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Probabilistic Model and Solution Algorithm for the Electricity Retailers in the Italian Market

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Abstract

The paper considers the problem of maximizing the profits of a retailer operating in the Italian electricity market. The problem consists in selecting the contracts portfolio and in defining the bidding strategy in the wholesales market while respecting the technical and regulatory constraints. A novel solution method based on a enhanced discovery of the search domain in the simulated annealing technique has been developed for its solution and a set of realistic test problems have been generated for its validation. The experimental results show that our method outperforms the standard simulated annealing by an improvement gap of 20,48% in average.

Key words: Probabilistic Models, Electricity Retailers, Hybrid Simulated Annealing Algorithm.

1. Introduction

With the deregulation of the electricity business, energy prices are no longer set by a regulator but by the market. As a result, both suppliers and consumers should construct their policies, while participating in the electricity market, on the basis of the profit maximization. With this view, the electricity is seen as any commodity (gas, oil, etc.) that is subject to the demand/offer rule. However, the power system is characterized by many complicating features the most important of which is the fact that electricity is not storable. Consequently, existing mathematical models for the profit optimization of other commodities cannot be applied to the context of power system and new models should be developed and applied. Such optimization problems become even more difficult whenever it concerns an operator who is allowed to buy and sell at the same time. This is the case of the electricity retailer (in Italian "*Grossisti*"); an operator that was introduced in Italy with the deregulation and whose role is to be an intermediary between sellers and buyers who are not interested in participating directly in the wholesales market. The retailers are allowed to buy and sell energy either through bilateral contracts or in the wholesales market. Moreover, the retailers in the Italian market have also the opportunity to trade the so-called *Green Certificates*. These are an environmental conservation tools introduced by the deregulation Order 79/99 that obliges each producer to dispatch a fraction of his production from renewal resources. A Green Certificate, having a value of 100 MWh and released by the system operator, represents the proof of such a green production that can be traded as contracts or in a specific market. It is worth noting here that the retailers in the Italian system are not allowed to have an own production capacity or to have a role in the distribution business.

In order to perform their mission, the retailers have to interact with the following electricity actors (figure 1):

- *Producers*: retailers can buy energy/capacity through bilateral contracts either from foreign suppliers or Italian producers;
- *Consumers*: retailers can sell energy/capacity through

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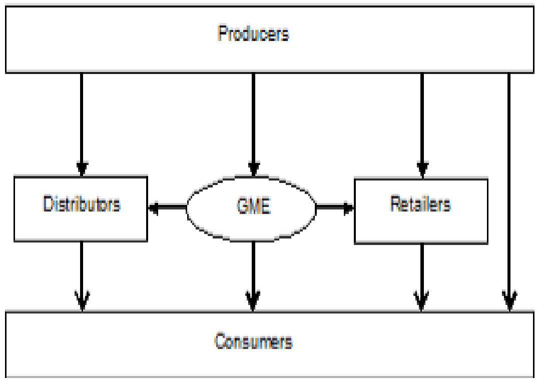

Fig. 1. **Main actors in the Italian electricity market**

bilateral contracts to all the Italian consumers;

- *GME*: the wholesales market represents another opportunity for the retailers to buy/sell energy, capacity, and green certificates. It is organized by the market operator *GME* (*Gestore del Mercato Elettrico*) with the technical support of the system operator called *Terna*.

More specifically, while Terna has the key role of ensuring the system's security and reliability, the GME is responsible for managing the short-term forward electricity exchange by using the auction as a market model for the price definition. The GME collects the offers/bids submitted by different players and provides the market clearing information. The market clearing price (MCP) and quantities (MCQ) for each time period are determined as the intersection of the aggregated supply and demand curves (Figure 2).

This paper has the objective of defining a profit maximization model for a retailer operating in the Italian electricity market and also developing a heuristic method for its solution. The field of the optimal management of power systems has represented for a long time a very active area of research. Besides the known problems of unit commitment ([7,28]), optimal power flow ([6,15]) and capacity expansion ([21,30]) that have had big importance for the electricity management both before and after the deregulation, many new problems have been generated by the liberalization process. Some of these problems arise as a revision of old models to make them suitable for the competitive context. Other problems describe the recent challenges faced by the different operators and should be defined from scratch. Literature is quite rich in addressing problems in both deterministic and stochastic frameworks. Examples of the first category include the optimization of the reservoirs in hydroelectric systems ([3,19]), the management of contingencies ([2,22]), etc. The list of the new challenging problems is quite long since each operator is involved with his specific models. A non exhaustive list may include the auction clearing ([4]) to be solved by the GME, the bidding strategy definition ([14,29]) and the contract selection ([9,20]) to be solved by both producers and consumers, and the energy pricing and tariff definition ([5,18]) that involve mainly the distributors. Many of these problems are discussed in recent books such as ([25–27]) and their mathematical formulations are presented in the review produced by Conejo and Prieto ([9]).

To the best of our knowledge only few papers have addressed the portfolio optimization problem for retailers. The review [9] includes a general discussion and a generic mathematical model to solve the decision problem to be faced by the retailers. Another general framework is also presented in [11]. A detailed model for the definition of optimal contracts for retailers has been proposed in [12]. However, this latter paper considers only a single client and moreover the resulting model presents some computational difficulties because of its nonlinear nature. A more robust (linear) model that takes into account the prices and demand uncertainties has been later proposed in [8]. Finally, the problem of definition of optimal bidding strategies for a pool retailer has been discussed in [10] by using a linear stochastic framework.

This paper provides an optimization framework and a solution approach for an Italian electricity retailer and proposes an integrated mathematical model for its profit maximization not only through the energy trading but also through the green certificates management. The paper is organized as follows: in sections 2 we introduce a deterministic model, in section 3 its probabilistic counterpart and in section 4 we discuss a hybrid simulated annealing method for their solution. Section 5 will be devoted to the experimental results and some concluding remarks will be drawn in the last section.

2. Deterministic Model

In developing the portfolio optimization model for an Italian retailer we only consider physical transactions without taking into account the financial tools. As a consequence, in developing our model we should consider that any quantity of energy that has been approved in the marketplace must be really dispatched. Moreover, it is necessary to take into account the topology of the network. The electric system in Italy is, indeed,

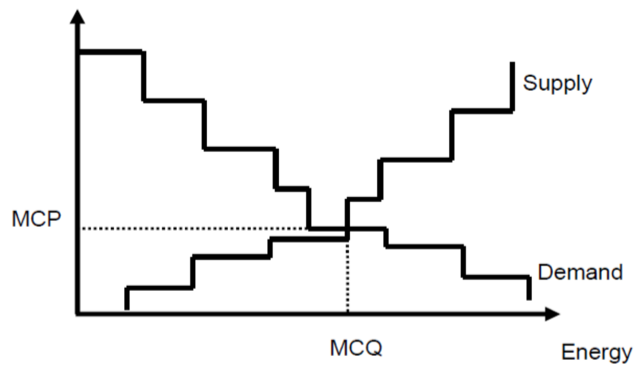

Fig. 2. **Price clearing in the electricity market**

divided into geographic zones that are connected by a limited transmission capacity which often causes congestion problems (Figure 3).

Fig. 3. **Transmission capacity between the geographic zones and border connections**

For this reason we will explicitly consider zonal index z in the model whenever it is appropriate and specially in indicating the consumption/emission point to which each energy transaction refers. Such information will possibly be used by the GME for clearing the energy markets whenever congestion happens. More specifically, the market clearing price will be unique if no congestion in the transmission network will be caused by the successful transactions, otherwise the GME defines different clearing prices one for each zone. A peculiarity of the Italian market consists in introducing a regulatory constraint imposing that, while zonal prices are allowed on the selling side, a uniform purchasing price has to be applied for all the zones of the Italian system for the first years of market operation. This feature introduces some modeling complexities into the optimization framework.

In what follows we describe our deterministic model taking into account the operative opportunities that a generic retailer has for making profits, i.e. bilateral contracts, wholesales market, and green certificate contracts. The proposed model is dynamic with respect to time and should cover a horizon period of one year following a rolling horizon basis. However, as suggested in [8], by using an appropriate time representation such a horizon can be divided into 72 periods. For the sake of simplicity, the problem is assumed to be separable with respect to time intervals and the model will be presented, thus, for a generic period without specifying any time index.

2.1. *Bilateral Contracts*

Bilateral contracts represent for the electricity operators and mainly for the retailers a risk aversion tool against the market price volatility. A retailer has, thus, always advantage to diversify his energy portfolio by designing a set of purchase and sales contracts. We assume here that the retailer designs for each contract a number of alternatives that may differ in price and/or quantity. Among these alternatives the model selects the set of contracts that maximizes the retailer's profits on the basis of the expected market clearing price.

Once a contract has been selected the retailer has the obligation to satisfy it, otherwise he will be subject to a penalty that is proportional to the missing quantity.

Thus, each bilateral contract will be totally characterized by its three attributes: price, quantity, and penalty. It is worth noting that the activation of a new contract will concern not only the operating time period but also future periods that are covered by the contract duration (two month, for example, as suggested in [8]). For those future periods the chosen energy quantity will be considered as problem data and will be included into the energy balance constraint.

In order to formulate the constraints related to the bilateral constraints we need to introduce the following notation:

Z	set of the geographic zones
CB^A	set of purchase bilateral contracts
CB^V	set of sales bilateral contracts
OPV^E	set of alternatives for each energy sales contract
OPA^E	set of alternatives for each energy purchase contract
$PRV^i_{h_z}$	selling price in alternative i of the energy sales contract h_z referred to an emission point located in zone z (€/MWh)
$QOV^i_{h_z}$	quantity in alternative i of the energy sales contract h_z referred to an emission point located in zone z (MWh)
$PNV^i_{h_z}$	penalty price for each dissatisfied energy unit i of the sales contract h_z (€/MWh)
$PRA^{i'}_{j_z}$	purchasing price in alternative i' of the energy purchase contract j_z referred to an emission point located in zone z (€/MWh)
$QOA^{i'}_{j_z}$	quantity in alternative i' of the energy purchase contract j_z referred to an emission point located in zone z (MWh)
$PNA^{i'}_{j_z}$	penalty price for each dissatisfied energy unit i' of the purchase contract j_z (€/MWh)

The decision variables corresponding to the bilateral contracts are:

$BINV^i_{h_z}$	binary variable that takes value 1 if alternative i of the sales bilateral contract h_z has been selected and 0 otherwise
$BINA^{i'}_{j_z}$	binary variable that takes value 1 if alternative i' of the purchase bilateral contract j_z has been selected and 0 otherwise
$FV^i_{h_z}$	fraction of energy satisfied out of alternative i of the sales bilateral contract h_z
$FA^{i'}_{j_z}$	fraction of energy satisfied out of alternative i' of the purchase bilateral contract j_z

The constraints on bilateral contracts can be written as:

$$FV^i_{h_z} \le BINV^i_{h_z} \qquad \forall h_z \in CB^V, \forall i \in OPV^E \quad (1)$$

$$FA^{i'}_{j_z} \le BINA^{i'}_{j_z} \qquad \forall j_z \in CB^A, \forall i' \in OPA^E \quad (2)$$

$$\sum_{i \in OPV^E} BINV^i_{h_z} \le 1 \quad \forall h_z \in CB^V \quad (3)$$

$$\sum_{i' \in OPA^E} BINA^{i'}_{j_z} \le 1 \quad \forall j_z \in CB^A \quad (4)$$

Constraints (1) and (2) ensure that the fraction of energy effectively satisfied is different from zero only if the corresponding contract has been selected, whereas constraints (3) and (4) impose that at most one alternative is selected for each contract h_z or j_z, respectively.

2.2. *Wholesales Market*

The wholesales electricity market includes four different auctions:

- Day Ahead Market (DAM): defines the preliminary dispatching programme. Operators participate by submitting energy supply offers and demand bids. Supply offers can be either simple (i.e. constituted by one price/quantity pair) or multiple (i.e. formed by up to four block components with the associated prices forming a non decreasing function).
- Adjustment market (AM): defines an updated preliminary dispatching programme. In this energy market both sellers and buyers are allowed to adjust their day-ahead schedules on the basis of the new information about the load forecast and the unit status.
- Dispatching Service Market (DSM): this market allows the definition of the final dispatching programme. Market participants submit offers/bids to increase or decrease injection or withdrawal for each elementary time period.
- Green Certificates Market (GCM): this is a weekly market that represents an occasion for the operators to exchange green certificates.

In order to simplify the model description, we suppose that all the offers/bids are simple and we restrict our attention only to DAM (*MGP* in Italian, i.e. "*Mercato del Giorno Prima*"). The extension to the other auctions is quite straightforward but involves additional variables and constraints without adding further insight or requiring any additional analytical developments for the application. The formulation of the constraints related to DAM requires the introduction of the following notation:

- I set of emission points for which the retailer is allowed to present offers;
- P set of consumption points for which the retailer is allowed to present bids;
- NV_{MGP} set of supply offers presented at the DAM;
- NA_{MGP} set of purchasing bids presented at the DAM;
- LT_{ij}^{MPG} energy transmission limit between zones i and j before the DAM (MW);
- P_z^{MGP} DAM clearing price in zone $z \in Z$ (€/MWh);
- PUN uniform purchasing price in the DAM auction calculated as the weighted average of zonal clearing prices (€/MWh);
- c_z contribution of zone z in forming the *PUN*;
- $QM_{i_z}^{MGP}$ maximum quantity of energy required in the consumption in the emission point i_z in zone z (MW);
- $QM_{p_z}^{MGP}$ maximum quantity of energy available for the DAM in the consumption point p_z in zone z (MW);
- Qm^{MGP} minimum quantity of energy that can be exchanged in the DAM (MW).

The decisions to be taken by the retailer are expressed by means of the variables:

- $QV_{h_{MGP}}^{i_z}$ quantity of energy to be specified in the supply offer h_{MGP} to the DAM referring to emission point i_z located in zone z (MWh);
- $PV_{h_{MGP}}^{i_z}$ selling price to be specified in the supply offer h_{MGP} to the DAM referring to emission point i_z located in zone z (€/MWh);
- $QA_{j_{MGP}}^{p_z}$ quantity of energy to be specified in the purchasing bid j_{MGP} to the DAM referring to consumption point p_z located in zone z (MWh);
- $PA_{j_{MGP}}^{p_z}$ purchasing price to be specified in the bid j_{MGP} to the DAM referring to consumption point p_z located in zone z (MWh);
- F_{ij}^{MGP} energy transmission flow between zones i and j resulting from the offers/bids presented to the DAM (MW).

The constraints can be written thus as:

$$\sum_{h_{MGP}} QV_{h_{MGP}}^{i_z} \le QM_{i_z}^{MGP}$$
$$\forall h_{MGP} \in NV_{MGP}, \forall i_z \in I, \forall z \in Z \tag{5}$$
$$\sum_{j_{MGP}} QA_{j_{MGP}}^{p_z} \le QM_{p_z}^{MGP}$$
$$\forall j_{MGP} \in NA_{MGP}, \forall p_z \in P, \forall z \in Z \tag{6}$$
$$Qm^{MGP} \le QV_{h_{MGP}}^{i_z}$$
$$\forall h_{MGP} \in NV_{MGP}, \forall i_z \in I, \forall z \in Z \tag{7}$$
$$Qm^{MGP} \le QA_{j_{MGP}}^{p_z}$$
$$\forall j_{MGP} \in NA_{MGP}, \forall p_z \in P, \forall z \in Z \tag{8}$$
$$P^{min} \le PV_{h_{MGP}}^{i_z} \le P_z^{MGP}$$
$$\forall h_{MGP} \in NV_{MGP}, \forall i_z \in I, \forall z \in Z \tag{9}$$
$$P^{max} \ge PA_{j_{MGP}}^{p_z} \ge PUN$$
$$\forall j_{MGP} \in NA_{MGP}, \forall p_z \in P, \forall z \in Z \tag{10}$$
$$F_{ij}^{MGP} \le LT_{ij}^{MGP}$$
$$\forall i, j \in Z, i \ne j \tag{11}$$

Constraints (5) and (6) ensure the respect of the maximum quantities available/required in the emission/consumption points. Constraints (7) and (8) impose a minimum energy quantity that can be exchanged in the DAM. Constraints (9) and (10) are the conditions on the offering/bidding price in order not to be only economically feasible but also successful in DAM. Specifically, a selling price referring to an emission point located in zone z will be economically feasible if it is higher than a minimum bidding price P^{min} to be chosen by the retailer and will be successful only if it is less than the clearing price of the same zone. Analogously, a purchasing price referring to a consumption point located in zone z will be feasible if it is less than a retailer's specified maximum price P^{max} and will be successful only if it is bigger than the uniform purchasing price. The last set of constraints (11) ensure the respect of the transmission limit between each pair of adjacent zones. The variables F_{ij}^{MGP} represent the total imbalance between the energy sold and purchased between each pair of zones i and j. Such variables are not thus independent since they can be expressed in terms of the quantities of energy exchanged in the DAM.

2.3. *Green Certificate Contracts*

Besides the weekly market organized by the GME, the green certificates (*CV* in Italian, i.e. "*Certificati Verdi*") can be exchanged through bilateral contracts giving another opportunity to the retailers to make profits. In order to model the constraints related to these contracts we introduce the following additional notation:

CBA^{CV}	set of green certificate sales contracts;
CBV^{CV}	set of green certificate purchase contracts;
OPV^{CV}	set of alternatives for a green certificate sales contract;
OPA^{CV}	set of alternatives for a green certificate purchase contract;
$PRV^{i}_{k_{CV}}$	selling price of alternative i of the green certificate sales contract k_{CV} (€/MWh);
$NOV^{i}_{k_{CV}}$	number of green certificates agreed in alternative i for the sales contract k_{CV};
$PRA^{i'}_{w_{CV}}$	purchasing price of alternative i' of the green certificate purchase contract w_{CV} (€/MWh);
$NOA^{i'}_{w_{CV}}$	number of certificates agreed in alternative i' for the purchase contract w_{CV}.

Analogously to the energy bilateral contracts, the decision variables here are:

$BINV^{i}_{k_{CV}}$	binary variable that takes value 1 if alternative i of the green certificate sales contract k_{CV} has been selected and 0 otherwise;
$BINA^{i'}_{w_{CV}}$	binary variable that takes value 1 if alternative i' of the green certificate purchase contract w_{CV} has been selected and 0 otherwise.

The conditions that the retailers should respect when trading green certificate contracts are the following:

$$\sum_{i \in OPV^{CV}} BINV^{i}_{k_{CV}} \leq 1 \quad \forall k_{CV} \in CBV_{CV} \tag{12}$$

$$\sum_{i' \in OPV^{CA}} BINA^{i'}_{w_{CV}} \leq 1 \quad \forall w_{CV} \in CBA_{CV} \tag{13}$$

$$\sum_{i', w_{CV}} NOA^{i'}_{w_{CV}} * BINA^{i'}_{w_{CV}} = \sum_{i, k_{CV}} NOV^{i}_{k_{CV}} * BINV^{i}_{k_{CV}} \tag{14}$$

Analogously to the bilateral contracts, constraints (12) and (13) ensure that at most one alternative is selected for each sales/purchase contract. Constraint (14) matches the number of sold and purchased green certificates exchanged.

Finally, the model should include the energy balance constraint:

$$\sum_{j_z, i'} QOA^{i'}_{j_z} * FA^{i'}_{j_z} + \sum_{p,j} QA^{p_z}_{j_{MGP}} = \sum_{h_z, i} QOV^{i}_{h_z} * FV^{i}_{h_z} + \sum_{i,h} QV^{i_z}_{h_{MGP}} + Past\_Contracts \tag{15}$$

that matches the total energy sold and that purchased by the retailer either through bilateral contracts or in the DAM. Such a balance should also take into account past contracts, expressed here simply by the term *Past_Contracts* that represents the difference between the energy sold and purchased through active contracts and that can be, thus, either positive or negative.

2.4. *Objective function*

The objective of the retailer is to maximize (over the whole time horizon) the profits defined as the difference between the revenues and the costs.

$$\begin{aligned}
\max \sum_{h_z, i} & [PRV^{i}_{h_z} * QOV^{i}_{h_z} * FV^{i}_{h_z} - \\
& PNV^{i}_{h_z} * QOV^{i}_{h_z} * (BINV^{i}_{h_x} - FV^{i}_{h_z})] - \\
& \sum_{j_z, i'} [PRA^{i'}_{j_z} * QOA^{i'}_{j_z} * FA^{i'}_{j_z} - \\
& PNA^{i'}_{j_z} * QOA^{i'}_{j_z} * (BINA^{i'}_{j_z} - FA^{i'}_{j_z})] + \\
& \sum_{z \in Z} [P^{MGP}_{z} * \sum_{i_z, h_{MGP}} QV^{i_z}_{h_{MGP}} - \\
& PUN * \sum_{p_z, j_{MGP}} QA^{p_z}_{j_{MGP}}] + \\
& \sum_{k_{CV}, i} \left[PRV^{i}_{k_{CV}} * NOV^{i}_{k_{CV}} * BINV^{i}_{k_{CV}}\right] - \\
& \sum_{w_{CV}, i'} \left[PRA^{i'}_{w_{CV}} * NOA^{i'}_{w_{CV}} * BINA^{i'}_{w_{CV}}\right]
\end{aligned}$$

The first summation refers to the revenues deriving from the sales of the energy bilateral contracts minus eventual penalty for any dissatisfied fraction. The second summation refers to the cost of buying energy through bilateral contracts to which we add eventual penalty for dissatisfied fraction of any contract. The third summation represents the profits that may result from the exchange of energy in the wholesales market. The last two summations represent the difference between the revenues deriving from the sales of the green certificates and the costs to be supported by the retailer for their purchases through bilateral contracts.

3. Probabilistic model

In the previous model, the zonal clearing prices and consequently the uniform purchasing price have been considered as known quantities. However, at the moment of presenting the offers/bids the retailer does not know the exact value of these prices and, thus, their probabilistic representation is necessary in order to have a more realistic formulation ([24]). For this reason, we assume that the zonal clearing prices can be represented as independent random variables having a normal distribution with mean value η_z and variance σ_z^2. Since the uniform purchasing price is a linear combination of the zonal prices, it will be also represented as a normal random distribution with mean and variance calculated as the linear combination of the mean values and the variances of the zonal prices, respectively.

The model we propose here is based on a chance constrained formulation: instead of imposing that the constraints (9)-(10) are satisfied for all the possible realizations of the random variables involved, we relax them by accepting that each of these constraints can be violated by a probability that does not exceed a chosen value α. Mathematically, such conditions can be expressed as:

$$\mathbf{P}[\, PA^{p_z}_{j_{MGP}} \geq PUN, \forall j_{MGP} \in NA_{MGP}, \forall p_z \in P, \forall z \in Z] \geq 1-\alpha \quad (16)$$

$$\mathbf{P}[\, PV^{i_z}_{h_{MGP}} \geq P^{MGP}_z, \forall h_{MGP} \in NV_{MGP}, \forall i_z \in I, \forall z \in Z] \geq 1-\alpha \quad (17)$$

This representation imposes a joint probability level p_{target} = (1-α) on the satisfaction of the price constraints over the whole set of constraints. The profit maximization model with constraints (16) and (17) is a probabilistic formulation and for its solution we need to develop its deterministic equivalent version. We focus here on constraint (16) but the extension of the results for constraint (17) is straightforward.

Let ξ denote the vector of all equal $n = |NA_{MGP}| * |P|$ components consisting in the independent random variables having a normal distribution. The mean and standard deviation vectors of the PUN are composed thus of n times the values μ and σ, respectively. Constraint (16) could be written as:

$$\mathbf{P}[A] \geq 1-\alpha \quad (18)$$

where A represents the event $A = \left\{PA^{p_z}_{j_{MGP}} \geq \xi\right\}$ and its complementary is $A^C = \left\{PA^{p_z}_{j_{MGP}} < \xi\right\}$.
By using the proprieties of the probability, we can write the stochastic constraint in the following equivalent way:

$$\mathbf{P}\left[A^C\right] = \mathbf{P}\left[PA^{p_z}_{j_{MGP}} < \xi\right] \leq \alpha$$

Now let indicate by F_ξ the cumulative distribution function (CDF) of the random vector ξ, then from the probability theory it is known that:

$$\mathbf{P}\left[PA^{p_z}_{j_{MGP}} \geq \xi\right] = \mathbf{F}_\xi\left(PA^{p_z}_{j_{MGP}}\right)$$
$$\Rightarrow \quad \mathbf{P}\left[PA^{p_z}_{j_{MGP}} < \xi\right] = 1 - \mathbf{F}_\xi\left(PA^{p_z}_{j_{MGP}}\right)$$

Thus, the probabilistic constraint can be expressed as:

$$1 - \mathbf{F}_\xi\left(PA^{p_z}_{j_{MGP}}\right) \leq \alpha$$

or equivalently, by using the proprieties of the CDF normal standard denoted by Φ_ξ, we get:

$$1 - \Phi_\xi\left(\frac{PA^{p_z}_{j_{MGP}} - \mu}{\sigma}\right) \leq \alpha \quad \forall j_{MGP} \in NA_{MGP}, \forall p_z \in P, \forall z \in Z$$

and, by reordering the terms, as:

$$\Phi_\xi\left(\frac{PA^{p_z}_{j_{MGP}} - \mu}{\sigma}\right) \geq 1-\alpha$$
$$\forall j_{MGP} \in NA_{MGP}, \forall p_z \in P, \forall z \in Z.$$

Now we are able to define the deterministic equivalent of the probabilistic constraint (16) that can be written as:

$$PA^{p_z}_{j_{MGP}} \geq \mu + R_{(1-\alpha)}\,\sigma = \mu + r\sigma \quad \forall j_{MGP} \in NA_{MGP}, \quad \forall p_z \in P, \forall z \in Z \quad (19)$$

in which $R_{(1-\alpha)} = r$ represents the value, called *r-value*, in correspondence of which the normal standard distribution Φ_ξ takes the probability value $(1-\alpha)$.

With this representation the profit maximization model, with constraints (19) substituting constraint (16), is a deterministic equivalent formulation depending on the r-value and that could be solved by conventional optimization methods in order to determine the optimal values of the variables $PA^{p_z}_{j_{MGP}}$. Such values will then be used to compute the probability $\mathbf{P}\left[PA^{p_z}_{j_{MGP}} \geq \xi\right]$, i.e. the CDF, in order to check whether the stochastic constraint (16) is satisfied with the desired level of probability. For this purpose we use a numerical method based on the multivariate integration technique of Genz to calculate the CDF value, that we denote by p, corresponding to the current solution ([13]). The solution will be considered acceptable only if the difference between the CDF value p and the desired probability p_{target} is below a small tolerance value ϵ, that is $|p - p_{target}| \leq \epsilon$. Otherwise, it will be necessary to determine a new value of the parameter r and solve again the deterministic equivalent problem with updated constraints (19). The procedure is repeated till the satisfaction of the target probability level.

We shall note that, by introducing the stochasticity into the model, the objective function will be slightly different from the one presented in the previous section. Specifically, the term:

$$\sum_z \left(P^{MGP}_z * \sum_{i_z, h_{MGP}} QV^{i_z}_{h_{MGP}} - PUN * \sum_{p_z, j_{MGP}} QA^{p_z}_{j_{MGP}} \right) \tag{20}$$

should be substituted by the term:

$$\sum_z [\eta_z * \sum_{i_z, h_{MGP}} QV^{i_z}_{h_{MGP}} - \mu * \sum_{p_z, j_{MGP}} QA^{p_z}_{j_{MGP}}] \tag{21}$$

where η_r indicates the mean value of the zonal price z and μ represents the mean value of the uniform purchasing price PUN.

To conclude this section we describe below the algorithm that allows us to update the multivariate r-value ([23]):

(1) Find the univariate r-value corresponding to the probability level $p_{target} = (1 - \alpha)$ by using the following approximation due to Abramowitz and Stegun ([1]):

$$r = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(1 - \alpha)$$

with $t = \sqrt{ln \frac{1}{(1-\alpha)^2}}$ where the error $|\epsilon(1 - \alpha)| < 4.5 \times 10^{-4}$. The value of the constants used are the following:

$$\begin{array}{ll} c_0 = 2.515517 & d_1 = 1.432788 \\ c_1 = 0.802853 & d_2 = 0.189269 \\ c_2 = 0.010328 & d_3 = 0.001308 \end{array}$$

Set $r = r_{target} = r_{lower}$.

(2) Set r_{upper} to a sufficiently big starting value.

(3) Solve the profit maximization problem and compute the corresponding CDF value. By using the approximation of step 1 find the univariate r-value $r = r_2$.

(4) Solve again the profit maximization problem using the r-value r_{lower} and compute the CDF value. Use the approximation of step 1 in order to compute the univariate r-value $r = r_1$.

(5) Compute the r-value r_{new} as follows:

$$r_{new} = r_{lower} + [(r_{target} - r_1)/(r_2 - r_1)](r_{upper} - r_{lower})$$

(6) Solve again the profit maximization problem using r_{new} and compute the corresponding CDF value, to be denoted by p, and then the univariate r-value $r = r_{temp}$ by using the approximation of step 1.

(7) If $|p - p_{target}| \leq \epsilon$ the algorithm terminates, otherwise go to step 8.

(8) If $r_{temp} \leq r_{target}$ then set $r_1 = r_{temp}$ and $r_{lower} = r_{new}$, otherwise set $r_2 = r_{temp}$ and $r_{lower} = r_{new}$.

(9) Go to step 5.

The first four steps are, indeed, the initialization part of the algorithm. The update of the r-value happens at steps 5 and 8 on the basis of a linear interpolation rule. The algorithm is stopped in step 7 as soon as the desired probability level is reached.

4. Solution method

For non trivial applications the deterministic equivalent model of the profit maximization problem is characterized by a high number of variables and constraints. The dimension of the problem increases, indeed, with the number of contracts, auctions, offers and bids, etc. Thus, solving the problem with exact solvers is generally incompatible with the timings of the market requirements. For this reason we developed a heuristic algorithm based on a variant of the simulated annealing (SA) method. The new heuristic, called hybrid simulated annealing method, consists in performing a better exploration of the neighborhood of the current solution. Before describing our method we present first the standard SA in order to fix the notation and to emphasize later the difference between the two variants.

4.1. *Simulated Annealing Algorithm*

Simulated annealing (SA) is a generic probabilistic meta-algorithm for the global optimization problem, namely locating a good approximation to the global optimum of a given function in a large search space. The name and inspiration come from *annealing* in metallurgy, a technique involving heating and controlled cooling of a material to increase the size of its crystals and reduce their defects. The heat causes the atoms to become unstuck from their initial positions (a local minimum of the internal energy) and wander randomly through states of higher energy; the slow cooling gives them more chances of finding configurations with lower internal energy than the initial one.

By analogy with this physical process, each step of the SA algorithm replaces the current solution by a random *nearby* solution, chosen with a probability that depends on the difference between the corresponding function values and on a global parameter T (called the *temperature*), that is gradually decreased during the process (after a fixed number of iterations) by a factor called *cooling ratio*. The dependency is such that the current solution changes almost randomly when T is large, but increasingly "downhill" as T goes to zero. The algorithm could be summarized as follows:

(1) Select a starting solution $current\_zeta$
(2) Define a search neighborhood of the starting solution
(3) Select a starting value of the temperature $T > 0$
(4) While the freezing temperature, the maximum number of iterations, and the maximum execution time are not reached:
   5. Generate randomly a new solution new_zeta in the neighborhood of $current\_zeta$
   6. Compute $\Delta\_zeta = new\_zeta - current\_zeta$
   7. If $\Delta\_zeta > 0$ then accept the new solution by setting $current\_zeta = new\_zeta$
   8. If $\Delta\_zeta \leq 0$ then accept the new solution with probability $\mathbf{P}(\Delta\_zeta) = e^{(\Delta\_zeta/T)}$
   9. Reduce the value of T by the cooling ratio
10. Return the best solution reached

This general algorithm can be applied to our profit maximization problem in the following way. The starting solution in step 1 can be determined by selecting arbitrarily one or more selling bilateral contract and one or more purchasing contract that ensure the feasibility of the solution. The definition of a search neighborhood in steps 2 and 6 consists in selecting randomly one alternative of a selling or a purchasing contract and change its status, i.e. set its corresponding decision variable to one if it was zero, and vice-versa.

Even though the SA technique has shown to be very efficient in the solution of the retailer's optimization problem, we have proposed and implemented a novel variant, called hybrid SA, that attempts, at each iteration, to explore in a better way the neighborhood of the current solution. The variant consists in generating randomly, in the neighborhood of $current\_zeta$, not just one solution but a sample of, say, k solutions that is sufficiently representative of the neighborhood. Among the generated solutions we select the one having the highest objective function that we denote by $best\_value$. On the other hand, we calculate the mean value m_{cur} and the standard deviation σ_{cur} of the k solutions and we assume that they follow a normal distribution. The difference between a function of these values $f(m_{cur}, \sigma_{cur})$ and the $best\_value$ will be then used in our implementation as measure of significativity (representativity) of the generated sample. If such a difference is higher than a given threshold then the sample is considered not sufficiently representative of the neighborhood and the generation of an additional number of k' solutions is necessary. The process is repeated till the satisfaction of the threshold on the difference $[f(m_{cur}, \sigma_{cur}) - best\_value]$. We set thus $new\_zeta = best\_value$ and the hybrid SA algorithm continues following the standard SA iterative scheme.

Hybrid Simulated Annealing Algorithm:

(1) Select a starting solution $current\_zeta$
(2) Define a search neighborhood of the starting solution
(3) Select a starting value of the temperature $T > 0$
(4) Set a *threshold* as measure of representativity of a sample in the neighborhood
(5) While the freezing temperature is not reached, the maximum number of iterations, and the maximum execution time are not reached:
   6. Generate randomly k new solution new_zeta in the neighborhood of $current\_zeta$
   7. Denote by $best\_value$ the solution having the highest objective function
   8. Compute the mean value m_{cur} and the standard deviation σ_{cur} of the k solutions
   9. Set $f(m_{cur}, \sigma_{cur}) = m_{cur} + 2 * \sigma_{cur}$
   10. If $f(m_{cur}, \sigma_{cur}) - best\_value > threshold$ then the solution having the highest objec-

tive function value is not representative of the neighborhood; back to step 6

11. If $f(m_{cur}, \sigma_{cur}) - best\_value < threshold$ then the solution having the highest objective function value is representative of the neighborhood; back to step 6
12. Set $new\_zeta = best\_value$
13. Compute $\Delta\_zeta = new\_zeta - current\_zeta$
14. If $\Delta\_zeta > 0$ then accept the new solution by setting $current\_zeta = new\_zeta$
15. If $\Delta\_zeta \leq 0$ then accept the new solution with probability $\mathbf{P}(\Delta\_zeta) = e^{(\Delta\_zeta/T)}$
16. Reduce the value of T by the cooling ratio
17. Return the best solution reached

The efficiency of the method clearly depends on the choice of the function $f(m_{cur}, \sigma_{cur})$ and that of the threshold value. Several functions $f(m_{cur}, \sigma_{cur})$ could be used such as for example $m_{cur} + 2 * \sigma_{cur}$, whereas the value to be assumed by the threshold depends on the cardinality of the set of solutions that could be generated randomly and, consequently, on the problem dimension. Such a value, that will be determined empirically as function of the number of the problem variables, will represent a trade-off between the necessity of generating a significant sample of each search domain and that of not spending a high amount of time in the solution of the generated problems.

5. Computational Experiments

Both the standard (SSA) and the hybrid SA (HSA) algorithms have been implemented in C language and by making use of ILOG CPlex© 8.1 for the solution of the MILP problems. The computation of the CDF values has been performed with the help of a Matlab function that implements the Genz's technique ([13]). The value of ϵ used for the determination of the r-value depends on the target probability level $(1-\alpha)$. For example, the value $(1-\alpha) = 0.8$ corresponds to $\epsilon = 0.005$, the value $(1-\alpha) = 0.999$ corresponds to $\epsilon = 0.0005$, etc. A starting temperature value of $T = 40.000$ has been defined empirically that then decreases every 100 iterations with a cooling rate of 0.8. The number of solutions to be generated in the search domain at each iteration of the HSA algorithm has been set to $k = 10$ and whenever the sample does not result to be significant an additional number of $k' = 5$ solutions will be generated. The threshold value measuring the sample significativity has been set to 5% for test problems with up to 20.000 variables, to 8% for up to 50.000 variable and to 10% for a higher number of decision variables. Finally, a penalty price corresponding to a further loss of 30% of the contract price has been used.

Table 1

Problem	Variables	Constraints	Binary Variables
P 1	2.480	1.209	1.300
P 2	5.300	2.629	1.050
P 3	1.780	859	350
P 4	3.020	1.489	650
P 5	10.200	5.269	3.000
P 6	7.400	3.679	2.100
P 7	10.600	5.319	700
P 8	7.800	3.729	2.100
P 9	9.400	4.719	2.400
P 10	10.200	5.099	2.800
P 11	11.000	5.539	3.200
P 12	11.800	5.979	3.600
P 13	12.600	6.419	4.000
P 14	13.400	6.859	4.400
P 15	14.200	7.299	4.800
P 16	14.680	7.359	4.800
P 17	15.000	7.399	4.800
P 18	16.200	8.079	5.200
P 19	17.400	8.759	5.600
P 20	18.900	9.609	6.100
P 21	19.700	9.709	6.100
P 22	21.700	10.809	6.100
P 23	23.000	11.209	6.100
P 24	24.400	11.859	7.600
P 25	25.900	12.709	8.100
P 26	26.700	12.809	8.100
P 27	28.600	13.709	8.100
P 28	30.500	14.609	9.100
P 29	38.100	18.109	11.100
P 30	45.700	21.609	13.100
P 31	53.300	25.109	15.100
P 32	60.900	28.609	17.100
P 33	68.500	32.109	19.100
P 34	76.100	35.609	21.100
P 35	83.700	39.109	23.100
P 36	93.700	46.109	24.000

Size of the test problems

In order to validate the profit maximization model and to test the performance of our solution method we have generated 36 test problems simulating the behavior of a retailer operating in the Italian electricity market by establishing bilateral and green certificate contracts and by participating in the DAM auction. The input data has been collected in such a way that the test problems could realistically represent the Italian applicative context. Indeed, most of these data have been collected from the information provided by Terna and GME referring to the first months of operation of the Italian market. The missing data has been collected from Nord Pool which is characterized by a similar jurisdiction with respect to the Italian market.

Table 1 reports the characteristics of our test problems on the basis of their size. For each test problem we report the total number of variables, the number of constraints, and the number of binary variables obtained as the sum of selling and purchasing contracts multiplied by the corresponding number of options. The number of selling/purchasing contracts considered varies from 35 to 1200 and the number of options goes from 5 to 10.

The experimental results reported in Table 2 show, from one side the validity of the profit maximization model and from the other side the superiority of the HSA method with respect to its standard counterpart. For all the test problems, indeed, the quality of the solution obtained by using our method is remarkably higher than that obtained with the SSA when both are executed for the same amount of time and for the same starting parameters.

Specifically, the improvement gap goes from a minimum value of 1,8% to a maximum of 54,7% with an average value of 20,48% over the 36 test problems when 20 minutes of execution time are allowed. This threshold time seems to be the most appropriate for the dynamics of the Italian electricity market. Nevertheless, greater execution times have been also considered and similar behavior has been observed in all our experiments and namely after 60 minutes when an average gap of 14,18 is reached over the 36 problems. This confirms that even though our method is forced to generate, at each iteration, a higher number of problems it is able to take advantage from discovering better the domain search in order to find better solutions. However, it is clear from the results that as the size of the problems increases the gap becomes less important and the time consumed in the solution of the generated problems penalizes, thus, the performance of the HSA method.

Another important result shown in Table 2 is related to the performance of SSA and HSA with respect to the exact solution obtained with Cplex after an unlimited amount of time. It is, indeed, worthwhile noting that, as the execution time increases from 20 to 60 minutes, the solution found by the algorithms becomes closer to the optimal solution because in this way the exploration of the solution space is more accurate. However, since 60 minutes are too long to be suitable with the market's dynamics (and the Cplex running time to get the exact solution as well) a feasible suggestion to improve the solution quality consists in focussing on the choice of the starting solution $current\_zeta$. One possibility to achieve this aim is to develop new techniques for the HSA based on the warm starting. This issue is left for further investigation.

6. Conclusions

The objective of this paper is to offer to the retailers operating in the Italian electricity market a decision support for their short and medium term activities. First, we analyzed the role of the retailers and their interaction with the other operators in the Italian context in order to define the profits opportunities behind establishing bilateral contracts and participating in the wholesales market. We proposed then a deterministic profit maximization model that ensures the contracts selection and the bidding strategy definition. Since the market clearing prices are not known before the bidding we model them as random variables and we provided a chance constrained formulation that allows the prices constraints to be violated by a probability that does not exceed a chosen value.

For the solution of the deterministic equivalent of the probabilistic formulation we developed a heuristic method that efficiently generates multiple solutions within the search domain of the standard simulated annealing technique. Experimental results on 36 realistic test problems have proved the validity of the profit maximization model and the superiority of the proposed method with respect to the standard simulated annealing algorithm. After 20 minutes of running time the improvement gap reaches a maximum value of 54,7% for small- and medium-scale problems but decreases for larger problems. The average improvement of our method with respect to the standard SA algorithm is 20,48% over the 36 generated problems. When 60 minutes of CPU time are allowed, the hybrid simulated annealing becomes able to find solutions that are quite close to the exact ones but computational times may be too long to be suitable with the market electricity timings. Many improvements for both the optimization model and the solution method are possible. The model can be enhanced by including the financial contracts that ensure an efficient management of the retailer's aversion towards the risk, whereas the performance of the HSA method can be improved by adopting a warm starting strategy.

Table 2

		20 min. of exec. time			60 min. of exec. time		
Problem	Exact	SSA	HSA	gap	SSA	HSA	gap
P 1	482.666,8	202.159,7	412.018,0	50,9	436.218,8	445.170,7	2,1
P 2	606.595,6	63.988,3	141.271,8	54,7	235.035,0	526.088,4	55,3
P 3	292.281,8	33.058,4	54.890,5	39,7	114.956,4	274.943,3	58,1
P 4	422.433,9	84.125,7	157.094,7	46,4	379.909,4	384.970,4	1,3
P 5	698.458,2	126.256,9	255.876,3	50,6	287.663,3	555.410,0	48,2
P 6	765.203,6	441.265,3	506.989,0	12,9	595.997,6	670.460,6	11,1
P 7	622.900,6	126.256,9	143.275,7	11,8	348.560,2	478.230,0	27,1
P 8	769.440,0	453.526,2	631.619,6	28,1	602.789,1	698.540,2	13,7
P 9	520.677,9	119.788,9	146.262,9	18,1	348.996,5	446.977,0	21,9
P 10	861.002,7	121.860,5	182.334,4	33,1	320.680,4	402.969,7	20,4
P 11	661.302,8	146.320,1	199.656,3	26,7	394.887,2	452.256,8	12,6
P 12	769.653,8	168.540,0	199.960,2	15,7	423.668,9	496.334,0	14,6
P 13	809.913,8	194.626,1	228.663,3	14,8	400.669,3	489.622,1	18,1
P 14	762.055,8	202.360,1	269.336,3	24,8	436.992,3	499.556,3	12,5
P 15	547.302,4	220.113,3	270.665,3	18,6	444.969,5	497.552,3	10,5
P 16	642.667,0	200.454,1	281.669,0	28,8	457.996,1	500.120,9	8,4
P 17	575.965,2	210.447,5	300.500,0	29,9	488.554,1	540.111,8	9,5
P 18	838.357,3	230.012,0	312.447,1	26,3	502.300,4	584.552,1	14,0
P 19	798.127,7	231.255,0	288.900,1	19,9	394.511,2	467.888,1	15,6
P 20	791.100,9	245.669,2	299.663,2	18,0	490.503,0	526.778,0	7,0
P 21	704.989,2	312.336,8	366.884,1	14,8	520.126,0	587.120,0	11,4
P 22	575.208,3	318.006,3	390.001,0	18,4	499.200,0	546.777,0	8,7
P 23	555.981,3	325.994,0	387.446,2	15,8	512.300,0	549.232,8	6,7
P 24	982.478,9	320.554,0	380.999,1	15,8	521.300,7	561.230,8	7,1
P 25	560.070,4	290.445,2	331.595,3	12,4	431.800,8	468.930,5	8,0
P 26	690.323,1	416.000,0	480.562,0	13,4	594.225,8	645.278,1	8,0
P 27	904.826,6	512.326,1	584.000,0	12,2	640.523,1	708.555,2	9,6
P 28	810.822,7	494.662,3	530.663,1	6,7	640.239,1	704.560,8	9,2
P 29	699.400.0	390.556,2	460.532,0	15,1	523.655,0	588.320,7	10,0
P 30	823.559,6	540.232,0	579.994,0	6,8	651.299,3	703.668,4	7,4
P 31	800.488,4	598.663,0	610.230,4	1,8	738.446,2	791.560,2	6,7
P 32	923.645,2	590.446,0	650.230,4	9,1	748.991,2	795.630,2	5,8
P 33	899.300,8	640.008,9	686.451,2	6,7	788.554,6	824.555,6	4,3
P 34	1.004.668,9	700.400,0	760.552,1	7,9	837.996,5	884.201,0	5,2
P 35	1.130.488,1	840.991,2	885.996,3	5,0	998.630,4	1.050.230,0	4,9
P 36	1.380.004,5	865.442,1	900.520,1	3,8	1.048.996,3	1.244.003,8	15,6

Hybrid vs. standard SA after 20 and 60 minutes of execution time

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