $-40''$ century^{-2} (Rochester, 1973). Associated with this is a loss of energy from the system

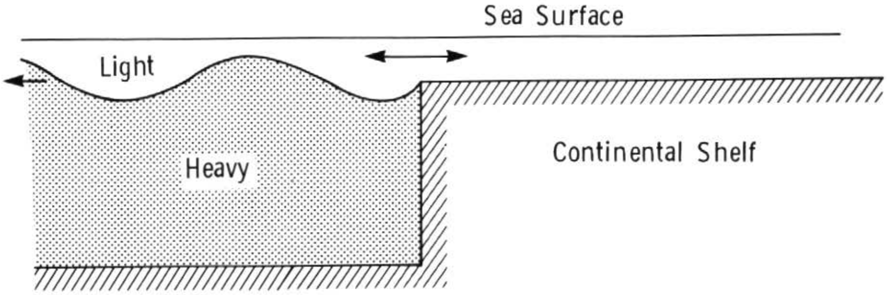

Figure 8

Internal tides of large amplitude and short wavelength are generated as the forces associated with the surface tides try to push the water of a stratified ocean up onto the continental shelf.

at a rate of about $5x10^{12}$ watts, and a reduction in the earth's rotation rate enough to lengthen the day by about one second in 30,000 years. Solar tides change each of these figures fractionally.

It is likely that this energy dissipation occurs in the ocean, rather than in the moon or the solid earth (Munk, 1968), and turbulent friction in shallow seas appears to be the most likely sink, though it has not been shown that this can account for all of the dissipation. Any remaining dissipation might occur through generation of internal tides elsewhere in the ocean, and loss of energy from these (by internal "whitecaps") would result in mixing of the ocean. It is intriguing that only 10 per cent or so of the tidal dissipation would account for the increase in potential energy associated with the degree of vertical mixing that is thought to occur in the deep ocean.

Tidal Power

Harnessing the tides could never produce more than a tiny fraction of man's energy requirements. However, in some areas with particularly high tides tidal power could provide a useful supplement to other types of power.

This is not the place to discuss the arguments for and against tidal power, or even the related scientific problems, other than that of the effect on the tidal regime already mentioned. An irrelevant but amusing point was made by Professor P. B. Fellgett in a letter to the Times of London on November 27, 1973. He pointed out that a tidal power scheme taps the rotational energy of the earth, which is non-renewable, whereas over the time scale required to extract this energy (at least tens of millions of years) fossil fuels are renewable!

Paleo-tides

Associated with tidal friction is the slow recession of the moon from the earth. If we assume that tidal friction has changed only in accordance with the earth-moon separation, then extrapolation into the past brings the moon within a critical distance of the earth less than a billion years ago (Munk, 1968), in conflict with an age of the system very much greater than this. Reconstruction of the behaviour of the earth-moon system requires knowledge of how tidal friction has varied over the last few billion years.

The problem is also of importance over a shorter time scale. Records of the location of ancient eclipses tell one about changes in a linear combination of the earth's rotation rate and the moon's orbital velocity, but one cannot say with any confidence which has changed (Rochester, 1973). However, if it could be shown that tidal friction has been unchanged over the last 3,000 years, one could use the eclipse data, in combination with the modern value for the moon's secular acceleration to determine the change in the earth's rotation rate over the last 3,000 years.

So both on the short and long time-scales, we would like to know how ocean tides have varied in the past. Reconstruction of ocean basins and ancient sea level combined with a numerical model is unlikely to be successful in view of our difficulties in modelling even modern tides. Do we have any other information?

In 1761 the Reverend Nevil Maskelyne and another leading astronomer, Charles Mason (of the Mason-Dixon line), travelled to the island of St. Helena in the South Atlantic to observe the transit of Venus across the face of the Sun. While there they made careful observations of sea level for 42 days. Cartwright (1972) has analysed their data and compared it with modern tidal conditions. He found that the semi-diurnal tides have not changed significantly in 200 years, but that the diurnal tide appears to have increased by about 5 per cent in amplitude, with a decrease in phase lag of about 9°. Cartwright found similar changes in the diurnal tides at the French port of Brest, for which 18th century data is available, and where the semi-diurnal tides also appear to have changed, decreasing by about 2.5 per cent.

On the other side of the Atlantic, Grant (1970) has described fascinating geological evidence which suggests that the large tidal range in the Bay of Fundy has built up over the last 4,000 years.

All this data emphasises the sensitivity of ocean tides to small changes in ocean configuration and depth, and suggests that significant changes in tidal friction may have occurred even over the last few thousand years. But accurate hindcasting is not currently possible.

Tidal Currents

Tidal currents play a major role in shaping the coastline and sedimentary patterns on the sea floor, not just in the steady to and fro motion of the water, powerful though this may be, but also through the action of "residual currents" that can be generated by inertial effects. For example, at the head of the Bay of Fundy, the tide enters Minas Basin as a jet, with a velocity up to 8 knots, but on the outflow the current is more uniform (Tee, 1974, personal communication). The net effect is an average circulation, the residual current, as illustrated in *Figure 9*, with a strength of up to 1½ knots. These tidally generated residual currents may be an important part of the current system in larger areas, such as the southern part of the North Sea.

Earth Tides

The rise and fall of the earth's surface at tidal frequencies, with an amplitude of tens of centimetres, causes variations in gravity, strain and tilt (Melchior, 1966; Lambert and Bower, 1974; AGU, 1973). These can now be measured to startling accuracy (e.g., tilt to better than 10^{-9} radians). The dominant fact is the interdependence of earth and ocean tides. Indeed, in a major contribution, Beaumont (1973) used measurements of M_2 tilt, at various sites in Nova Scotia, to discriminate not only between different crustal models, but also between different cotidal charts for the North Atlantic. An important recent proposal (Beaumont and Berger, 1974) is that changes in dilatancy, possibly associated with imminence of an earthquake, could be monitored by observing changes in earth tides.

Tidal periodicities have also been claimed for other geophysical phenomena, such as eruption of volcanoes (Hamilton, 1973).

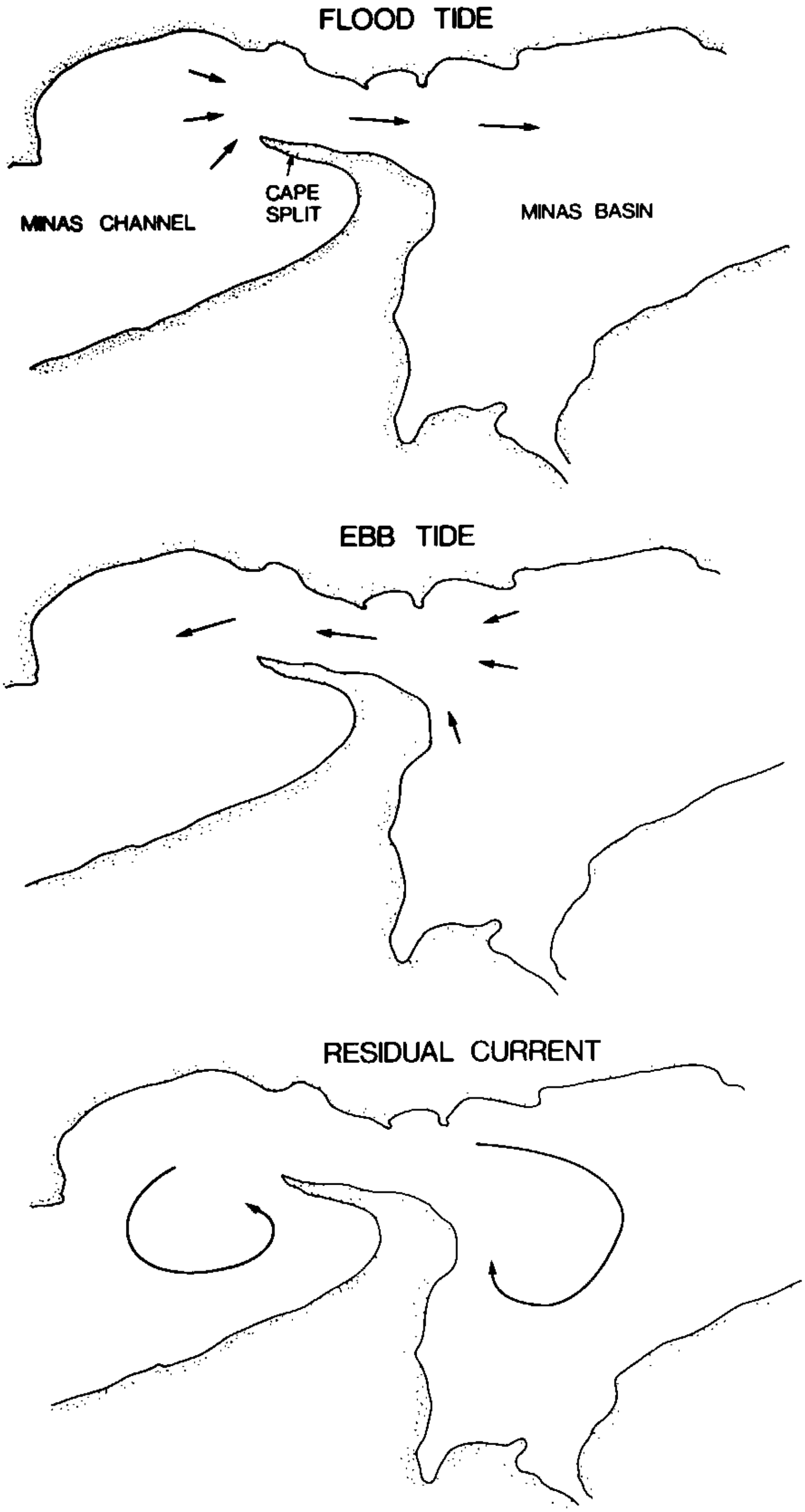

Figure 9
The tidal current in and out of Minas Basin becomes a jet as it flows past Cape Split. The average current over a tidal cycle shows two large eddies (Tee, 1974, personal communication).

Atmospheric Tides

Atmospheric pressure also varies with tidal periodicities, with a typical amplitude of one millibar or so. Semi-diurnal variations are a prominent feature of barograph records in the tropics, but are obscured by much larger meteorological fluctuations in temperate latitudes. The dominance of a 12 hour tide (S_2) in atmospheric pressure led to a difficulty in deciding whether the forcing is due to gravitational forces acting on the atmosphere, or to absorption of solar radiation. Basically, if the forcing is gravitational, a large lunar tide (M_2) should be present, or, if thermal, a large 24 hour tide should be generated. The problem has only been resolved in the last couple of decades (see Chapman and Lindzen, 1970, for a full discussion). The currently accepted theory is that the forcing is largely thermal, but the modes of oscillation excited at diurnal frequency are mostly restricted to the upper atmosphere, where, indeed, they may play an important role in mixing the atmosphere.

A more recent development (Hollingsworth, 1971) is that the lunar M_2 tide, while small, is probably forced mainly by the piston-like up and down motion of the M_2 ocean tide, rather than by direct gravitational attraction of the atmosphere by the moon!

Biological Phenomena

In the understanding of ocean tides, as in many other fields, the real experts live in Southern California, where they procreate on the sandy beaches. These little smelt-like fish, the grunion (*Leuresthes tenuis*) lay their eggs in the sand just after high tide on any of the three or so nights following spring tides (which are a day after full or new moon in this area). The eggs lie undisturbed until hatched and carried out to sea about ten days later as the tides build up to springs again (Idyll, 1969). In some unknown way, the grunion have been making their own tidal predictions, probably for as long as man has existed.

An exciting field of research, involving marine biology, ocean tides and geophysics, concerns the growth lines of corals and invertebrate skeletons. These show tidal and seasonal periodicities, and it has been suggested that the fossil record may thus contain information on the number of days in a month and in a year. Such information would then tell one about variations in the earth's rotation rate and the moon's orbit, and hence about variations of tidal friction. However, a recent review (Clark, 1974) is rather cautious, and emphasizes the uncertainties in interpretation even of modern growth lines.

Conclusion

The tide in the ocean is not just an isolated phenomenon affecting water level in harbours, but interacts strongly with other processes in the ocean, the solid earth and the atmosphere. The foundations of tidal studies were laid hundreds of years ago, but the subject is currently very active, with many old problems to be solved or unsolved, and new riddles to be discovered.

Acknowledgements

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