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Approximable 1-Turn Routing Problems in All-Optical Mesh Networks

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Abstract

In all-optical networks, several communications can be transmitted through the same fiber link provided that they use different wavelengths. The MINIMUM ALL-OPTICAL ROUTING problem (given a list of pairs of nodes standing for as many point to point communication requests, assign to each request a route along with a wavelength so as to minimize the overall number of assigned wavelengths) has been paid a lot of attention and is known to be $\mathcal{NP}$–hard. Rings, trees and meshes have thus been investigated as specific networks, but leading to just as many $\mathcal{NP}$–hard problems.

This paper investigates 1-turn routings in meshes (paths are allowed one turn only). We first show the MINIMUM LOAD 1-TURN ROUTING problem to be $\mathcal{NP}$–hard but 2-APX (more generally, the MINIMUM LOAD k-CHOICES ROUTING problem is $\mathcal{NP}$–hard but k-APX), then that the MINIMUM 1-TURN PATHS COLOURING problem is 4-APX (more generally, any d-segmentable routing of load L in a hypermesh of dimension d can be coloured with $2d(L-1)+1$ colours at most). >From there, we prove the MINIMUM ALL-OPTICAL 1-TURN ROUTING problem to be APX.

Key words: minimum load routing, minimum path colouring, all-optical networks, mesh, 1-turn routing, approximation algorithms.

1. Introduction

In optical networks, links are optical fibers. Wavelength Division Multiplexing (WDM) is a technique (see for instance [1]) that proposes to take advantage of the huge optical fiber bandwidth by allocating a unique frequency to each communication. Several communications can simultaneously use the same fiber as long as their wavelengths are different, while no expensive wavelength conversion is needed when traversing nodes.

In this context, networks are called *all-optical networks*. They can be viewed as graphs and ***communication requests*** as pairs of nodes. We call ***communication instance*** any graph together with a family of communication requests (a pair of nodes may appear more than once in the family). Given some communication instance, the ***all-optical routing problem*** is then formulated as: to each communication request assign some path connecting its two nodes, that is find a ***routing*** for this instance, and to each of these paths, assign some colour in such a way that no two paths using a common edge bear the same colour, thas is turn the routing into an ***all-opticall routing***.

Given a communication instance, two natural optimization problems arise: find an all-optical routing which minimizes the overall number of colours assigned to paths, namely the ***minimum all-optical routing problem***, and find an all-optical routing which minimizes the number of paths having to traverse a common edge, namely the ***minimum load routing problem*** [2]. See figure 1 for an example. The minimum number of distinct colours is clearly an upper bound to the minimum achievable load but the difference cannot be bounded by a constant in general [3,4]. Note that if network nodes are converters, that is if any path can change its colour at any node, the minimum all-optical routing problem reduces to the minimum load routing problem.

It is known that there is no $(loglogM)^{1-\epsilon}$ approximation for the unirected congestion minimization problem unless $NP \subset ZPTIME(n^{polylogn})$, where M is the size of the graph and ϵ is any positive constant [5] (while, in the directed case, there is no $c\log(\log(n))$-approximation algorithm for this problem unless

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[2] This problem is can be formulated as an integer multicommodity flow problem (see for instance [2]).

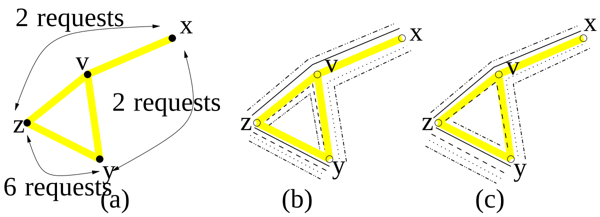

Fig. 1. Figure (a) shows a communication instance I. Figure (b) and (c) show all-optical routing R_b and R_c resp. which are solutions to I. R_c is a minimum all-optical routing for I, but R_b is not (R_b, resp. R_c, makes use of 6 colours, resp. 5). On the other hand, R_b is a minimum load routing for I while R_c is not (R_b makes every link support 4 colours, R_c makes link zy support 5 colours).

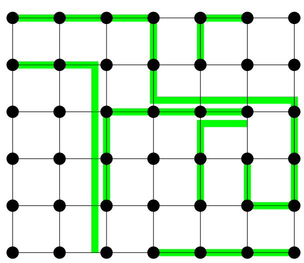

Fig. 2. A mesh with 6 rows and 7 columns. Every path but one is a 1-turn path (i.e. granted one change of direction at most).

$NP \subseteq D-\ TIME(n^{O(\log(\log(\log(n))))})$ [6]). The minimum all-optical routing problem is $\mathcal{NP}$*–hard* in general, whether graphs are directed [7] or not [8,9,7]. Moreover, restricted to directed graphs, the problem is known to be *No-APX* [10, corollary 3.1.5] (for more about approximation theory see [11]). Focussing on specific network topologies, namely when networks are linear, rings, stars, spiders and trees of rings, makes these problems range from polynomial to $\mathcal{NP}$*–hard*, whether *APX* or not, these results sometimes depending heavily on whether the graph is directed or not [12–17]. A more detailed summary can be found in [18].

Meshes are networks with a grid pattern (nodes are organized in rows and columns). A mesh with 6 rows and 7 columns is shown in figure 2. Indeed, when all-optical networks are concerned, meshes have been considered as real competitive solutions among current metropolitan topologies [10,19,20]. Restricted to meshes, the minimum all-optical routing problem is still $\mathcal{NP}$*–hard* [8]. To our knowledge, it is not known whether it is *APX* (at least, if it is d*–APX*, then one must have $d \geq 2$ [8]), and the best result is a $poly(\ln\ln N)$ approximation algorithm on meshes of $N \times N$ nodes [9].

This paper is devoted to the ***all-optical 1-turn routing problem***, the restriction of the all-optical routing problem in non directed meshes where routings are to be made of paths which are allowed one change of direction at most. These paths, which we call ***1-turn paths*** (see figure 2), are commonly used in meshes, see for example [21,22].

It turns out that even so restricted, the minimum all-optical 1-turn routing problem is $\mathcal{NP}$*–hard*. Actually, this result must have been known (for instance a proof can be derived from [23] where communication instances on rings are mapped on meshes), though it seems not to have been published as such. None the less, we provide a genuine proof and then prove the minimum all-optical 1-turn routing problem to be *APX* by providing an 8-*APX* algorithm (the best performance guarantee with a constant ratio known to us up till now) which follows straightforwardly from combining *APX* results for each of the two following steps, where I is some communication instance:

- step 1: compute some 1-turn routing R for I
- step 2: assign colours to the paths of R to make it an all-optical routing

Connecting these two steps, routing loads play a central role. First, given some positive integer k, when each request of a communication instance to the all-optical problem is given k paths from which its connecting path must be chosen, the minimum load routing problem becomes the ***minimum load*** k***-choices routing problem***. We show this problem to be $\mathcal{NP}$*–hard* but k-APX, from which follows that the minimum load 1-turn routing problem is 2-APX. Then, we show that given the paths of a routing of load L, one can colour these paths into an all-optical routing using no more than $4L - 3$ colours (actually, this stems from a more general result dealing with so-called direction segmentable routings in meshes of dimension d introduced in the appropriate section), which leads to the 8-approximation algorithm mentioned above.

The sequel is organized as follows.

- Section 2 is devoted to load routing problems, where the minimum load 1-turn routing problem is proved to be 2-*APX*.
- Section 3 is devoted to the minimum path colouring problem in d-dimensional meshes when restricted to some special paths, yielding the minimum 1-turn path colouring problem to be 4-*APX*.
- Section 4 is devoted to the all-optical 1-turn routing problem, where the minimum all-optical 1-turn routing problem is then proved to be 8-*APX*.

We conclude in section 5.

2. Load 1-turn routing problems

Given two positive integers L and k, the ***L-load k-choices routing problem*** is the decision problem defined as follows:

instance: a communication instance I **and** to each request $r = \{a, b\}$ in I, the assignment of at most k paths joining a and b in the I network

question: is there a routing of load L for I such that each request r from I is satisfied by a path assigned to r ?

and we call accordingly the derived minimization problem the ***minimum load k-choices routing problem***.

2.1. *The L-load 1-turn routing problem $\mathcal{NP}$-completeness*

It turns out that the L-load 1-turn routing problem is in $\mathcal{P}$ when $L = 1$ and otherwise $\mathcal{NP}$*–complete*. Our proofs refer to the celebrated SATISFIABILITY problem whose restriction as 3–*SAT* is $\mathcal{NP}$*–complete* (for instance, see [24, p. 39, p. 48]) while its 2–*SAT* restriction is in $\mathcal{P}$ (for instance, see [25, p. 185]).

2.1.1. $L = 1$

A straightforward reduction of the 1-load 2-choices routing problem to 2–*SAT* yields the following:

Proposition 1 *The 1-load 2-choices routing problem is in $\mathcal{P}$.*

Proof: We reduce the 1-load 2-choices routing problem to 2–*SAT*.

Assume $R = \{r_i | 1 \le i \le n\}$ is the set of requests of some instance I of a 1-load 2-choices routing problem such that P_0^i and P_1^i are the two paths assigned to the request r_i for $1 \le i \le n$. Using R as a set of boolean variables, we define C as the set of 2-clauses which, in turn, are defined for each pair $\{i, j\}$ with $1 \le i, j \le n$, according to three possible events:

- $\{\neg r_i, \neg r_j\}$ when P_1^i and P_1^j share a common edge
- $\{r_i, r_j\}$ when P_0^i and P_0^j share a common edge
- $\{\neg r_i, r_j\}$ when P_1^i and P_0^j share a common edge

One can check that there is a solution to the 1-load 2-choices routing problem instance if and only if there is a solution to the 2–*SAT* problem instance associated with C (for instance, one can associate assigning the value true to r_i with choosing path P_1^i). We conclude

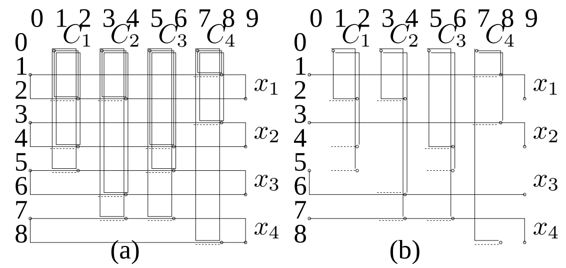

Fig. 3. Let $C = \{C_1, C_2, C_3, C_4\}$ with $C_1 = \{x_1, x_2, \neg x_3\}$, $C_2 = \{x_1, x_3, \neg x_4\}$, $C_3 = \{x_2, \neg x_3, \neg x_4\}$ and $C_4 = \{\neg x_1, \neg x_2, x_4\}$. Figure (a) shows the communication instance I associated with C and figure (b) shows a 2–load 1-turn routing solution to I. In figure (a) each "horizontal" (resp. "vertical") rectangle bears the two possible 1-turn paths satisfying the communication request associated with one of the variables x_1, x_2, x_3 and x_4 (resp. to one of the literals of clauses C_1, C_2, C_3 and C_4, with vertical rectangles being grouped according to the clause to which the literal they stand for belongs). "Blocking requests" are depicted with dotted lines.

from the fact that the set of clauses C can be computed in polynomial time.

Noticing that there are at most two possible 1-turn paths joining any two vertices in a mesh, the following straightforwardly stems from proposition 1:

Corollary 2 *The 1-load 1-turn routing problem is in $\mathcal{P}$.*

2.1.2. $L \ge 2$

Theorem 3 *The L-load 1-turn routing problem is $\mathcal{NP}$–complete for $L \ge 2$.*

Proof: We reduce 3–*SAT* to the L-load 1-turn routing problem. We assume $L = 2$ (the proof is easily extended for $L > 2$ by solely adding a convenient number of so-called "blocking requests" as defined below).

Clearly the problem is in $\mathcal{NP}$. Using a reduction of 3–*SAT*, we prove it to be $\mathcal{NP}$*–complete*. Let C be some instance of 3–*SAT* with $C = \{c_1, c_2, ...c_m\}$, a set of 3-clauses over the set of boolean variables $X = \{x_1, x_2, ...x_n\}$. Let $M_{[(2n)\times(2m+1)]}$ be the mesh whose rows are numbered from 0 to $2n$ and whose columns are numbered from 0 to $2m + 1$. Finally, let I be the instance of the 2-load 1-turn routing problem defined as follows:

- to each variable x_i, we assign the request $r_i = \{(2i-1, 0), (2i, 2m + 1)\}$
- to each positive literal $l \in c_j$, with $l = x_i$, we assign the request $r_{i,j} = \{(0, 2j-1), (2i, 2j)\}$ together with

a so-called "blocking request" $blk_{i,j} = \{(2i, 2j - 1), (2i, 2j)\}$

- to each negative literal $l \in c_j$, with $l = \neg x_i$, we assign the request $r'_{i,j} = \{(0, 2j-1), (2i-1, 2j)\}$ together with a so-called "blocking request" $blk'_{i,j} = \{(2i-1, 2j-1), (2i-1, 2j)\}$

Then one can check that there is some truth assignment satisfying C if and only if there is a 2-load 1-turn routing solution to I. We conclude by considering that the instance I of L-load 1-turn routing problem associated with C can be computed in polynomial time.

2.2. *The minimum load 1-turn routing problem approximation*

Clearly, theorem 3 yields the following:

Corollary 4 *The minimum load 1-turn routing problem is $\mathcal{NP}$–hard.*

also, since restricting paths in a mesh to be 1-turn paths turns routing problems into 2-choices routing problems:

Corollary 5 *The minimum load k-choices routing problem is $\mathcal{NP}$–hard.*

We now show this problem to be APX.

Theorem 6 *The minimum load k-choices routing problem is k-APX.*

Proof: The scheme of the proof is: define the problem as an integer linear programming problem, relax the integer constraint, then round straightforwardly a real optimal solution. Details are as follows.

Let I be some instance of the minimum load k-choices routing problem. Let $R = \{r_i\}_{1 \le i \le n}$ be the set of requests from I. To each request r_i is associated a set $P_i = \{p^i_1, p^i_2, ..., p^i_{k_i}\}$ of k_i feasible paths in the network G, with $k_i \le k$. Selecting path p^i_j to join end-nodes of request r_i if and only if $x^i_j = 1$ yields a one-to-one mapping between routing solutions to I and solutions to the integer linear programming instance defined as:

$$x^i_j \in \{0,1\} \text{ for all } i, j, 1 \le i \le n, 1 \le j \le k_i$$

$$\sum_{j=1}^{k_i} x^i_j = 1 \text{ for all } i, 1 \le i \le n$$

$$z \ge \sum_{e \in E(p^i_j)} x^i_j \text{ for every edge } e \text{ of the network } G$$

$$\text{objective: minimize } z$$

For every edge e of the network G, let $\pi(e) = \sum_{e \in E(p^i_j)} x^i_j$, let $\pi^*_{\mathbb{N}}$ denote the optimal value of π, and let $\pi^*_{\mathbb{R}}$ be the optimal value of π when relaxing, for all $i, j, 1 \le i \le n, 1 \le j \le k_i$, integer condition $x^i_j \in \{0,1\}$ to real condition $x^i_j \in [0,1]$. Obviously $\pi^*_{\mathbb{R}} \le \pi^*_{\mathbb{N}}$.

For all $i, j, 1 \le i \le n, 1 \le j \le k_i$, assume a^i_j to be the value of x^i_j in an optimal solution to the relaxed linear programming problem and define :

$$b^i_j = \begin{cases} 1 \text{ if } a^i_j = \max_{1 \le h \le k_i} a^i_h \\ 0 \text{ otherwise} \end{cases}$$

(for a given $i, 1 \le i \le n$, if more than one b^i_j is equal to 1, set all of them but one at 0).

Now, as $\max_{1 \le j \le k_i} a^i_j \ge \frac{1}{k}$, letting $\pi^{algorithm}_{\mathbb{N}}$ denote the load associated with the $(b^i_j)_{1 \le i \le n, 1 \le j \le k_i}$ solution yields the following :

$$\frac{\pi^{algorithm}_{\mathbb{N}}}{\pi^*_{\mathbb{N}}} \le \frac{k\pi^*_{\mathbb{R}}}{\pi^*_{\mathbb{N}}} \le \frac{k\pi^*_{\mathbb{R}}}{\pi^*_{\mathbb{R}}} = k$$

We conclude by noting that the size of the linear programming instance is polynomially related to the size of the minimum load k–choices routing instance.

Remark 7 *One can show that the approximation analysis in the theorem above is tight [18].*

Restricting again k–choices routings to 1-turn routings in meshes, theorem 6 yields the following.

Corollary 8 *The minimum load 1-turn routing problem is 2–APX.*

The 2 approximation factor expressed in corollary 8 might be improved upon, but, applying the gap technique (see [26] for instance) to theorem 3 with $L = 2$ yields:

Corollary 9 *If the minimum load 1-turn routing problem is d–APX for some constant d, then $d \ge 3/2$.*

3. The 1-turn paths colouring problem

Given some routing R solution to a given communication instance I, the **conflict graph** induced by R is the graph G whose nodes are the paths of R, with two paths being adjacent in G when they have at least one edge in common.

Let d be an integer such that $d \ge 2$ and let n_1, n_2, ... n_d be non-negative integers. Let $M_{[n_1 \times n_2 \times ... \times n_d]}$ denote the hypermesh where, with $0 \le i_k, j_k \le n_k$ for all $k \in [1, d]$, nodes $x = (i_1, ...i_d)$ and $y = (j_1, ...j_d)$

are adjacent iff $i_k = j_k$ for all $k \in [1, d]$ but one, say k^*, for which $|i_{k^*} - j_{k^*}| = 1$, the edge xy being called an **edge of direction** k^*. A **mesh of dimension** d is a graph M isomorphic to such a $M_{[n_1 \times n_2 \times \ldots \times n_d]}$, and, for $k \in [1, d]$, $E_k(M)$ denotes the set of edges of M which are of direction k.

Let P be a path in some hypermesh M of dimension d. If for all $i \in [1, d]$ the set $E_i(G) \cap E(P)$ induces a path in M, then P is said to be a **direction-segmentable path**. A routing in a hypermesh whose every path is direction-segmentable is a **direction-segmentable routing**.

Lemma 10 *If G is the conflict graph of some direction-segmentable routing R on a hypermesh of dimension d, then $E(G) \leq d(L-1)(n - \frac{L}{2})$ where n is the number of vertices of G and L is the load of R.*

Proof: For every $i \in [1, d]$, let G_i be the subgraph of G induced by conflicts which occur along direction i only, and let L_i be the maximum load on edges of $E_i(G)$. Then G_i is an interval graph therefore a chordal graph and therefore has a perfect elimination ordering[27, pages 6 and 50]. It follows from there that the number of edges of G_i is less or equal to $f_n(k) = (k-1)(n-\frac{k}{2})$ where n is the number of vertices of G_i and k is the maximum size of a clique. On the other hand, G_i being an interval graph, any clique of maximum size in G_i is of size L_i. Thus $|E(G_i)| \leq (L_i - 1)(n - \frac{L_i}{2})$. As $f_n(k)$ is a non-decreasing function when $k \leq n$ and as $L_i \leq L$ for all $i \in [1, d]$, it follows that $|E(G_i)| \leq (L-1)(n - \frac{L}{2})$. One concludes the proof considering that $|E(G)| = \sum_1^d |E_i(G)|$.

Lemma 11 *If G is the conflict graph of a direction-segmentable routing R on a hypermesh of dimension d, one of the nodes of G is of degree at most $2d(L-1)$, where L is the load of R.*

Proof: The average node degree in G is $\frac{2 \times E(G)}{n}$, where n is the number of vertices of G. One can conclude from lemma 10.

Theorem 12 *Any direction-segmentable routing in a hypermesh of dimension d can be coloured in polynomial time using at most $2d(L-1)+1$ colours, where L is the routing load.*

Proof: By induction on the number n of paths in the routing R. The result is straightforward if $n = 1$. As colouring the routing is equivalent to colouring the nodes of its conflict graph, let $n > 1$ and let G be the conflict graph induced by R. From lemma 11, some node p in G is of degree $2d(L-1)$ at most. Let R' be the routing obtained from R by suppressing the path p, G' be the conflict graph induced by R', and L' be the load of R'. By the induction hypothesis, G' can be coloured using $2d(L'-1)+1$ colours at most, thus $2d(L-1)+1$ colours at most. Considering the degree of p yields the result.

As an interesting special case, theorem 12 yields:

Corollary 13 *Any 1-turn routing in a mesh can be coloured in polynomial time using at most $4L-3$ colours, where L is the routing load.*

4. The all-optical 1-turn routing problem

Given some positive integer k, let the k***-all-optical 1-turn routing problem*** be the decision problem defined as follows: given some communication instance in a mesh, is there an all-optical 1-turn routing for this instance which uses k colours at most?

Theorem 14 *For any $k \geq 2$, the k-all-optical 1-turn routing problem is $\mathcal{NP}$–complete.*

Proof: We take advantage of the proof of theorem 3 and we assume $k = 2$ (as for theorem 3, the proof is easily extended to $k \geq 2$). Let C be some instance of 3–*SAT* and let I be the communication instance associated with C in the proof of theorem 3. One can check that I can be satisfied using 2 colours if and only if there is a 2-load 1-turn routing which satisfies I, that is, as in the proof of theorem 3, if and only if C is satisfiable. Which leads to the conclusion.

Given a communication instance I and a 1-turn routing S for this instance, let $\pi(S)$, resp. $\omega(S)$, denote the load, resp. the number of colours, used by S. Similarly, let $\pi(I)$, resp. $\omega(I)$, denote the load of a minimum load 1-turn routing for I, resp. the number of colours used by a minimum all-optical 1-turn routing for I. As mentioned before, one has $\pi(S) \leq \omega(S)$, and therefore $\pi(I) \leq w(I)$ as well.

Theorem 15 *The minimum all-optical 1-turn routing problem is 8–APX.*

Proof: Let I be some communication instance whose network is a mesh, let S be a routing for I computed by a 2-approximation minimum load 1-turn routing algorithm whose existence is asserted by theorem 8, and let

$c(S)$ be the number of colours used by a path colouring algorithm using at most $4 \times \pi(S)$ colours, whose existence is asserted by theorem 13.

We then have $c(S) \leq 4 \times \pi(S) \leq 4 \times 2 \times \pi(I)$, and we conclude with the general inequality $\pi(I) \leq \omega(I)$.

5. Conclusion

In general, the minimum all-optical routing problem and the minimum load routing problem are both $\mathcal{NP}$–*hard*, and it is not known whether they are *APX* or not, while the minimum path colouring problem is both $\mathcal{NP}$–*hard* and no-APX. Restricting these problems to meshes does not change their complexity status. In this paper, we restricted these three problems to 1-turn routings in meshes.

Regarding load routing problems, we proved the L-load 1-turn routing problem to be in $\mathcal{P}$ when $L = 1$ and otherwise $\mathcal{NP}$–*complete*, and we provided a 2-APX algorithm to solve the associated minimizing problem.

Regarding the minimum 1-turn path colouring problem, we proved it to be 4-approximable, where L is the load of the path family, which is an improvement over several previous results known to us (namely, 8-approximation algorithms [28,22,29,21]). This result stems from a result expressed for *dimension-segmentable paths* in *meshes of dimension d*.

Regarding the minimum all-optical 1-turn routing problem, and due to the indirect proof of the result, we think the constant asserted in the 8-APX result (see theorem 15) can be improved upon.

Last, it is worth noting that, not surprisingly, some results can be extended from meshes to tori.

References

[1] Bruno Beauquier, Jean-Claude Bermond, Luisa Gargano, Pavol Hell, Stéphane Pérennes, and Ugo Vaccaro. Graph problems arising from wavelength-routing in all-optical networks. In *Proceedings of 2nd IEEE Workshop on Optics and Computer Science*, Geneva, Switzerland, apr 1997. IEEE Press. part of IPPS'97.

[2] Alexander Schrijver. *Combinatorial Optimization*, volume C of *Algorithms and Combinatorics*. Springer, 2003.

[3] Alok Aggarwal, Amotz Bar-Noy, Don Coppersmith, Rajiv Ramaswami, Baruch Schieber, and Sudan Madhu. Efficient routing and scheduling algorithms for optical networks. In ACM-SIAM, editor, *Proceedings of the 5th Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 412–423, Arlington, Virginia, USA, jan 1994.

[4] Alok Aggarwal, Amotz Bar-Noy, Don Coppersmith, Rajiv Ramaswami, Baruch Schieber, and Sudan Madhu. Efficient routing in optical networks. *Journal of the ACM*, 6(43):973–1001, nov 1996.

[5] Matthew Andrews and Lisa Zhang. Hardness of the undirected congestion minimization problem. In *37th Annual ACM Symposium on Theory Of Computing (STOC'O5)*, Hunt Valley, Maryland, USA, 2005. ACM e-store. CD-ROM.

[6] Julia Chuzhoy and Joseph Naor. New hardness results for congestion minimisation and machine scheduling. In *36th Annual ACM Symposium on theory of Computing (STOC'O4)*, pages 28–34, Chicago, Illinois, USA, June 2004. ACM press.

[7] Thomas Erlebach and Klaus Jansen. Scheduling of virtual connections in fast networks. In *Proceedings of the 4th Parallel Systems and Algorithms Workshop (PASA'96)*, pages 13–32. World Scientific Publishing, 1997.

[8] M. R. Kramer and J. Leeuwen. The complexity of wire-routing and finding minimum area layouts for arbitrary VLSI circuits. *Advances in Computing Research*, 2:129–146, 1984.

[9] Yuval Rabani. Path coloring in the mesh. In *FOCS*, pages 400–409, 1996.

[10] Bruno Beauquier. *Communications dans les réseaux optiques par multiplexage en longueur d'onde*. PhD thesis, Université de Nice - Sophia Antipolis, jan 2000.

[11] G. Ausiello, P. Crescenzi, G. Gambosi, V. Kann, A. Marchetti-Spaccamela, and M. Protasi. *Complexity and Approximation*. Springer, 2003.

[12] Thomas Erlebach and Klaus Jansen. The complexity of path coloring and call scheduling. *Theoretical Computer Science*, 255:33–50, 2001.

[13] Prabhakar Raghavan and Eli Upfal. Efficient routing in all-optical networks. In *Proceedings of the 26th ACM symposium on Theory of Computing (STOC'94)*, pages 134–143. ACM Press, may 1994.

[14] Milena Mihail, Christos Kaklamanis, and Satish Rao. Efficient access to optical bandwidth - wavelength routing on directed fiber trees, rings, and trees of rings. In *Proceedings of 36th IEEE Annual Symposium on Foundations of Computer Science (FOCS'95)*, pages 548–557. IEEE Press, oct 1995.

[15] Thomas Erlebach. *Scheduling Connections in Fast Networks*. PhD thesis, Technische Universit?t München, 1999.

[16] Gordon Wilfong and Peter Winkler. Ring routing and wavelength translation. In *Annual ACM-SIAM Symposium on Discrete Algorithms*, volume 9, pages 333–341. ACM-SIAM, 1998.

[17] Christos Kaklamanis and Pino Persiano. Efficient wavelength routing on directed fiber trees. In *Lecture Notes in Computer Science*, volume 146, pages 444–473. Springer Verlag, 1996.

[18] Guillaume Bagan, Olivier Cogis, and Jérôme Palaysi. Approximable all-optical routing problems in all-optical mesh networks (revisited). Technical report, LIRMM, Univ. Montpellier 2, CNRS, 161 rue Ada, 34 392 Montpellier Cedex 5, may 2007. url : http://hal-lirmm.ccsd.cnrs.fr/lirmm-00150160.

[19] Thierry Chich. *Optimisation du routage par déflexion dans les réseaux de télécommunications métropolitains*. PhD thesis, École Normale Sup?rieure de Lyon, dec 1997.

[20] Heiko Schröder, Ondrej Sýkora, and Imrich Vrt'o. Optical all-to-all communication for some product graphs. In *Proceedings of the 24th Seminar on Current Trends in Theory and Practice of Informatics*, 1997.

[21] Sandeep N. Bhatt, Gianfranco Bilardi, Geppino Pucci, Abhiram Ranade, Arnold L. Rosenberg, and Eric J. Schwabe. On bufferless routing of variable length messages in leveled networks. *Transactions on Computers*, 45(6):714–729, june 1996.

[22] Jérôme Palaysi. Approximability of paths coloring problem in mesh and torus networks. In *Proceedings of the Second Colloquium on Mathematics and Computer Science: Algorithms, Trees, Combinatorics and Probabilities*, pages 213–222, University of Versailles-St-Quentin (France), sep 2002.

[23] Thomas Erlebach and Klaus Jansen. Call scheduling in trees, rings and meshes. Unpublished report, presumably long version to "Call Scheduling in Trees, Rings and Meshes" in Proceedings of the 30th Hawaii International Conference on System Sciences (HICSS-30), IEEE Computer Society Press, pages 221-222, 1997.

[24] M. R. Garey and D. S. Johnson. *Computers and Intractability : A Guide to the Theory of NP-Completeness*. Freeman, 1979.

[25] Christos H. Papadimitriou. *Computational Complexity*. Addison-Wesley Publishing Company, 1994.

[26] J. K. Lenstra and D. B. Shmoys. *Scheduling Theory and its Applications*, chapter 1 Computing Near-Optimal Schedules. John Wiley and Sons, 1995.

[27] Andreas Brandst?dt, Van Bang Le, and Jeremy P. Spinrad. *Graph classes*. SIAM, 1999.

[28] Mustapha Kchikech and Olivier Togni. Paths coloring algorithms in mesh networks. In C.S. Calude et al., editor, *DMTCS 2003*, pages 193–202, 2003.

[29] T. Erlebach and K. Jansen. Conversion of coloring algorithms into maximum weight independent set algorithms in communication networks. In Carleton Scientific, editor, *ARACNE 2000*, pages 135–146, 2000.

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