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Performance of Simulated Annealing, Tabu Search, and Evolutionary Algorithms for Multi-objective Network Partitioning

C. Gil [a] , R. Baños [a] , M.G. Montoya [a] and J. Gómez [b]

[a]Dept. Arquitectura de Computadores y Electrónica, Universidad de Almería, La Cañada de San Urbano s/n, 04120 Almería (Spain)

[b]Dept. Lenguajes y Computación, Universidad de Almería, La Cañada de San Urbano s/n, 04120 Almería (Spain)

Abstract

Most real optimization problems often involve multiple objectives to optimize. In single-objective optimization there exists a global optimum, while in the multi-objective case no optimal solution is clearly defined but rather a set of solutions, so called Pareto-optimal set. Thus, the goal of multi-objective strategies is to obtain an approximation to this set. However, the majority of this kind of problem cannot be solved exactly as they have very large and highly complex search spaces. In recent years, meta-heuristics have become important tools for solving multi-objective problems encountered in industry as well as in the theoretical field. Thus far, there exist many comparative studies about the performance of evolutionary algorithms, but are few the papers dealing with non-evolutionary strategies. The goal of this paper is to analyze the performance of both paradigms in a realistic problem. In concrete, we have adapted five multi-objective meta-heuristics, based on Simulated Annealing, Tabu Search, and Evolutionary Methods, to solve the Network Partitioning Problem.

Key words: Multi-objective Meta-heuristics, Simulated Annealing, Tabu Search, Evolutionary Computation, Network Partitioning.

1. Introduction

Multi-objective optimization problems (MOP) require taking into account multiple objectives at the same time. In most cases, these objectives are in conflict, i.e., the improvement of one objective implies the deterioration of others. Usually, MOPs are solved with conventional single-objective methods by using scalarizing sum functions. Recently, the Pareto optimality concept [1] has been used by many authors in the design of multi-objective meta-heuristics (MOMHs) to solve MOPs. The majority of these papers are based on extending evolutionary algorithms to treat several objectives at the same time. However, other non-evolutionary based methods, like hill climbing, simulated annealing, tabu search, etc., have also been successfully presented. Thus far, the number of comparative studies in real optimization problems is very limited.

With the purpose of developing an adequate comparison, we propose to solve the network partitioning problem. This problem, included in the category of NP-complete [2], consists of dividing the nodes of a network into several balanced sub-domains, such that the number of paths connecting nodes of different sub-domains is minimized. Thus far, almost none of the papers related to this problem consider the simultaneous optimization of both objectives, but it is usual to consider one of them as a constraint. This multi-objective perspective is treated here.

Section 2 describes the five MOMHs we have adapted to solve the network partitioning problem. Section 3 formally describes how represent networks by graphs, and how to partition them by using the graph partitioning model. Section 4 presents the results obtained by them in several test instances, and the metrics used in the comparison. Finally, Section 5 contains the conclusions drawn by this paper.

2. Solving MOPs using Multi-Objective Meta-Heuristics

Given a MOP with $K \geq 2$ objectives to optimize, instead of giving a scalar value to the objective function $f_{1 \ldots K}(s)$, a partial order is defined according to

Email: C. Gil [cgil@ace.ual.es], R. Baños [rbanos@ace.ual.es], M.G. Montoya [mari@ace.ual.es], J. Gómez [jgomez@ual.es].

Pareto-dominance relations [1]. A solution s_1 is said to *dominate* another s_2 ($s_1 \prec s_2$) when s_1 is better than s_2 in at least one objective, and not worse in the others. Two solutions are called *indifferent* or *incomparable* ($s_1 \sim s_2$) if neither s_1 *dominates* s_2, nor s_2 *dominates* s_1. The set of *non-dominated* solutions (ND) constitute the so-called Pareto optimal set, which usually contains not one solution, but several. As all the objectives are equally important, the aim of multi-objective optimization is to find this entire set (or a representative sample of it). In the literature there are comparative studies of different issues of MOMHs [3,4]. In what follows we describe the methods we have implemented in this paper to solve the network partitioning problem.

2.1. *Serafini's Multi-Objective Simulated Annealing (SMOSA)*

One of the earliest MOLSAs was *Serafini's Multi-Objective Simulated Annealing (SMOSA)* [5]. In single objective Simulated Annealing (SA) [6], better neighboring solutions are always accepted, whereas worsening solutions are accepted with a certain probability, which is dependent on a parameter, called temperature. In a multi-objective context a new solution ($s*$) is accepted if it *dominates* the current one (s); and is accepted with a certain probability, if it is *dominated* by s. This probability is usually determined by including the degree of improvement/deterioration in the quality of the solution, and the temperature variable in the Metropolis function [7]. However, there is a special case to keep in mind when both solutions are *indifferent* ($s \sim s*$). Serafini [5] suggested several transition probabilities for this case, obtaining good results. In theory, SA makes possible to reach global optimality.

2.2. *Ulungu's Multi-Objective Simulated Annealing (UMOSA)*

Another important SA-based MOMH is *Ulungu's Multi-Objective Simulated Annealing (UMOSA)* [8]. Unlike SMOSA, where weights in the scalarizing function can be dynamically adapted at runtime, UMOSA executes separate runs by using fixed weights (see parameter λ in formula (1)). Thus, each run of UMOSA generates a set of *non-dominated* solutions. After the execution of the algorithm with different weights for each objective, all the *non-dominated* sets are joined together in a global set. The benefit of this method is obtained when the number of executions with different combinations of weights is large enough. However, the runtime of UMOSA increases according to the number of separate runs. The test problem used to evaluate UMOSA was a multi-objective formulation of the knapsack problem. In the conclusions [8], Ulungu suggested future research in order to compare their strategy versus others like [9], question treated in this paper.

$$c(s) = \lambda_1 \cdot f_1(s) + \lambda_2 \cdot f_2(s) + \cdots + \lambda_K \cdot f_K(s) \quad (1)$$

2.3. *Czyzak's Pareto Simulated Annealing (PSA)*

A population-based version of SMOSA is *Pareto Simulated Annealing (PSA)* [9]. PSA is based on accepting neighboring solutions with a certain probability, which, like SMOSA and UMOSA, depends on the temperature parameter. However, while SMOSA and UMOSA use only one solution in the optimization process, PSA uses several. In PSA, each solution dynamically modifies its weights in the objective function, which is an attempt to assure adequate dispersion of the *non-dominated* solutions. Results obtained by PSA are compared with SMOSA in the multi-objective knapsack problem (see [9]). However, no comparison is provided considering other meta-heuristics.

2.4. *Hansen's Multi-Objective Tabu Search (MOTS)*

Tabu Search (TS) [10] is an optimization method which repeatedly moves from the current solution to the best in the neighborhood, while trying to avoid being trapped in a local optimum by maintaining a list of tabu movements. An extension of TS for MOO is *Multi-Objective Tabu Search (MOTS)* [13]. It works with a set of current solutions which are simultaneously advanced towards the *non-dominated* front (like PSA). These solutions are upgraded using a TS acceptance criterion. In MOTS, the weighting values are adaptively modified during the search process. The number of solutions changes according to the dominance rank among solutions. Thus, if the rank is high, the solutions dominate each other in the objective space, and then the number of solutions is decreased. However, a low average rank indicates that the non-dominated solutions are well-spread. Experimental analysis of MOTS [11] is focused to evaluate the variation in the quality of the solutions according to the length of the tabu list. Nevertheless, this strategy is not compared with other methods.

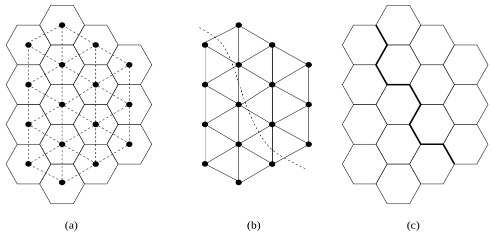

Fig. 1. (a) Cellular Network, (b) Partition in the nodal graph, (c) Partition in the cellular network.

2.5. *Knowles' Pareto Archived Evolution Strategy (PAES)*

Previous methods are based in local search. However, it seems to be interesting the evaluation of the performance of evolutionary methods. In concrete, last MOMH we have adapted to solve the Network Partitioning Problem is the Pareto Archived Evolution Strategy (PAES) [12], proposed by Knowles et al.. PAES, also called (1+1)PAES, is a Multi-Objective Evolutionary Algorithm (MOEA) that uses a single solution during the optimization process. In PAES one parent generates by mutation one offspring. The offspring is compared with the parent, and according to dominance relations, the parent or the offspring continue the search. Knowles [12] proposed the Adaptive Grid Archiving (AGA), which maintains the diversity of the solutions in the external archive of non-dominated solutions (ND). The crowding procedure used in AGA works by dividing up the objective space occupied by the individuals of the population into different rectangular areas, called grid regions. The number of grid regions, which is constant during the search process, is set according to a parameter called *div*. However, as the location of the solutions changes during the search process, the space, the location, and size of the grid regions in the objective space also vary in runtime. The goal is to obtain a set of non-dominated solutions (ND) so that the number of solutions in the same grid is minimized, which indicates that ND is well-spread. PAES also tries to expand the non-dominated solutions in order to build a extensive set.

3. Graph Partitioning as Tool to Partitioning Networks

Since the first heuristic method was proposed to partitioning graph/networks [12], the interest for this problem has increased much. In fact, the number of works related to the partitioning of multiple types of networks, like heterogeneous communication networks [14], Cellular networks [15], Wireless and Mobile networks [16], VLSI networks [17], Neural networks [18], Circuits [19], etc., is very extensive.

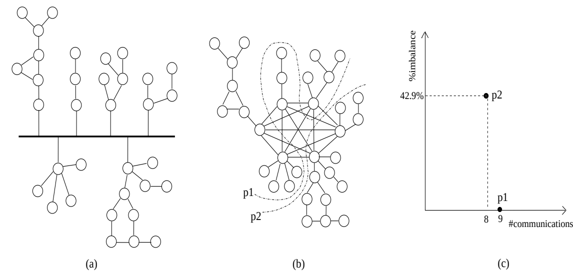

Fig. 2. (a) LAN Network, (b) two partitions, (c) representation in the solution space.

It is known that graphs model a large variety of real problems [20]. One of them is the Graph Partitioning Problem (GPP). The translation of networks to graphs is direct: in terms of network partitioning, vertices in the graph represent nodes in the network, while edges correspond to connections among nodes. An intuitive application of partitioning appears in cellular networks. Figure 1(a) shows a small cellular network and its associated graph. The dotted lines connect the cells with a common face. Figure 1(b), shows a possible partition of the associated graph, whose translation to the cellular network is represented in Figure 1(c). Figure 2 shows as other network is modeled by graphs.

Definition 1. Given an undirected graph, $G = (V, E)$, where V is the set of vertices, $|V| = n$, and E is the set of edges which determines the connectivity of V. The GPP consists of dividing V into SG balanced sub-graphs, $V_1, V_2, \ldots, V_{SG}$, such that $V_i \cap V_j = \phi, \forall i \neq j$; and $\sum_{sg=1}^{SG} |V_{sg}| = |V|$. The imbalance degree is defined by the maximum sub-graph weight, $M = \max(|V_{sg}|), \forall sg \in [1, SG]$. In the single-objective formulation of the GPP, the aim is minimize the communications, while the imbalance defined by M is considered a constraint. Thus, if the maximum allowed imbalance is x%, the partition must verify that $M \leq ((n/SG) * ((100 + x)/100))$.

Most strategies proposed [21] to solve the single-objective GPP use the multilevel paradigm [22] in combination with another optimization technique [23,**?**]. The main handicap of this model is the high dependency on the imbalance constraint. In the example drawn in Figure 2(c) solutions $p1$ and $p2$ are indifferent, reason why both are returned to be used according to the practical application. If, for example, we plan to assign a powerful server to each sub-network and the communications are very slow (little bandwidth, high latency, etc), minimization of communications are more important than imbalance. For this situation, partition $p2$ would be chosen. On the contrary, if the servers are not so powerful and the network quality is high, it would be more advisable to minimize the imbalance, i.e., partition $p1$ is the most suitable. Thus, this multi-objective formulation allows to obtain a collection of solutions which are very useful in the decision making process.

Some authors have proposed multi-objective formulations of the GPP [25,26]. A typical approach is based on creating a mathematical function as the weighted sum of the objective functions [25]. In this scheme, the choice of the weighting values is used to determine the relative importance of the objectives. The main weakness of this scheme is the impossibility of obtaining the Pareto-optimal solutions, due to it is extremely difficult to assign adequate weights to the objectives. An interesting method to overcome this disadvantage is to use the Pareto-dominance concept [1], which has been successfully applied to a large variety of MOPs. In the GPP problem, we can reference the work of Rummler and Apetrei [26], where authors adapted a MOEA (SPEA [27]) to solve the GPP. They tested the performance of his adaptation, obtaining unsatisfactory results. They remarked that the main reason for these disappointing results is the redundancy in the representation of the solutions for this problem, which results in the evolutionary operators, mainly the crossover operator, not to work as well as in other problems. Their experimental results indicated that the use of a local search procedure allows for improvement in the quality of the solutions in comparison with the SPEA adaptation. This is the main reason why we use the methods described in Section 2. In concrete, previous studies [19] have demonstrated the good behavior of SA and TS in the single-objective formulation of this problem. Further, as PAES works with a single solution, crossover operator is not applied, avoiding the problems that happened in the adaptation proposed in [26].

4. Experimental Results

The executions have been performed in a 2.4 GHz processor with 512 Mbytes of RAM memory. The programs were implemented in standard C using the *gcc-3.3.2* compiler of Linux Fedora Core 1. As our goal is to evaluate the quality of the MOMHs described above in the partitioning of large networks, we have used a set of test graphs. These graphs, that belong to a public domain set [21] frequently used to compare single-objective graph partitioning algorithms, have also been used to evaluate network partitioning algorithms [14]. Table 1 details the vertices degrees (maximum, minimum, and average). All them have thousands of vertices (nodes) and edges (connections among nodes).

Table 1

graph	|V|	|E|	min	max	avg
add20	2395	7462	1	123	6.23
3elt	4720	13722	3	9	5.81
uk	4824	6837	1	3	2.83
add32	4960	9462	1	31	3.82
crack	10240	30380	3	9	5.93
wing_nodal	10937	75488	5	28	13.80

Test graphs.

4.1. *Parameter Setting*

In what follows we describe the parameter configuration we have used in the experiments. The initial solutions are obtained by using the GGA strategy [28]. This procedure starts from a randomly selected vertex, which is then assigned to the first sub-graph, as their adjacent vertices. This recursive process is repeated until this sub-graph reaches n/SG vertices. From this point, the following visited vertices are assigned to a new sub-graph. When all the vertices are assigned to a certain sub-graph, the procedure finishes. As the position of the initial vertex determines the structure of the primary partition, its random selection offers a very useful diversity.

As we commented previously, SMOSA, UMOSA, and PAES use a single solution to perform the search process, while PSA and MOTS use a set of solutions (notice that this value is fixed in PSA, and variable in MOTS). In both cases, the population size has been set to $|P| = 100$. The maximum size of the non-dominated set has also been set to $|ND| = 100$. The initial temperature (T_i), and cooling rate (T_{cr}) for SMOSA, UMOSA, and PSA are $T_i = 100$, $T_{cr} = 0.995$. In UMOSA, the number of runs with different weighting values has been set to 10. Weights in function (1) have been assigned as we detail now: $\lambda_{communications} = 1.0$ for the first execution, $\lambda_{communications} = 0.9$ in the second one,

Table 2

	SMOSA		UMOSA		PSA		MOTS		PAES	
SMOSA	*add20*	*3elt*	0.000	0.000	0.142	0.545	0.000	0.571	0.636	0.400
	uk	*add32*	0.600	0.625	0.000	0.857	0.571	0.250	0.000	0.900
	crack	*wing_nodal*	0.000	0.000	0.000	0.000	0.200	0.714	0.000	1.000
UMOSA	1.000	0.400			1.000	0.181	0.000	0.857	1.000	1.000
	0.250	0.286			0.111	0.000	0.517	0.500	0.500	0.900
	0.875	1.000			0.875	1.000	0.800	1.000	1.000	1.000
PSA	0.556	0.400	0.000	0.000			0.000	0.714	1.000	0.800
	1.000	0.000	0.900	0.500			0.857	0.000	1.000	0.700
	0.875	0.957	0.000	0.000			0.600	0.714	0.000	1.000
MOTS	0.222	0.200	0.429	0.500	0.143	0.181			0.818	0.200
	0.250	0.286	0.300	0.125	0.111	0.000			0.500	0.300
	0.125	0.000	0.000	0.000	0.000	0.000			0.000	0.000
PAES	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.571		
	0.000	0.000	0.200	0.000	0.000	0.000	0.571	0.000		
	0.375	0.000	0.000	0.000	0.000	0.000	0.600	0.571		

Results using Metric C *in each test graph.*

downing to $\lambda_{communications} = 0.0$ in the last run. Note that $\lambda_{imb} = 1.0 - \lambda_{communications}$. The number of iterations of each run has been set to 1838. This value corresponds to the number of iterations needed by the SA-based MOMHs to fall below $t = 0.01$ with these annealing parameters. PAES also executes the same number of iterations. The results here shown correspond to the partitioning of the test graphs into $SG = 16$ subgraphs. With the purpose of comparing the quality of these MOMHs with single-objective strategies [21], we have considered an additional constraint which consists of discarding those solutions with an imbalance greater than 5% (i.e., $M \leq 1.05$).

4.2. *Performance Measures*

The results of computational experiments will be compared using the S and C metrics proposed in [27]. In what follows we describe them:

Definition 2. *Coverage of two sets* (C). Let X, X' be two subsets of solutions. The function C maps the ordered pair (X, X') to the interval [0,1]. The value $C(X, X') = 1$ means that all points in X' are *dominated* by or *indifferent* to the points of X. Figure 3(a) shows that the set X covers most of the solutions of X'.

$$C(X, X') \leftarrow \frac{|a' \in X'; \exists a \in X : a \preceq a'|}{|X'|} \qquad (2)$$

Definition 3. *Average size of the space covered* (S). Given a set of solutions, $X = (x_1, x_2, \ldots, x_n)$, the function $S(X)$ returns the average volume enclosed by the union of the polytopes p_1, p_K, where each p_i is formed by the intersections of the following hyperplanes arising out of x_i, along with the axes: for each axis in the objective space, there is a perpendicular hyperplane passing through the point $(f_1(x_i), \ldots, f_K(x_i))$. In the bi-dimensional case, each p_i represents a rectangle defined by the points $(0, 0)$ and (f_1, f_2). Thus, the smaller this average volume is, the better the approximation to the (unknown) Pareto-optimal front.

$$S(X) \leftarrow \frac{\sum_{i=1}^{|X|}(\prod_{k=1}^{|K|} \frac{f_k(x_i)}{max(f_k(X))})}{|ND|} \qquad (3)$$

Let us consider the non-dominated solutions of set X, in Figure 3(b). Each solution x_i encloses an area of size [*communications*(x_i)**imbalance*(x_i)]. Metric S determines the quality of the non-dominated sets by preferring the smaller enclosed area. As the non-dominated sets usually have a different number of solutions, it is necessary to normalize this value to obtain an average enclosed area (see function (3)). In Figure 3(b) it is clear that the average area covered by X is smaller than the area of X', which leads us to think that X is closer to the (unknown) Pareto-optimal front than X'. It is worth noting that, as both objectives (*communications* and *imbalance*) have different scales, it is necessary to define the work area. For this reason, we establish a maximum *imbalance* $M \leq 1.05$ (see Definition 1). This value also allows us to compare the multi-objective approaches analyzed here with the

Table 3

graph	MOMH	S	$S_{norm.}$	communications	communications$_{norm.}$
add20	SMOSA	0.329	2.511	2525	1.137
	UMOSA	0.189	1.443	**2221**	**1.000**
	PSA	0.371	2.832	2403	1.082
	MOTS	**0.131**	**1.000**	3964	1.785
	PAES	0.438	3.344	2773	1.249
3elt	SMOSA	0.056	8.000	**741**	**1.000**
	UMOSA	**0.007**	**1.000**	922	1.244
	PSA	0.069	9.857	764	1.031
	MOTS	0.279	39.857	2989	4.034
	PAES	0.118	16.857	1945	2.625
uk	SMOSA	0.078	1.279	249	1.078
	UMOSA	0.177	2.901	410	1.775
	PSA	**0.061**	**1.000**	**231**	**1.000**
	MOTS	0.211	3.459	697	3.017
	PAES	0.145	2.377	627	2.714
add32	SMOSA	0.034	1.062	**191**	**1.000**
	UMOSA	0.090	2.813	440	2.304
	PSA	**0.032**	**1.000**	309	1.618
	MOTS	0.092	2.706	5538	28.994
	PAES	0.352	11.000	1125	5.890
crack	SMOSA	0.139	8.176	2670	1.743
	UMOSA	**0.017**	**1.000**	**1532**	**1.000**
	PSA	0.041	2.412	1761	1.149
	MOTS	0.047	2.765	3332	2.175
	PAES	0.039	2.294	3000	1.958
wing_nodal	SMOSA	0.151	12.583	16956	1.601
	UMOSA	**0.012**	**1.000**	**10588**	**1.000**
	PSA	0.048	4.000	15745	1.487
	MOTS	0.085	7.083	34347	3.244
	PAES	0.045	3.750	23890	2.256
average	SMOSA	-	4.122	-	1.026
	UMOSA	-	**1.000**	-	1.130
	PSA	-	2.587	-	**1.000**
	MOTS	-	6.850	-	5.871
	PAES	-	4.859	-	2.266

Comparing MOMHs: SMOSA, UMOSA, PSA, MOTS, and PAES

Table 4

	SMOSA	**UMOSA**	**PSA**	**MOTS**	**PAES**
SMOSA		0.204	0.257	0.384	0.489
UMOSA	0.635		0.528	0.612	0.900
PSA	0.631	0.233		0.481	0.750
MOTS	0.181	0.226	0.073		0.303
PAES	0.081	0.033	0.000	0.386	

Average results using Metric C.

single-objective results detailed in [21]. The *communications* limit is defined by the worst initial solution in this objective. Therefore, the work area of the GPP is $([0, \max\_communications)], [0, \max\_imbalance])$.

4.3. *Analysis of the Results*

Table 2 shows the comparison among MOMHs for the Metric C, considering the test graphs of Table 1 in the same order. In average (see Table 4) the non-dominated solutions obtained by UMOSA dominates more than half of the non-dominated solutions of the other strategies. PSA also obtains good solutions for this metric, mainly versus SMOSA, MOTS, and PAES. On the contrary MOTS and PAES are the worst in this comparison, which indicates the good performance of the SA-based MOMHs.

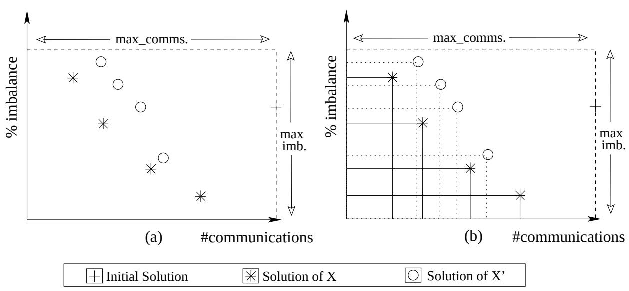

Fig. 3. Graphical explanation of the metrics used over two non-dominated sets.

In addition to the Metric C, we compare these approaches by using the metric S. First and second numerical columns of Table 3 show the absolute and normalized enclosed area, respectively. Last row of this table details the average normalized results. We can observe the best performance is obtained by UMOSA. These conclusions go in the same way that the obtained for metric C.

The column *communications* shows the minimum communication volume for this objective obtained by any of the non-dominated sets for these test graphs. As we can see in last column, SA-based MOMHs always obtain, in average, the best results. It is important to notice that, although we are solving the multi-objective formulation, the results obtained are close to the best known solutions for the single-objective case [21].

Previously, we have analyzed the results obtained by the MOMHs over six test graphs. However, there are cases were statistic results do not offer enough information, because some of them can obtain good results in some metrics, but not in others. Thus, in addition to the numerical analysis, it is also interesting to study graphically the fronts obtained. Figure 4 shows the non-dominated solutions obtained by the MOMHs in the partitioning of all the test graphs into $SG = 16$ sub-graphs, in the same order that appear in Table 1. These figures help us to support the conclusions previously obtained. For example, in first graphic (add20) we see as UMOSA dominates all the other methods, excepting MOTS, whose solutions are non-dominated by the other fronts. This graphical conclusion is related to the results obtained in Table 2. In addition, these figures display as population-based MOMHs (PSA and MOTS) obtain larger non-dominated sets. Finally, we indicate that the non-population approaches are faster than the population-based versions (one order of magnitude). In all cases the runtimes are less than one hour in the computer described above.

5. Summary and Conclusions

This paper proposes a novel multi-objective formulation of the network partitioning problem, which simultaneously optimizes the load balancing among sub-networks, and the amount of communication among nodes belonging to different sub-domains. Further, we have adapted five of the most important multi-objective meta-heuristics proposed until now with the aim to obtain high quality non-dominated fronts. The results obtained in several test graphs, which model networks, indicate that simulated annealing outperforms the partitions obtained by tabu search, and evolutionary methods. Comparing the first group, UMOSA obtains the best performance, thanks to it executes several separate runs with a diversified set of weights. However, the number of non-dominated solutions obtained by the population-based versions, like PSA, is often higher. In same cases, the quality of the partitions are very close to those obtained by other methods in the single-objective formulation. These conclusions give useful information in order to hybridize some of these strategies, taking advantage of their particular characteristics. This experimental analysis also facilitates the multi-objective treatment of other optimization problems.

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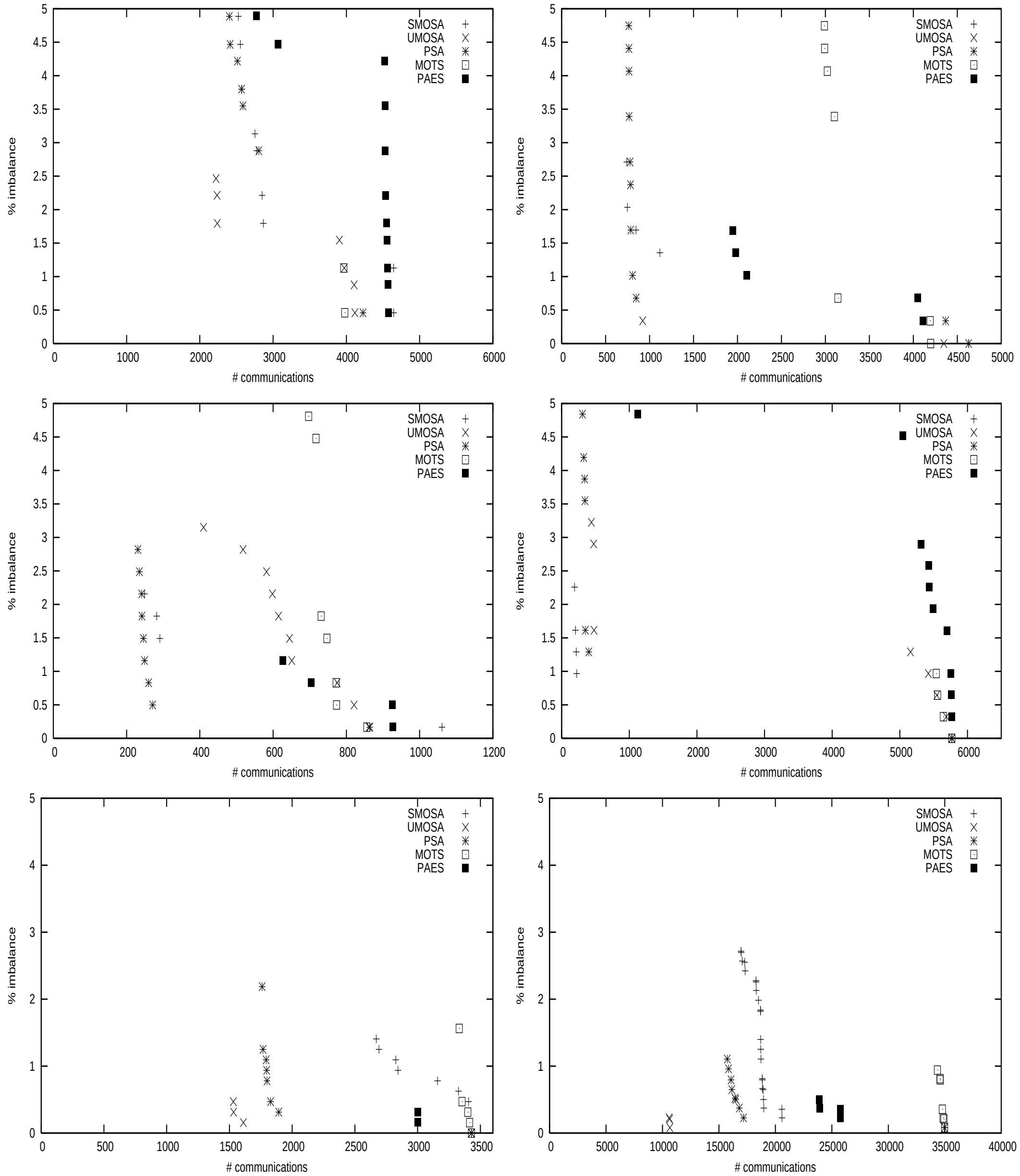

Fig. 4. Graphical representation of the solutions in all the test graphs (SG=16).

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