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The α-reliable shortest path problem

P. Beraldi and F. Guerriero

Department of Electronics, Informatics and Systems, University of Calabria, 87030 Rende (CS) - Italy

Abstract

Many real-life applications, arising in transportation and telecommunication systems, can be mathematically represented as shortest path problems. The deterministic version of the problem, where a deterministic cost is associated to each arc and the configuration of the network (nodes and arcs) is assumed to be known in advance, is easy to solve and has been extensively studied. However, in real applications, costs are typically not known a priori and may be subject to significant uncertainty. In addition, due to failure, maintenance, natural disasters, weather conditions, etc., some arcs could not be available causing a change of the network configuration. In this paper we introduce a variant of the shortest path problem under uncertainty, that concerns the situation in which for each arc two different states are possible (i.e. operating and failed states) and the aim is to find the path connecting a given pair of nodes with a sufficiently large probability α and such that the total cost is minimized. The problem can be formulated as a large scale integer programming model with knapsack constraints. For its solution a heuristic approach has been designed and implemented. Preliminary numerical experiments have been carried out on a set of randomly generated test problems.

Key words: Shortest path, stochastic programming, heuristic approach.

1. Introduction

Graphs and networks can be used to model many important problems in engineering, business, physical and social science ([1]).

Among the network optimization problems, the shortest path problem plays a crucial role. Indeed, many real applications arising in computer, telecommunication, urban traffic, logistic systems can be represented and solved as shortest path problems.

The deterministic version of the problem is defined on a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, \ldots, n\}$ denotes the set of nodes, and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ the set of arcs. To each arc $(i, j) \in \mathcal{E}$ is associated a nonnegative scalar cost c_{ij}, representing either the traversal time of the arc or the arc length. Given two distinct nodes o (referred to as origin node) and d (referred to as destination node), a path from node o to node d is a sequence of nodes $\Pi^{(od)}= \{o = i_1, \ldots, i_l = d\}$, $l \geq 2$ such that $(i_k, i_{k+1}) \in \mathcal{E}$ for $k = 1, \ldots, l-1$.

The shortest path problem concerns finding the cheapest way to connect o to d. From a mathematical standpoint, the problem can be defined as follows:

Email: F. Guerriero [guerriero@deis.unical.it].

$$\min c^T x \tag{1}$$

$$Ax = b \tag{2}$$

$$x \in \{0,1\}^{|\mathcal{E}|} \tag{3}$$

where A is the incidence matrix associated to $\mathcal{G}$ and the vector b is defined as follows:

$$b_i = \begin{cases} 1 & if \text{ i=o} \\ -1 & if \text{ i=d} \\ 0 & otherwise \end{cases}$$

It is well known that, because of the unimodularity property of the constraint matrix A, the integer constraints (3) can be relaxed.

In the formulation (1)–(3), the network configuration (i.e. cost coefficients, nodes and arcs) is assumed to be known in advance. It is evident that, even though the deterministic version of the problem can be easily solved, the assumptions made are rather restrictive, making model (1)–(3) not adequate to model real life problems. In effect, uncertainty, either associated with physical phenomena that are inherently random or with predictions or estimations of reality, is pervasive in all real applications ([6]). In particular, in the case of the shortest path problem uncertainties could arise from a variation of either the cost coefficients or the network structure.

The shortest path problem under arc length uncertainties has been studied by different authors [7,9,11,13–15,19,21,22,25–27,30] and different models for dealing with uncertainty in data have been proposed. In particular, in [13,21,22], data uncertainty is structured by taking arc length as interval ranges defined by known upper and lower bounds and without assuming any probability distribution, whereas in [9,15,30], uncertainty is modelled by means of a discrete scenario set. In this case, each scenario represents a potential realization of the arc lengths which occurs with a given probability level. Finally, in [7,11,14,19,25,26],and [27], possibility models, where arc lengths are represented by fuzzy numbers, are used to deal with arc length uncertainties.

Uncertainties related to variations of the network structure may be caused by different factors, which are typically classified in abnormal events as serious disasters, huge accidents, large-scale maintenance work, and normal events as traffic accidents, adverse weather conditions (see for example, [3,17,18]). Typically, uncertainty in the network structure is led to the uncertainty in the arc state, which is modelled by Bernoulli random variables taking only to possible outcomes, representing either an $operating$ or a $failed$ state ([17]).

Path problems under arc state uncertainties have been addressed within the network connection reliability framework ([8,17]). More specifically, given a network where each arc is assumed to be operating or failed, connectivity reliability is related to the probability that nodes remain connected. A special case of connectivity reliability is the terminal reliability, that concerns the existence of a path between a specific pair of nodes ([3,8,17,18]). In this case, the objective is to find the probability that a pair of nodes in the network remains connected, when one or more arcs are obstructed or unavailable. The terminal reliability problem has been also analyzed in [16]. In particular, the paper considers the general case of dependent probability failure and proposes a solution approach to find the best possible bounds on the probability of an operating path between a given pair of nodes.

Another problem, somehow related to the connection reliability issue and mainly addressed in the context of the telecommunication systems, is the reliability constrained least-cost problem ([31]). Two different parameters, reliability and cost, are associated with each arc of the network. The reliability of a path is defined as the probability that all the arcs are available simultaneously, and it is equal to the product of the arc reliability, whereas the cost is simply computed as the sum of the costs of the arcs of the path. The aim is to find a path between a given pair of nodes such that its reliability is greater than a given value and the cost is minimized. To solve such a NP-hard problem a heuristic approach has been proposed by the authors in [31]. The basic idea is to define, for each arc, a new parameter obtained as weighted sum of the reliability value and the cost and to compute the shortest path according to this new value hoping that the resulting path satisfies the reliability requirement and the cost is as little as possible.

In this paper we introduce a variant of the shortest path problem under arc state uncertainty. We consider a directed network and we assume that known cost coefficients are associated to arcs, whereas arc states are uncertain. Given a pair of nodes o and d and a reliability level α, the problem concerns determining the shortest path which, despite the random state of network, connects o to d with reliability α. In what follows, the problem under investigation will be referred to as α-reliable shortest path problem (α- $\mathcal{RSPP}$ for short). To model this problem, we adopt a scenario based approach. We observe that, even though somehow related to other shortest path problems under uncertainty referred above, to the best of our knowledge, the α-$\mathcal{RSPP}$ has not been addressed by the scientific literature in the proposed form.

The α-$\mathcal{RSPP}$ model can be viewed as an interesting generalization of the shortest path problem under uncertainty. In many problems of practical interest, arising in the field of transportation and telecommunication, the network's structure can be uncertain and the proposed model can be applied as useful modeling tool. For example, in data routing problems, defined on telecommunication networks with randomly failing arcs, it is of main concern to ensure that information is delivered from some source node to some destination node with a sufficiently large probability α ([29]). Other potential applications in the telecommunication field may regard the point to point network connectivity ([23]) and the path selection in ad hoc networks ([28]).

In the transportation networks, the evaluation of the travel time reliability plays a crucial role, in order to provide drivers with accurate route guidance information and to find the shortest paths, connecting origins and destinations, especially under conditions of varying demands and limited capacities ([4,**?**]). In this context, the proposed α-$\mathcal{RSPP}$ model might be useful in the evaluation of the operational reliability of the system, since it allows to treat, by scenarios, the more general case of dependent link failures.

The rest of the paper is organized as follows. In Section 2 we introduce the α-reliable shortest path problem. The inherent complexity of the problem poses the crucial issue of designing efficient solution approaches. In Section 3 we present a heuristic solution strategy. Preliminary computational results are reported in Section 4, while conclusions and future research directions are drawn in Section 5.

2. Model definition

In order to model the α-reliable shortest path problem we adopt a scenario approach ([6]). We assume that the state of each arc $(i,j) \in \mathcal{E}$, is represented by a Bernoulli random variable $\omega_{i,j}$ taking the value 1 if the corresponding arc is operating and 0 otherwise. Thus, in the formulation (1)–(3), the deterministic incidence matrix A is replaced by its random counterpart $A(\omega)$. We note that no assumption on independence of arc states is made. We assume that the "random state" of the network can be described by finitely many scenarios, each of them representing a snapshot of the network situation. Representation by scenarios is frequently adopted in strategic models where the knowledge of the possible uncertain outcomes in the future is obtained through expert's judgments and only a finite number of possible realizations are considered in detail. In the following, we shall denote by $\mathcal{S} = \{1,\ldots,S\}$ the index set for scenarios, each of them occurring with a given probability level p_s, and by A^s the corresponding incidence matrix.

We shall assume that there always exists a feasible solution (i.e., the destination can be reached starting from the origin, whatever scenario will materialize). This is a standard assumption in stochastic programming (i.e., the problem has complete recourse) and can be made without loss of generality, since it is possible to ensure complete recourse in any problem, by considering constraints violation penalty costs ([6]).

It is evident that requiring the satisfaction of the flow conservation constraints

$$A^s x = b \tag{4}$$

for all the scenarios $s \in \mathcal{S}$, corresponds to determine the shortest path of reliability 1. Such a solution would result very expansive taking also into account scenarios particularly unfavorable that could occur with a very low probability. Since the determination of the shortest path is typically a strategic decision, that should be taken "here and now" without full information about the future state of the network, it would result more appropriate to operate in a reliability perspective by looking for the shortest path with a given reliability level α. The choice of the value α depends on the specific application at hand. Eventually, it would be possible to find the shortest paths for different reliability levels and choose the one that guarantees the best cost-reliability trade off.

In the following, we illustrate as the α-$\mathcal{RSPP}$ can be mathematically defined.
Let us consider the set $\mathcal{K} = \{1,\ldots,K\}$ whose elements $k \in \mathcal{K}$ are defined as

$$k \subseteq \{1,\ldots,S\}, \sum_{s\in k} p_s \geq \alpha \tag{5}$$

Then, the α-$\mathcal{RSPP}$ can be formulated as follows:

$$\min c^T x \tag{6}$$

$$\bigcup_{k\in\mathcal{K}} \bigcap_{s\in k} \{A^s x = b\} \tag{7}$$

$$x \in \{0,1\}^{|\mathcal{E}|} \tag{8}$$

Problem (6)-(8) is clearly nonconvex. In effect, two sources of nonconvexity are merged: the former related to the binary restriction on the decision variables and the latter to the disjunctive constraint (7). By adopting a standard technique (Big-M approach) used in disjunctive programming ([2]), (6) can be rewritten by introducing binary variables. In particular, once replaced the equality constraints $A^s x = b$ by two inequality constraints $A^s x \geq b$ and $A^s x \leq b$, we introduce a vector $M \in R^{|\mathcal{N}|}$ such that, for scenarios $s = 1,\ldots,S$,

$$A^s x + M \geq b \quad \text{and} \quad A^s x - M \leq b.$$

In addition, we introduce a vector y of binary variables whose components y_s, $s = 1,\ldots,S$ take value 0 if the corresponding set of constraints has to be fulfilled and 1 otherwise. Thus, problem (6)-(8) can be equivalently rewritten as:

$$\min c^T x \tag{9}$$

$$A^s x + M y_s \geq b \quad s = 1,\ldots,S \tag{10}$$

$$A^s x - M y_s \leq b \quad s = 1,\ldots,S \tag{11}$$

$$\sum_{s=1}^{S} p^s y_s \leq (1-\alpha) \tag{12}$$

$$y_s \in \{0,1\} \quad s = 1,\ldots,S \tag{13}$$

$$x \in \{0,1\}^{|\mathcal{E}|} \tag{14}$$

We observe that (12)-(13) define a binary knapsack constraint which assures the violation of the stochastic constraints for a subset of scenarios whose cumulative probability is less than the complement of the imposed reliability level.

It is evident that the full integer programming problem turns out to be very difficult to solve: the number of flow conservation constraints is duplicated in the number of scenarios, which is typically a very large number for real life applications. Nevertheless, the introduction of the knapsack restriction causes the loss of the total unimodularity property of the constraint matrix.

3. The solution approach

The inherent complexity of problem (9)-(14) poses the crucial question of designing efficient solution approaches. It is worthwhile noting that the reformulation (9)-(14) suggests a straightforward solution strategy. Rather than attacking the full formulation of the problem, we may divide the solution approach into two phases. In the first one, we derive all the feasible solutions of the knapsack constraint, whereas in the second one we use such solutions to select different subproblems to solve. The optimal solution of the original problem will be the best solution among those obtained by solving the different subproblems.

This simple approach has the great advantage of allowing the total unimodularity property of the constraint matrix to be regained. On the contrary, it suffers from the disadvantage imposed by the exact nature of the method, i.e. requiring the determination of all the feasible solutions of the knapsack constraint. The number of these solutions can be really huge for a reasonable number of scenarios, making prohibitive the application of this approach. For example, in the case of equiprobable scenarios the number of feasible solutions is given by:

$$\binom{S}{V},$$

where $V = \lfloor (1-\alpha) * S \rfloor$.

The considerations introduced above show that the complexity of the problem limits the applicability of exact solution methods to problems of small size. Nevertheless, no known exact approach seems to be directly applicable to efficiently solve the considered problem. The definition of a specific approach which exploits the particular structure of the problem is beyond the scope of the paper and is the subject of ongoing research. In the following we shall focus on heuristic strategies and we present our proposal.

3.1. *The two-stage heuristic strategy*

The proposed heuristic strategy can be seen as a particularization of the approach introduced above. Rather than enumerating all the feasible solutions of the knapsack constraint, we select one (or a limited number) of such solutions according to some specific criteria.
The method is based on the solution of two problems defined at two different stages.

In the first stage we solve the following knapsack problem:

$$\max \sum_{s=1}^{S} \gamma_s y_s \tag{15}$$

$$\sum_{s=1}^{S} p^s y_s \leq (1-\alpha) \tag{16}$$

$$y_s \in \{0,1\} \quad s = 1, \ldots, S \tag{17}$$

Let us denote by y^* the optimal solution of the problem. On the basis of the values of y^*, we define the set $\bar{\mathcal{S}} = \{s = 1, \ldots, S \mid y_s^* = 0\}$, which is used to define the second stage problem:

$$\min c^T x \tag{18}$$

$$A^s x = b \quad \forall s \in \bar{\mathcal{S}} \tag{19}$$

$$x \in \{0,1\}^{|\mathcal{E}|} \tag{20}$$

We observe that different heuristic strategies may be defined by specifying the values of the parameters γ_s in (15). In the following, we present two strategies: the simple strategy and the cost based one.

In the simple strategy ($\mathcal{SS}$) the values of γ_s are fixed to 1 for all the scenarios. Thus, the aim of model (15)-(17) is to find the solution of knapsack problem that minimizes the cardinality of the set $\bar{\mathcal{S}}$. The main drawback of this simple strategy is that no information about the different scenarios is used to perform the selection. In effect, minimizing the number of flow conservation constraints to satisfy, does not necessary lead to a good choice. For example, if the probability value associated to each scenario is the same, then the number of feasible solutions with the same objective function value can be really large.

The cost based strategy ($\mathcal{CS}$) has been defined with the aim to overcome the drawback of the simple approach. In particular, for each scenario s, the weight γ_s is computed by taking into account the cost of the path associated to that specific scenario. This choice of the values of γ_s is motivated by the following simple observation.

Let us denote by z^s the cost of the path associated to the scenario s. It is evident that, given a subset of scenarios $\bar{\mathcal{S}}$, $z^* \geq \max\{z^s, s \in \bar{\mathcal{S}}\}$, where z^* denote the optimal solution over $\bar{\mathcal{S}}$.

Among all the feasible solutions of the knapsack problem, the cost based approach provides the solution associated to the subset $\bar{\mathcal{S}}$ with minimal weight.

It is worthwhile observing that enlarging the search space would greatly improve the quality of the sub-optimal solution. In our case, this can be accomplished by exploring multiple feasible solutions. To this aim, we have defined the following basic scheme: once determined the optimal solution of the knapsack problem, starting from that solution we try to determine other ones by performing a simple swap of one or more objects. Eventually, it is possible to define a threshold on the number of solutions to generate.

More specifically, let y^* be the optimal solution of the knapsack problem and let $\tilde{\mathcal{S}}$ denote the subset of scenarios such that $y^*_s = 1$, $\forall s \in \tilde{\mathcal{S}}$.

In the simple case in which the swap involves only one object and equiprobable scenarios are considered, starting from y^* another feasible solution $\tilde{y}$ can be obtained by executing the following operations:

- Select an index $\tilde{s} \in \tilde{\mathcal{S}}$.
- Choose an index $\bar{s} \in \mathcal{S} \setminus \tilde{\mathcal{S}}$.
- Set $\tilde{y}_{\tilde{s}} = 0$.
- Set $\tilde{y}_{\bar{s}} = 1$.
- Set $\tilde{y}_s = y^*_s$, $\forall s \in \mathcal{S} \setminus \{\tilde{s}, \bar{s}\}$.

Following the strategy described above, the total number of different feasible solutions, that can be determined starting from y^*, is equal to $(\mid \tilde{\mathcal{S}} \mid * \mid \mathcal{S} \setminus \tilde{\mathcal{S}} \mid)$.

In the case in which a different probability level is associated to each scenario and only one object is considered for the swapping, in order to generate, starting from the optimal solution y^*, another feasible solution, chosen the index $\tilde{s} \in \tilde{\mathcal{S}}$, the index $\bar{s} \in \mathcal{S} \setminus \tilde{\mathcal{S}}$ has to be selected in such a way that the condition $p_{\bar{s}} \leq p_{\tilde{s}}$ is satisfied.

In the general case, in which the swap operation involves $\eta \leq \min(\mid \tilde{\mathcal{S}} \mid, \mid \mathcal{S} \setminus \tilde{\mathcal{S}} \mid)$ objects, starting from y^* a feasible solution $\tilde{y} \not\equiv y^*$ is determined as follows.

(1) Select two subsets of scenarios $\bar{\mathcal{S}}$ and $\hat{\mathcal{S}}$, that satisfy the following conditions:
   - $\bar{\mathcal{S}} \subseteq \mathcal{S} \setminus \tilde{\mathcal{S}}$ and $\hat{\mathcal{S}} \subseteq \tilde{\mathcal{S}}$;
   - $\mid \hat{\mathcal{S}} \mid = \mid \bar{\mathcal{S}} \mid = \eta$;
   - $\sum_{\bar{s} \in \bar{\mathcal{S}}} p_{\bar{s}} \leq \sum_{\hat{s} \in \hat{\mathcal{S}}} p_{\hat{s}}$.

(2) Set $\tilde{y}_{\hat{s}} = 0, \forall \hat{s} \in \hat{\mathcal{S}}$;

(3) Set $\tilde{y}_{\bar{s}} = 1, \forall \bar{s} \in \bar{\mathcal{S}}$;

(4) Set $\tilde{y}_s = y^*_s$, $\forall s \in \mathcal{S} \setminus \{\hat{\mathcal{S}} \cup \bar{\mathcal{S}}\}$.

4. Numerical illustration

In this section we report on preliminary computational experiments carried out on a set of test cases. We observe that the lack of a library of problems to use as benchmark has initially posed the problem of generating suitable instances. To this aim a scenario generator has been designed and implemented. It takes as input an $initial$ network and generates a set $\mathcal{S}$ of scenarios that satisfies the following conditions:

- the flow conservation constraints are satisfied for all scenarios;
- the set $\mathcal{S}$ contains non-dominated scenarios.

For a detailed description of the scenario generator the reader is referred to [5]. The characteristics of the test problems are reported in Table 2. For each test problem, we indicate the number of nodes, arcs and scenarios. The initial networks have been generated with the public-domain program Netgen ([24]). All arc costs have been chosen according to an uniform distribution from the range $[1, 100]$. For all the considered test problems, the probability levels have been randomly generated and normalized.

Table 1

Problem	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{E}\rvert$	$\lvert\mathcal{S}\rvert$
Test1	100	500	30
Test2	200	1000	40
Test3	300	6000	40
Test4	500	8000	50
Test5	1000	10000	60

Characteristics of the test problems

The state-of-the-art LINGO 8.0 ([20]) has been used to solve integer programs. More efficient codes, based for example on the Dijkstra method ([10]) could be used to solve the shortest path problems within the heuristic scheme. However, the choice of a general purpose solver has been motivated by the aim of making the results reproducible and of evaluating the speed-up provided by the heuristic method over the exact counterpart. All the experiments have been carried out on an Intel Pentium Centrino 1.6 GHz, 1 GB RAM, under the Microsoft Windows XP Professional operating system.

The numerical results have been collected for different values of the reliability level α, in particular for $\alpha = 0.75, 0.80, 0.85, 0.90, 0.95, 0.99$.

The following figure shows the optimal objective function value versus the reliability level for the problem Test4.

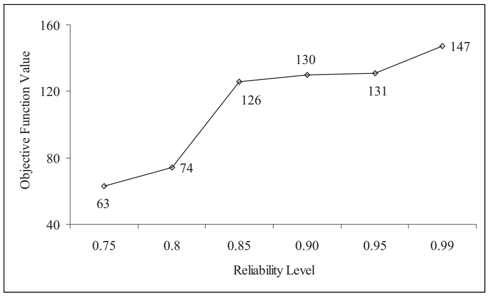

Fig. 1. Objective Function Value versus Reliability Level for the problem Test4

As expected, the higher the value of α the higher the objective function value. We observe, however, that even for high values of α, a reduction in the objective function value with respect to the case of $\alpha = 1$ may be registered (similar behavior has been observed for the other test problems). The choice of the reliability level is up to the decision maker, who, on the basis on his/her experience and the specific problem at hand, will decide the appropriate value also evaluating the cost-reliability trade-off.

In the evaluation of the performance of a heuristic approach two different parameters are typically considered: the computational effort and the solution quality. The first parameter is meant as the speed of computation of the heuristic approach over the exact counterpart. Obviously, the solution of the full integer programming problem is computationally demanding and the lack of a specific exact method makes the comparison of little value. For completeness, we note that the speed-up factor is about 15 % for Test1 and that larger savings are achieved as the size of the instance is increased.

The solution quality has been evaluated by computing the relative percentage error ϵ, defined as follows:

$$\epsilon = \frac{\tilde{z} - z^*}{z^*} * 100,$$

where z^* represents the cost of the optimal solution, whereas $\tilde{z}$ is the cost of an approximate solution determined by the heuristic method.

In Table 3, for each instance, we report the relative percentage error of the best heuristic solution (between the $\mathcal{SS}$ and the $\mathcal{CS}$ strategies) with respect to the optimal solution.

The preliminary numerical results seem to be encouraging (see Table 3). They show that the cost of the heuristic solution is higher than that of the optimal one only in few cases and, in addition, the worsening of solution quality is limited (i.e., 3.63, 8.55, 2.68 and 3.65%). Obviously further experiments are necessary to confirm this favorable trend. This is beyond the scope of this contribution whose main aim is the proposal of a new model and its validation by a preliminary testing phase.

A last consideration concerns the comparison of the two heuristic strategies. Figures 2-6 show the objective function values of $\mathcal{SS}$ and $\mathcal{CS}$ strategy for the different values of α. The analysis of the results shows that, as expected, $\mathcal{CS}$ always outperforms $\mathcal{SS}$ in terms of solution quality.

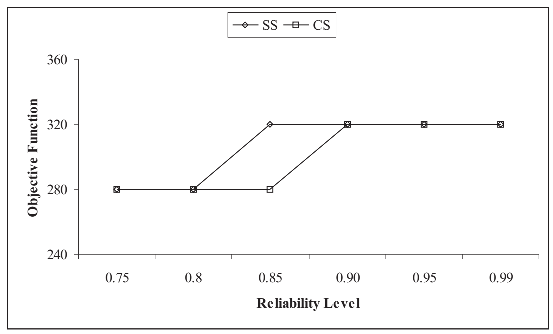

Fig. 2. $\mathcal{SS}$ versus $\mathcal{CS}$ strategy for Test1

5. Conclusion

In this paper we have addressed the α-reliable shortest path problem, a variant of the shortest path problem under arc uncertainty. Given a pair of nodes and a reliability level α, the objective is to determine the shortest path which, despite the random state of the network, connects the selected nodes with reliability α.

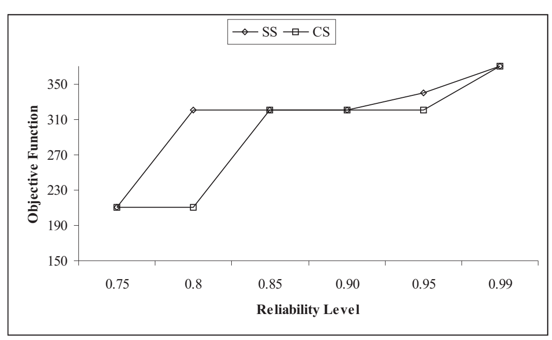

Fig. 3. $\mathcal{SS}$ versus $\mathcal{CS}$ strategy for Test2

Table 2

Problem	$\alpha = 0.75$	$\alpha = 0.8$	$\alpha = 0.85$	$\alpha = 0.9$	$\alpha = 0.95$	$\alpha = 0.99$
Test1	0.00%	0.00%	0.00%	3.63%	0.00%	0.00%
Test2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Test3	0.00%	8.55%	0.00%	0.00%	0.00%	0.00%
Test4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Test5	2.68%	0.00%	3.65%	0.00%	0.00%	0.00%

Percentage error ϵ of the best heuristic solution with respect to the optimal solution.

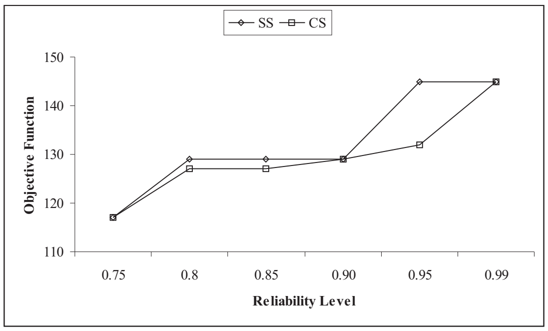

Fig. 4. $\mathcal{SS}$ versus $\mathcal{CS}$ strategy for Test3

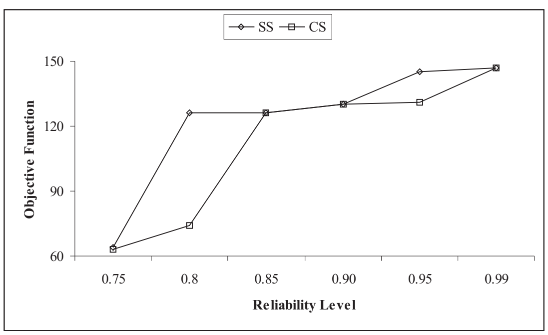

Fig. 5. $\mathcal{SS}$ versus $\mathcal{CS}$ strategy for Test4

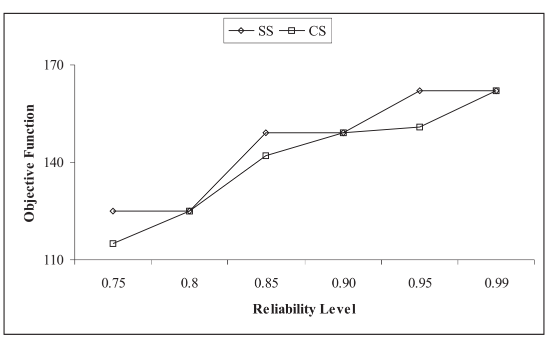

Fig. 6. $\mathcal{SS}$ versus $\mathcal{CS}$ strategy for Test5

The resulting model is a large scale integer programming problem with knapsack constraints. For its solution a two-stage heuristic scheme, based on a partial enumeration of the feasible solutions of the knapsack constraint, has been designed and implemented. Preliminary numerical experiments have been carried out on a set of randomly generated test problems. The promising results prompt to enrich the computational phase also considering larger size instances derived from real applications.

Furthermore, to better evaluate the performance of the heuristic method, the design of an exact approach, which takes full advantage of the specific structure of the problem, is required. Its implementation is the subject of ongoing research.

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